

Unsteady hydromagnetic flow of an ionized gas between parallel porous plates with Hall currents

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ABSTRACT

The theoretical problem of an unsteady viscous flow of an ionized gas with constant properties in the presence of an applied magnetic and electric field through a horizontal channel bounded by two parallel porous plates, subject to an uniform suction, applied normal to the plates, by taking Hall currents in to account is investigated. The transpiration velocity distributions are assumed to vary periodically with time about a non-zero velocity. The closed-form solutions for the velocity distributions, such as primary and secondary velocity distributions, also the corresponding mean velocities are reported for small periodic frequency parameter when both plates are made up of non-conducting and conducting porous materials. These results are depicted graphically to elucidate the interesting features of the flow pattern in presence of Hall currents. We note that, when the motion is in steady state and for non-porous plates, these results coincide with those of Sato [20]. Also, it is noticed that, the velocity distributions thus obtained are found to be the independent of s (the ratio of electron pressure to the total pressure) in case of non-conducting porous plates and are depending on 's' for the case of conducting plates.

Keywords: MHD Unsteady flow, Ionization, Hall currents, porous boundaries.

INTRODUCTION

Magnetohydrodynamics is currently undergoing a period of great enlargement. It is well known that a number of astronomical bodies, namely, the sun, the planets, the magnetic stars, pulsars etc., possess fluid interior and at least surface magnetic field. Hence, any flow phenomenon occurring in a celestial body take place under the influence of an external magnetic field. The interest in these problems generates from their importance in liquid metals, electrolytes and ionized gases. The mechanism of conduction in ionized gases in the presence of a strong magnetic field is different from that in a metallic substance. The electric current in ionized gases is generally carried by electrons which undergo successive collisions with other charged or neutral particles. In the ionized gases, the current is not proportional to the applied potential except when the electric field is very weak. However, in the presence of strong electric field, the electrical conductivity is found to be affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced. Hence, the current is reduced in the direction normal to both electric and magnetic fields. This phenomenon is known as Hall Effect. Due to this Hall current, the electrical conductivity of the fluid becomes anisotropic and this causes a secondary flow in magnetohydrodynamic primary flows. Hall current is of great importance in many astrophysical problems, Hall accelerator and flight MHD as well as flows of plasma in a MHD power generator. The stationary and non-stationary unsteady flows of an electrically conducting viscous incompressible fluid flow through porous plates in presence of Hall currents have been well established with increasing interests both from the practical needs of the astronautics, plasma physics and aero-space engineering by many investigators, namely, Muhuri [14], Lajpat Rai[11], Pop[18,17], Datta and Jana [4], Mazumdar [13], Debnath et al [6], Rao and Krishna [19], Bharali and Borkakati [1], Tiwari and Kamal singh [25], Seth And Ghosh [21], Hossain and Rashid [8], Siva Prasad et al [23], Chand et al. [10] and Rajasekhar et al. [15] under varied conditions and of different geometrical situations. Even though, these studies through porous

plates were carried out in different ways, it seems that the studies concerning unsteady viscous incompressible flows of an ionized gas between two parallel porous plates have been appeared only a few in literature. As such studies are expected to be useful for the boundary layer control in the field of aerodynamics, space science and in nuclear fusion research etc. Also, these studies are useful to carry out experiments with ionized gases to produce power on a large scale in stationary plants with large magnetic fields, such as in MHD generators, Hall accelerators etc. So in this paper, an attempt has been made to study the effects of the behaviour of flow parameters in an unsteady hydromagnetic viscous flow of an ionized gas between two infinite parallel porous plates, taking Hall currents into account under the influence of a uniform transverse magnetic field, following the analysis of Sato [20], Raju and Rao [12], by including the porosity at the plates. Exact solutions have been obtained for both the primary and secondary velocity distributions, also their corresponding mean velocities in two cases, that is, when both plates are made up of non-conducting and conducting porous materials. The profiles of the distributions are plotted after obtaining the numerical values for different sets of values of the governing parameters involved and discussed in detail by analyzing parameters such as, Hartmann number M , Hall parameter m and suction parameter λ .

FORMULATION AND SOLUTION OF THE PROBLEM

Consider an unsteady viscous fully developed flow of an ionized gas with constant properties through a horizontal channel bounded by two parallel porous plates (infinite in extent along x -and z -directions) subject to the uniform suction v_0 , applied normal to the plates, that is along y -direction. The fluid is driven by a constant pressure gradient. In the coordinate system choosing the origin midway between the plates, the x -axis is taken in the direction of hydrodynamic pressure gradient in the plane parallel to the channel plates but not in the direction of flow. A parallel uniform magnetic field of strength B_0 is applied in the y -direction. It is assumed that the induced magnetic field is negligible in comparison with the applied field, under the assumption that the magnetic Reynolds number is small, such that $\bar{B} = (0, 0, B_0)$. The assumption is justified because the magnetic Reynold's number of a partially ionized fluid is very small. Since, the fluid is subjected to a constant section v_0 applied normal to both the plates and hence, if (u, v, w) are the velocity components in the fluid, then the equation of continuity $\nabla \cdot \bar{q} = 0$ gives $v = -v_0(v_0 > 0)$, where $\bar{q} = (u, v, w)$. The height of the channel is denoted by $2h$ and the width is assumed to be very large in comparison with the channel height $2h$. All physical quantities except pressure become functions of y and t only, as the walls are infinite in extent along x -and z -directions. Further, to simplify the theoretical analysis, the following assumptions as in Sato [20], Raju and Rao [12] are considered: (i) the density of gas is always constant, (ii) the ionization is in equilibrium, which is not affected by the applied magnetic and electric fields, (iii) the effect of space charge is neglected, (iv) the flow is fully developed and stationary, that is $\partial/\partial t = 0$ and $\partial/\partial x = 0$ except $\partial p/\partial x \neq 0$, (v) the magnetic Reynolds number is small (so that the externally applied magnetic field is undisturbed by the fluid), namely the induced magnetic field is small compared with the applied field. Therefore, components in the conductivity tensor are expressed in terms of B_0 and (vi) the flow is two-dimensional, namely $\partial/\partial z = 0$.

