Wet dark fluid in spherical symmetric space-time admitting conformal motion

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ABSTRACT

In this paper, we have examined the wet dark fluid matter in the spherical symmetric space-time admitting one-parameter group of conformal motions. Also, we have discussed the properties of the solution obtained.

Keywords: Spherical symmetry, wet dark fluid and conformal motion.

INTRODUCTION


General relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by Einstein equations. Symmetries of geometrical/Physical relevant quantities of this theory are known as collineations and the most useful collineation is conformal killing vector defined by

\[ \mathcal{L}_\xi g_{ij} = \xi_{i;j} + \xi_{j;i} - \psi g_{ij}, \quad \psi = \psi(x^i), \]

where \( \mathcal{L}_\xi \) signifies the Lie derivative along \( \xi^i \) and \( \psi=\psi(x^i) \), is the conformal factor. In particular, \( \xi \) is a special conformal killing vector, if \( \psi_{,ij} = 0 \) and \( \psi_{,i} \neq 0 \). Here \((;)}\) and \((,)}\) denote covariant and ordinary derivatives respectively.

The paper is outlined as follows:
In Sec.2, we have obtained Einstein field equations for static spherically symmetric distribution of wet dark fluid admitting one-parameter group of conformal motions. In Sec.3, the solutions of the Einstein field equations are obtained for wet dark fluid. At the end, the properties of the solution obtained are discussed in concluding section.
MATERIALS AND METHODS

2. Field Equations

The most general static line element with spherical symmetry is given by

$$ds^2 = -e^{2\lambda}dt^2 - e^{2\nu}dr^2 - r^2(1 - e^{2\lambda}r^2)\sin^2\theta d\phi^2 + e^{2\nu}dt^2,$$ (1)

where $\lambda$ and $\nu$ are functions of $r$ alone and $x^{1,2,3,4} = r, \theta, \phi, t$.

Also, Einstein field equations can be expressed as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi T_{ij}$$ (2)

where $T_{ij}$ is energy momentum Tensor for wet dark fluid (WDF) and $\Lambda$ is the cosmological constant.

We have

$$T_{ij} = (p_{WDF} + \rho_{WDF})u_iu_j + p_{WDF} g_{ij}$$ (3)

together with

$$g_{ij}u^iu^j = 1,$$ (4)

where $u^i$ is the four-velocity vector of the fluid, $p_{WDF}$ and $\rho_{WDF}$ are the pressure and energy density of wet dark fluid respectively.

Here, we shall use geometrized units so that $8\pi G = c = 1$.

Then using line element (1), from (2) and (3), we get

$$e^{-\lambda}\left(\frac{\nu'^2}{2} + \frac{1}{r^2} + \Lambda\right) = \rho_{WDF},$$ (5)

$$e^{-\lambda}\left(\frac{\nu'^2}{2} + \frac{1}{r^2} + \Lambda\right) = p_{WDF}$$ (6)

and

$$\frac{e^{-\lambda}}{2}(\nu'^2 + 2 + \frac{\nu'^2 - \lambda' - \nu'\lambda'}{r} - \frac{\nu'\lambda'}{2}) + \Lambda = p_{WDF}$$ (7)

where primes denote differentiation w. r. t. $r$.

Now, we shall assume that space-time admits a one-parameter group of conformal motions (Aktas and Yilmaz [6]) i.e.

$$\mathcal{E}_\xi g_{ij} = \xi_{ij} + \xi_{,ij} = \psi g_{ij},$$ (8)

where $\mathcal{E}_\xi$ signifies the Lie derivative along $\xi$ and $\psi$ is an arbitrary function of $r$. In particular, $\xi$ is a special conformal killing vector, if $\psi_{,i} = 0$ and $\psi_{,i} \neq 0$. Here $(;)$ and $(,)$ denote covariate and ordinary derivatives respectively.

Conformal killing vectors provide a deeper insight into the space-time geometry and facilitate generation of exact solutions to the field equations.

Using (1) and (8) by virtue of spherical symmetry, we get the following expressions:

$$\xi^i u^j = \psi,$$ (9)

$$\xi^4 = c_1 = constant,$$ (10)

$$\xi^1 = \frac{\psi r}{2}$$ (11)

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where a comma denotes partial derivatives.