Using these assumptions, the governing equations of motion and current are formulated as follows for the two-dimensional problem of an unsteady MHD flow of neutral fully ionized gas in presence of Hall currents through two parallel porous plates.

The equations of motion and current for the unsteady flow of neutral fully-ionized gas valid under the above mentioned conditions and assumptions as in Spitzer [24] are simplified as

$$-\left[1 - s\left(1 - \frac{\sigma_1}{\sigma_0}\right)\right] \frac{\partial p}{\partial x} + \rho v \frac{\partial^2 u}{\partial y^2} - \rho v_0 \frac{\partial u}{\partial y} + B_0 [-\sigma_1(E_z + uB_0) + \sigma_2(E_x - wB_0)] = \rho \frac{\partial u}{\partial t}, \quad (1)$$

$$s \frac{\sigma_2}{\sigma_0} \frac{\partial p}{\partial x} + \rho v \frac{\partial^2 w}{\partial y^2} - \rho v_0 \frac{\partial w}{\partial y} + B_0 [\sigma_1(E_x - wB_0) + \sigma_2(E_z + uB_0)] = \rho \frac{\partial w}{\partial t}, \quad (2)$$

These equations are to be solved subject to the boundary conditions:

$$u(-h) = 0, \quad (3)$$

$$u(h) = \varepsilon \cos(\omega t), \varepsilon \ll 1, \quad (4)$$

$$w(-h) = 0, \tag{5}$$

$$w(h) = \varepsilon \sin(\omega t), \quad \varepsilon \ll 1. \tag{6}$$

In the above equations, u and w are x - and z - components of the velocity \bar{V} , known as the primary and secondary velocity distributions respectively. E_x, E_z , and J_x, J_z are the x - and z - components of electric field \bar{E} and current density \bar{J} respectively. Here, $s = p_e/p$ is the ratio of the electron pressure to the total pressure; the value of s is 1/2 for neutral fully-ionized plasma and approximately zero for a weakly-ionized gas. Also p is pressure, ρ the density, ν the kinematic viscosity, where ε (amplitude) is a small constant quantity such that $\varepsilon \ll 1$ and ω is frequency of oscillation.

$$\sigma_1 = \frac{\sigma_0}{1 + m^2}, \tag{7}$$

$$\sigma_2 = \frac{\sigma_0 m}{1 + m^2}, \tag{8}$$

and Hall parameter, $m = \frac{\omega_e}{\left[\frac{1}{\tau} + \frac{1}{\tau_e} \right]}$, (9)

where ω_e is the gyration frequency of electron; τ and τ_e are the mean collision time between electron and ion, electron and neutral particles respectively; σ_1, σ_2 are the modified conductivities parallel and normal to the direction of electric field. The above expression for m which is valid in the case of partially-ionized gas agrees with that of fully-ionized gas when τ_e approaches infinity.

Then the eqs.(1) and (2) are non-dimensionalized, using the characteristic length h and velocity $u_p = - \left(\frac{\partial p}{\partial x} \right) \left(\frac{h^2}{\rho \nu} \right)$, the notations u, w for u/u_p and w/u_p and y for y/h , t for $\frac{\nu}{h^2} t$, $\omega = \frac{\omega h^2}{\nu}$ (frequency), the

Hartmann number M , which is defined as $M^2 = \frac{B_0^2 h^2 \sigma_0}{\rho \nu}$ and λ (Porous parameter or Suction number) = $(h v_0)/\nu$. (10)

Using the above transformations, the following non-dimensional forms of equations are obtained as:

$$k_1 + \frac{d^2 u}{dy^2} - \lambda \frac{du}{dy} - \frac{M^2}{1 + m^2} (m_z + u) + \frac{m M^2}{1 + m^2} (m_x - w) = \frac{du}{dt}, \tag{11}$$

$$k_2 + \frac{d^2 w}{dy^2} - \lambda \frac{dw}{dy} + \frac{M^2}{1 + m^2} (m_x - w) + \frac{m M^2}{1 + m^2} (m_z + u) = \frac{dw}{dt}, \tag{12}$$

$$I_x = \left(\frac{1}{1 + m^2} \right) (m_x - w) + \left(\frac{m}{1 + m^2} \right) (m_z + u) - \frac{s}{H_a^2} \left(\frac{m}{1 + m^2} \right), \tag{13}$$

$$I_z = \left(\frac{1}{1 + m^2} \right) (m_z + u) - \left(\frac{m}{1 + m^2} \right) (m_x - w) + \frac{s}{H_a^2} \left(1 - \frac{m}{1 + m^2} \right), \tag{14}$$

where

$$k_1 = 1 - s \left(1 - \frac{1}{(1 + m^2)} \right), \quad k_2 = -s \left(\frac{m}{(1 + m^2)} \right), \quad m_x = E_x/(B_0 u_p), \quad m_z = E_z/(B_0 u_p).$$

The corresponding boundary conditions are:

$$u(-1) = 0, \tag{15}$$

$$u(1) = \varepsilon \cos(\omega t), \quad \text{where } \varepsilon \ll 1, \tag{16}$$

$$w(-1) = 0, \quad (17)$$

$$w(1) = \varepsilon \sin(\omega t), \quad \varepsilon \ll 1. \quad (18)$$

For simplicity, introducing the complex notations:

$$q = u + iw, \quad k = k_1 + ik_2, \quad E = m_x + im_z;$$

the eqs.(17) and (18) can be written in complex form as:

$$\frac{d^2 q}{dy^2} - \lambda \frac{dq}{dt} + \left\{ M^2 \left(\frac{im-1}{1+m^2} \right) \right\} q = -k - \left(\frac{m+i}{1+m^2} \right) M^2 E + \frac{dq}{dt}. \quad (19)$$

Further, I_x and I_z are defined in non-dimensional form by writing $I_x = J_x / (\sigma_0 B_0 u_P)$ and $I_z = J_z / (\sigma_0 B_0 u_P)$ respectively, and these are given in complex notation as

$$I = I_x + iI_z = \frac{(m+i)}{(1+m^2)} \left(q - iE - \frac{s}{M^2} \right) + \frac{is}{M^2}. \quad (20)$$

The non-dimensional electric field E is to be determined by boundary conditions at large x and z .

SOLUTIONS OF THE PROBLEM

In order to solve the eq.(19) subject to the boundary conditions eqs. (15) to (18) for obtaining the primary and secondary velocity distributions, we assume that their solutions are of the form:

$$q = q_0 + \varepsilon e^{i\omega t} q_1, \quad \text{where } q_0 = u_0 + iw_0 \quad \text{and} \quad q_1 = u_1 + iw_1, \quad (21)$$

as in the study of Helmy's [9]. Then separating the real and imaginary parts of the resulting solution, we obtain the solutions for u_0 , w_0 , u_1 and w_1 , where, $u_0(y)$ and $w_0(y)$ are the primary and secondary velocity distributions in basic steady state case; $u_1(y)$ and $w_1(y)$ are the corresponding time dependent components. The solutions for both primary and secondary velocity distributions are obtained in two cases of study, that is, when the plates are made up of (i) non-conducting and (ii) conducting porous materials and are given in the following sections **(21)** and **(21)** as in the analysis of Sato [20] and Raju and Rao [12].

Non-conducting porous plates:

When the side plates are kept at large distance in z -direction and are made up of the non-conducting porous materials, then the induced electric current does not go out of the channel but circulates in the fluid. Therefore, an additional condition for the current defined in non-dimensional form is obtained by

$$\int_0^1 I_z dy = 0. \quad (22)$$

If the insulation at large x is also assumed, another relation is obtained as

$$\int_0^1 I_x dy = 0. \quad (23)$$

The above two conditions of insulation at large distance are physically realized, for example, when the flow of ionized gas is surrounded by the cold non-conducting gas. Constants in the solutions for velocity distributions are determined by these two conditions. And it is seen that, the solutions for velocity and current distributions q and I , respectively are all independent of the partial pressure of electron gas 's' and are obtained as

$$q = q_0 + \varepsilon e^{i\omega t} q_1 = a_{13} \left(a_7 e^{a_5 y} - a_8 e^{a_6 y} + 1 \right) + \varepsilon e^{i\omega t} \left(b_3 e^{b_1 y} + b_4 e^{b_2 y} \right) \quad (24)$$

$$I = \left(\frac{m+i}{1+m^2} \right) \left(\frac{q}{q_m} - 1 \right), \quad (25)$$

the mean velocity for u and w in the complex notation is given by

$$\begin{aligned}
 q_m &= u_m + iw_m \\
 &= \int_0^1 q dy \\
 &= a_{11}a_{13} + \varepsilon e^{i\omega t} b_5,
 \end{aligned} \tag{26}$$

in which u_m and w_m are the mean velocities of the primary and secondary velocity distributions u and w respectively. Further, by separating the real and imaginary parts from the eqs. (24) and (26), we get the independent solutions for both primary and secondary velocity distributions, and their corresponding mean velocities respectively.