Using equations (9) to (12), we obtain

\[ e^y = c_2 r^2, \]

\[ e^x = \left( \frac{c_3}{\psi} \right)^2 \]

and

\[ \xi^t = c_2 \delta^t_1 + \left( \frac{\psi^t}{r} \right) \delta^t_1 \]

where \(c_2\) and \(c_3\) are the constants of integrations.

Substituting (13) and (14) into (5) - (7), we get

\[ \frac{1}{r^2} \left( 1 - \frac{\psi^2}{c_2^2} \right) - \frac{2 \psi \psi'}{c_2^2 r} - \Lambda = \rho_{WDF}, \]

\[ \frac{1}{r^2} \left( 1 - \frac{3 \psi^2}{c_2^2} \right) - \Lambda = -p_{WDF}, \]

and

\[ \frac{\psi^2}{c_2^2 r^2} + \frac{2 \psi \psi'}{c_2^2 r} + \Lambda = p_{WDF}. \]

RESULTS AND DISCUSSION

3. Solutions of Field Equations

From (17) and (18), we get

\[ 2 \psi \psi' - \frac{2}{r^2} \psi^2 = -\frac{c_2^2}{r^2} \]

Putting \(\psi^2 = z\), we get

\[ \frac{dz}{dr} - \frac{2}{r} z = -\frac{c_2^2}{r}. \]

This is the first order linear differential equation having solution

\[ z = \frac{c_2^2}{2} - kr^2, \]

where \(k (> 0)\) is the constant of integration.

Therefore the general solution of equation (19) is given by

\[ \psi^2 = \frac{c_2^2}{2} - kr^2. \]

Subtracting (16) and (17), we get

\[ \rho_{WDF} + p_{WDF} = \frac{2 \psi^2}{c_2^2 r^2} - \frac{2 \psi \psi'}{c_2^2 r} = \frac{1}{r^2}. \]
Subtracting (16) and (18), we get

\[
\rho_{WDF} - p_{WDF} = \frac{1}{c^2} \frac{2y^2}{c^2 r^2} - \frac{4\psi\psi'}{c^2 r} - 2\Lambda = \frac{6k}{c^2} - 2\Lambda, \tag{24}
\]

From (23) and (24), we obtain

\[
\rho_{WDF} = \frac{1}{2r^2} + \frac{3k}{c^2} - \Lambda \tag{25}
\]

and

\[
p_{WDF} = \frac{1}{2r^2} - \frac{3k}{c^2} + \Lambda. \tag{26}
\]

Using (13) and (14), the space-time geometry of wet dark fluid (i.e. the line element given by (1)) becomes

\[
ds^2 = -\frac{c^2}{y^2} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + c^2 r^2 dt^2. \tag{27}
\]

The equation of state for Wet Dark Fluid (WDF) is given by

\[
p_{WDF} = \gamma (\rho_{WDF} - \rho^*), \tag{28}
\]

where the parameters \( \gamma \) and \( \rho^* \) are taken to be positive and \( 0 \leq \gamma \leq 1 \), and it is a good approximation for many fluids including water, where the internal attraction of the molecules make negative pressure possible.

If we set \( \Lambda = \frac{3k}{c^2} \), then for \( \rho^* = 2\Lambda - \frac{6k}{c^2} \) and \( \gamma = 1 \), the equation of state for WDF (28) is satisfied.

Further

\[
p_{WDF} = \rho_{WDF}. \tag{29}
\]

WDF has two components:

One behaves as a cosmological constant and other as standard fluid with equation of state \( p = \gamma \rho \) (Mishra and Sahoo [5]).

Further, from (23)

\[
\rho_{WDF} + p_{WDF} = \frac{1}{r^2} \geq 0. \tag{30}
\]

This implies that WDF will not violate the strong energy condition.

**CONCLUSION**

From (25) and (26), it is clear that \( \rho_{WDF} \) and \( p_{WDF} \) are not regular at centre of sphere \((r = 0)\) and we do not have finite radius of the sphere for which \( p_{WDF} = 0 \).

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**REFERENCES**