Conducting porous plates:

When the side plates are made up of conducting materials and are short-circuited by an external conductor, the induced electric current flows out of the channel. In this case no electric potential exists between the side plates. If we assume zero electric field also in the x - and z - directions, then we have $m_x = 0$, $m_z = 0$. Constants in the solution are determined by these two conditions and it is observed that the solutions for u , w , I_x and I_z are all depend on 's' and are given in complex notations as

$$q = a_9 e^{a_5 y} + a_{10} e^{a_6 y} + a_{12} + \varepsilon e^{i\omega t} (b_3 e^{b_1 y} + b_4 e^{b_2 y}), \tag{27}$$

$$I = \left(\frac{m+i}{1+m^2} \right) \left(q - \frac{s}{M^2} \right) + \frac{i s}{M^2}, \tag{28}$$

the mean velocity for u and w in the complex notation is given by

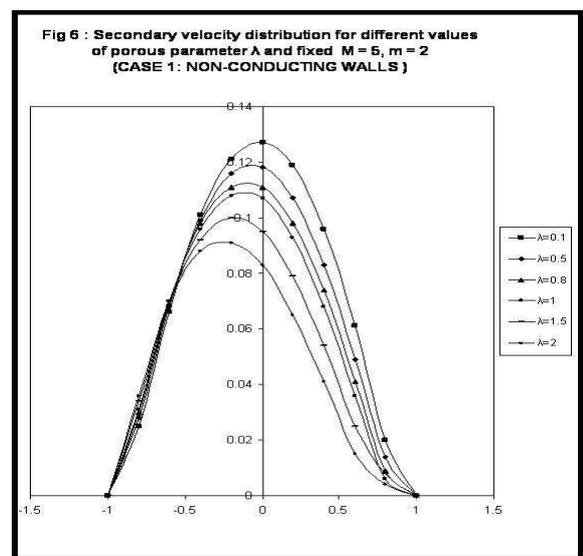
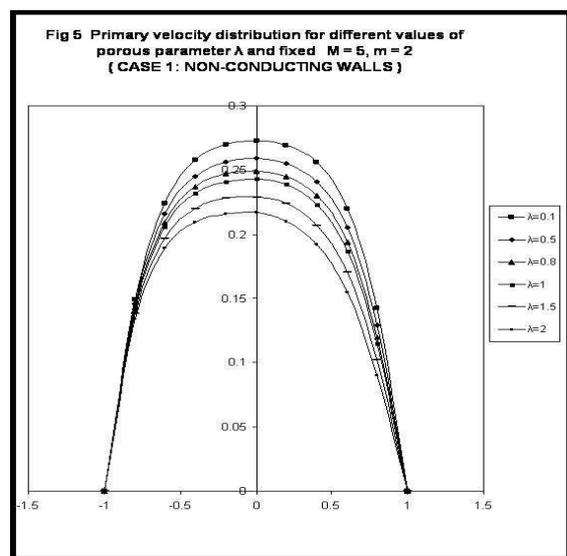
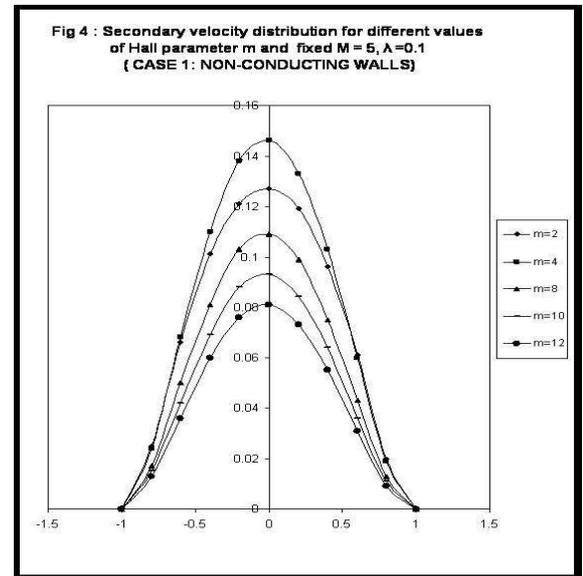
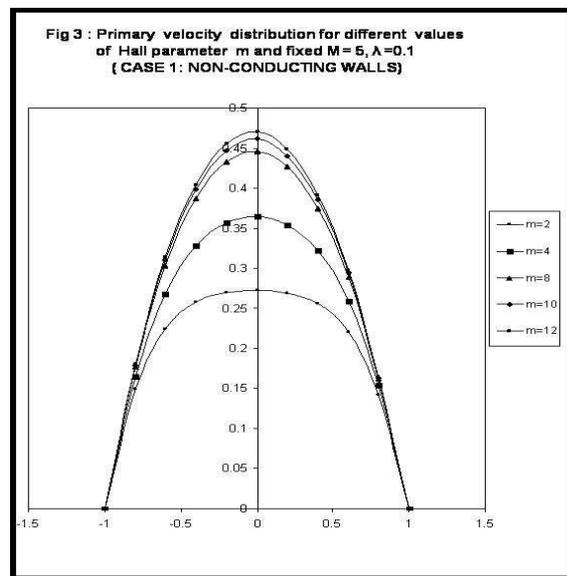
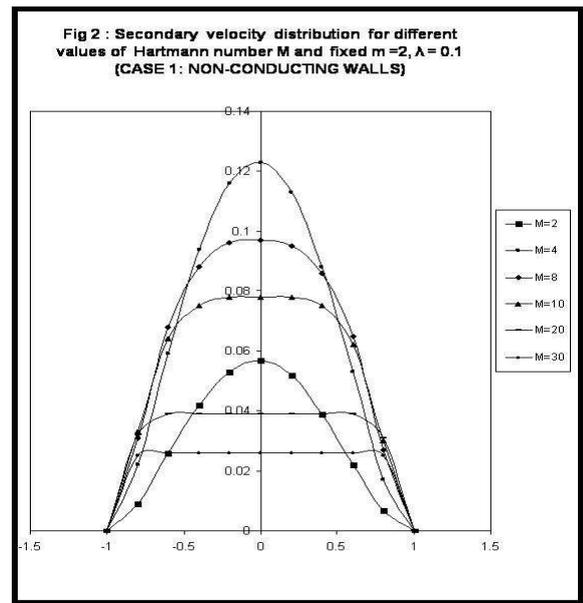
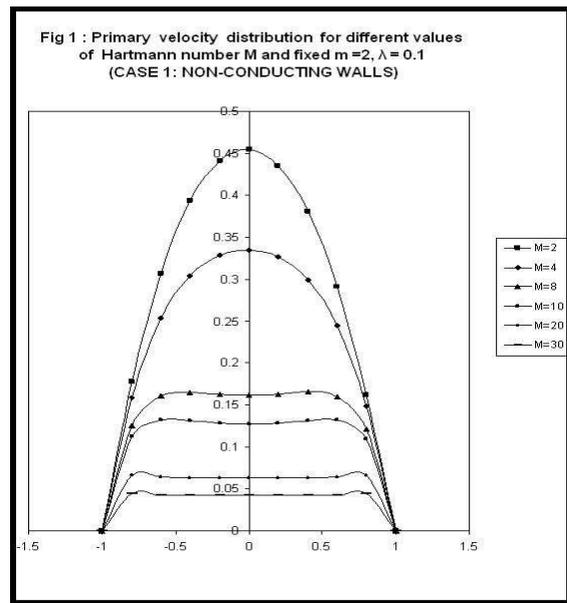
$$q_m = a_{11} a_{12} + \varepsilon e^{i\omega t} b_5. \tag{29}$$

Where the constants involved in the above solutions are given by

RESULTS AND DISCUSSION

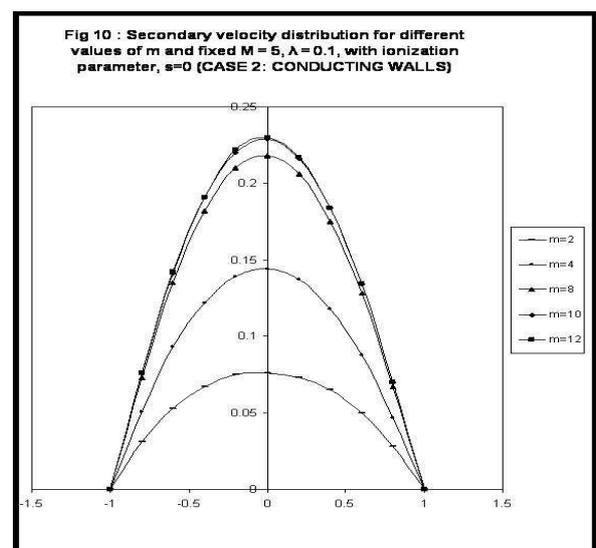
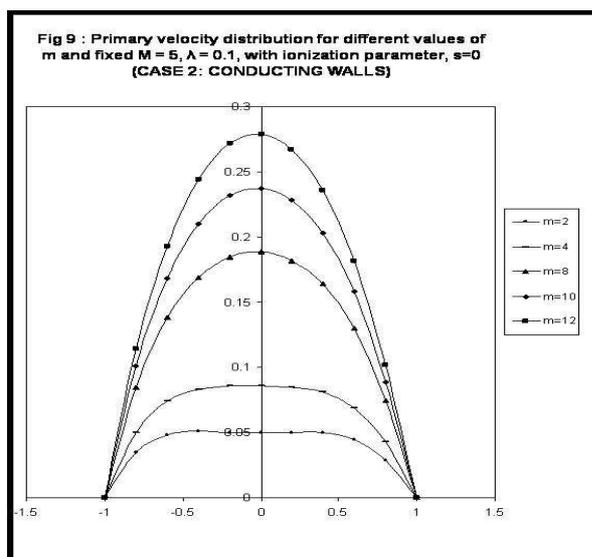
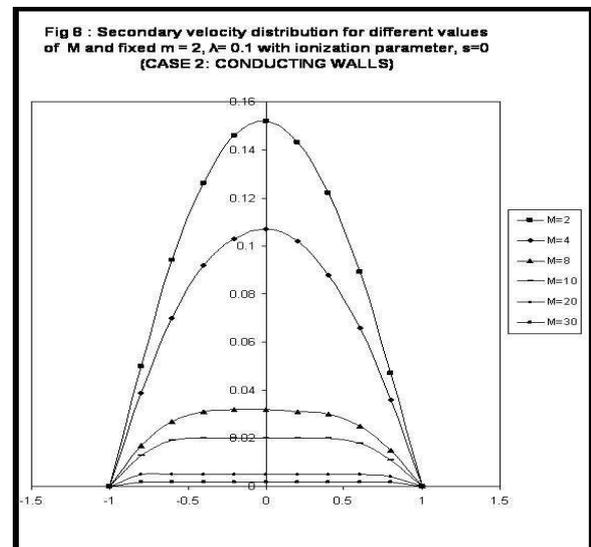
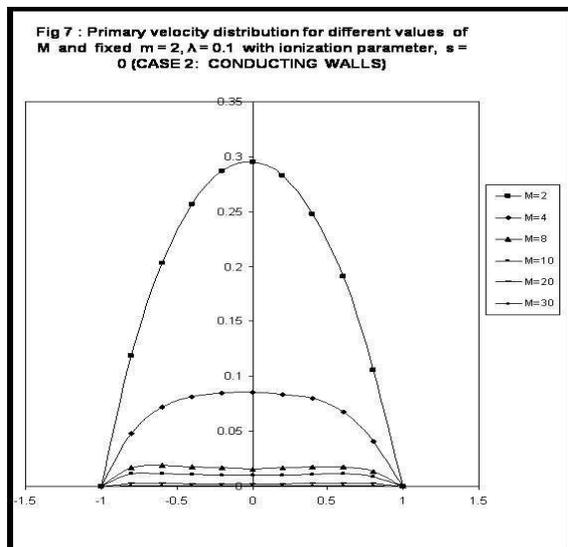
The closed-form solutions for the velocity distributions, such as primary and secondary velocity distributions, that is, u and w respectively, also the corresponding mean velocities are reported for small ε , the coefficient of exponent of periodic frequency parameter when both plates are made up of non-conducting and conducting porous materials. Computational values for various sets of values of the governing parameters involved are determined for primary and secondary velocity distributions, also the corresponding mean velocities. The results are depicted graphically in figures. 1– 24 for primary and secondary velocity distributions, also their mean velocities to elucidate the interesting features of the flow pattern in presence of Hall currents. We note that, when the motion is in steady state and $\lambda = 0$ (i.e., for non-porous plates), these results are coincide with those of Sato [20]. Also, an interesting point to be noted here that, the velocity distributions thus obtained are found to be the independent of s (the ratio of electron pressure to the total pressure) in case of non-conducting porous plates and are depending on 's' for the case of conducting plates.

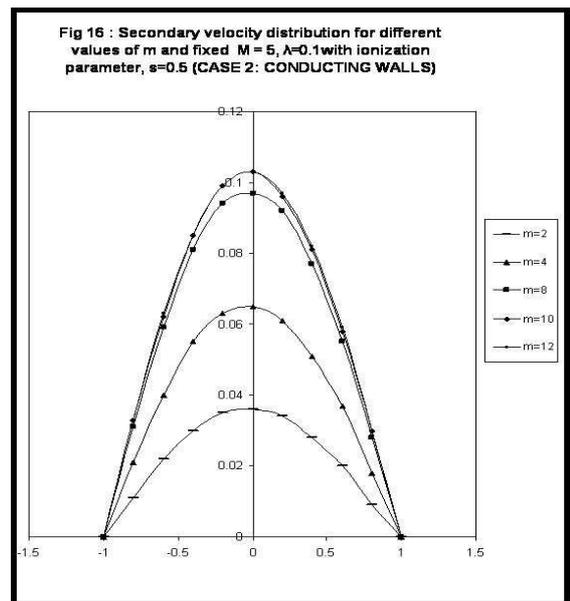
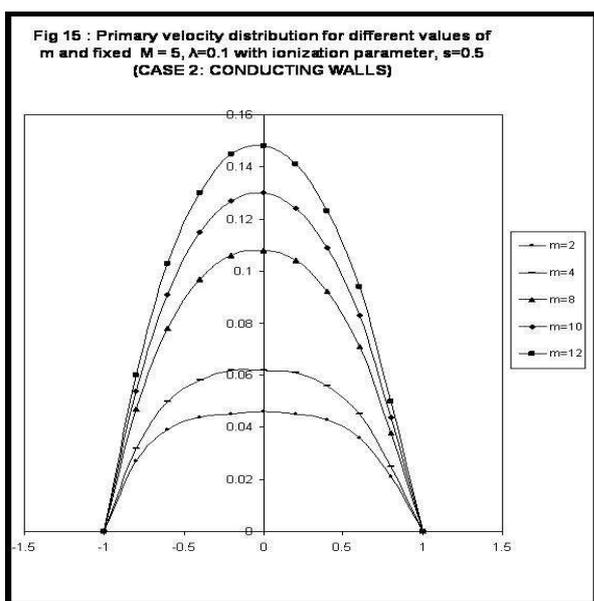
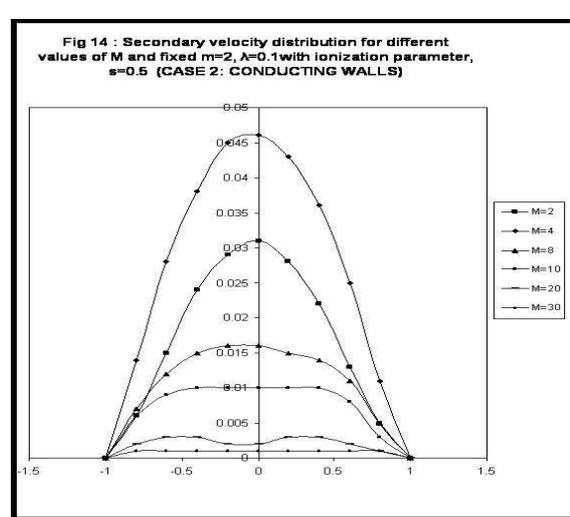
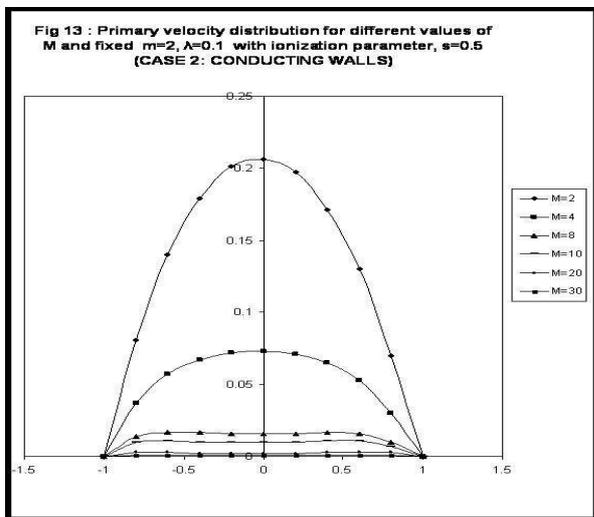
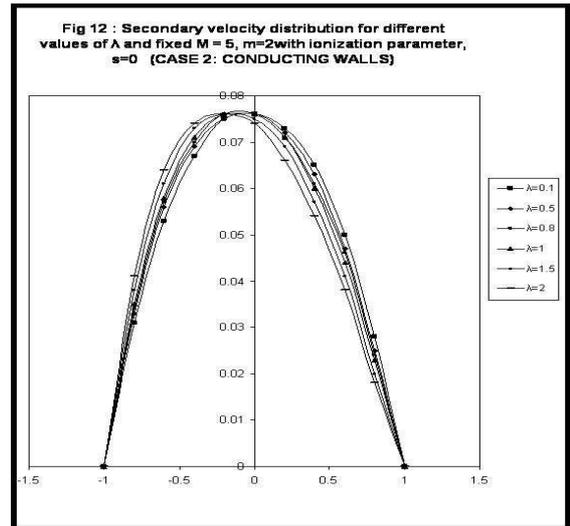
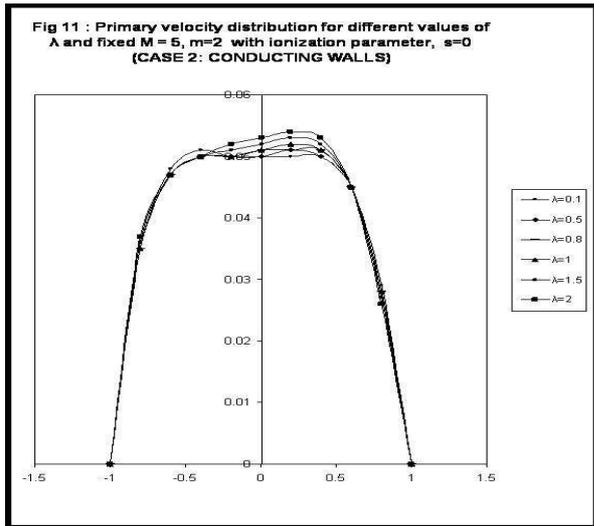
Figs. 1 to 6 show the velocity distributions, such as primary and secondary distributions for different values of the Hartmann number M , Hall parameter m and porous parameter λ , in the case when the porous plates are non-conducting. From Fig 1, it is noticed that the primary velocity distribution decreases as the Hartmann number M increases for fixed values of Hall parameter m and porous parameter λ . From fig 2, it is observed that the secondary velocity distribution increases as Hartmann number M increases and then decreases the same when $M > 4$ near the center of the channel, but this distribution increases at nearer to both the plates as M increases. From fig 3, it is noticed that, an increase in Hall parameter enhances the primary velocity distribution for fixed values of M and λ . But from fig 4, it is found that an increase in Hall parameter increases the secondary velocity distribution and then decreases the same when $m > 4$ for fixed values of M and λ . The effect of porous parameter λ , on the primary and secondary velocity distributions is depicted in figures 5 and 6 respectively. From figure 5, it is seen that the primary velocity distribution decreases as the porous parameter λ increases for fixed values of M and m . From fig.6, it is observed that the secondary velocity distribution decreases at the center of the channel as λ increases. But the same is increasing at the lower plate where as it decreases at the upper plate.

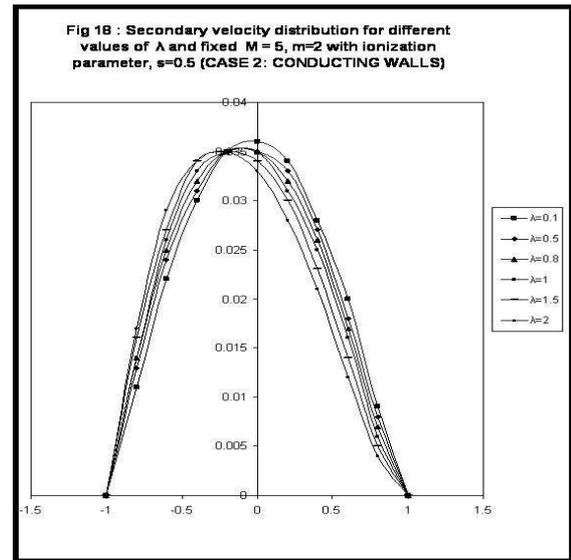
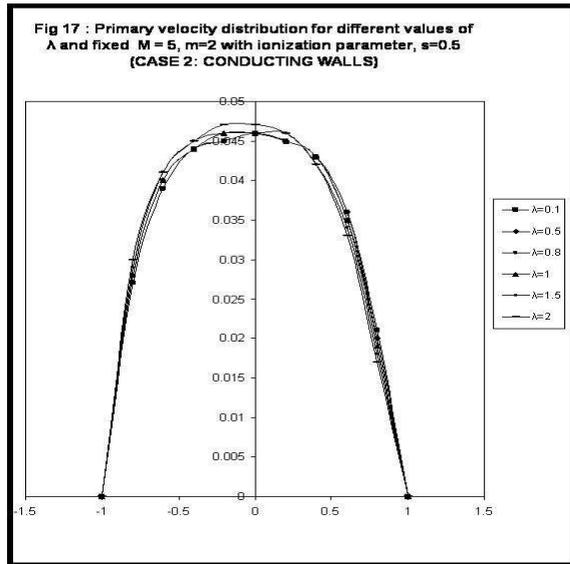


Figs. 7 to 18 show the velocity distributions, such as primary and secondary distributions for different values of the Hartmann number M , Hall parameter m and porous parameter λ , in the case when the porous plates are conducting and for ionization parameter $s = 0$ and 0.5

From figs 7 and 8, it is noticed that both the primary and secondary velocity distributions decrease as the Hartmann number M increases for fixed values of m and λ . From figs 9 and 10, it is seen that both primary and secondary velocity distributions increase as Hall parameter m increases for fixed M and λ . From figs 11, it is observed that primary velocity distributions decrease as the Porous parameter λ increases for fixed M and m . From fig 12, it is noticed that the secondary velocity distribution increases at near to the lower wall and it decreases that at the upper wall as λ increases for fixed values of M and m . From figs 13, it is seen that primary velocity distributions decreases as Hartmann number M increases for fixed m and λ . From fig 14, it is seen that the secondary velocity distribution increases as Hartmann number M increases up to $M = 2$ to 4 there after the distribution decreases for fixed m and λ . From fig 15, it is noticed that, the primary velocity distribution decreases as Hall parameter m increases for fixed values of Hartmann number M and porous parameter λ . From fig 16, it is noticed that, the secondary velocity distribution increases as Hall parameter m increases for fixed M and λ . From fig 17, it is noticed that the primary velocity distribution increases at near to the lower wall and it decreases that at the upper wall as λ increases for fixed values of M and m . From fig 18, it is noticed that the secondary velocity distribution increases at near to the lower wall and it decreases that at the upper wall as λ increases for fixed values of M and m .



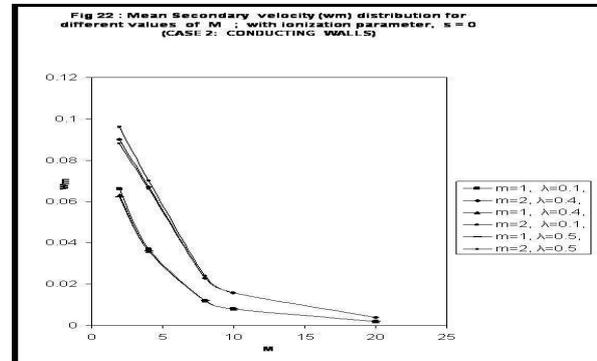
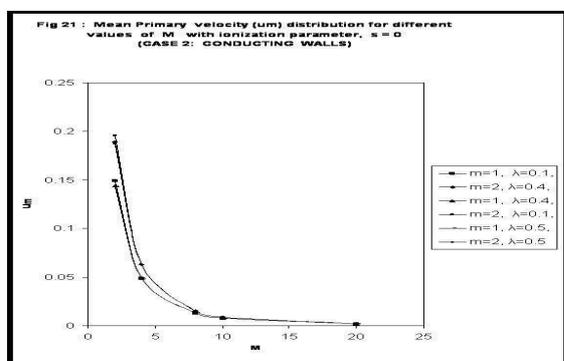
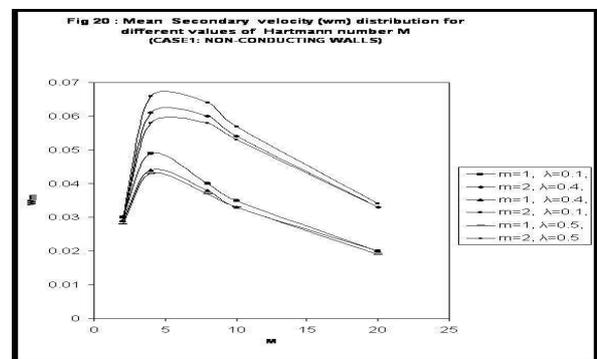
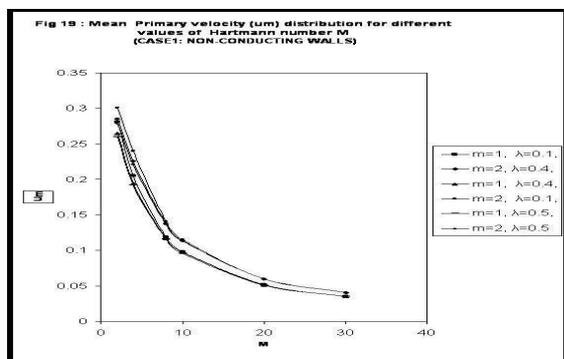


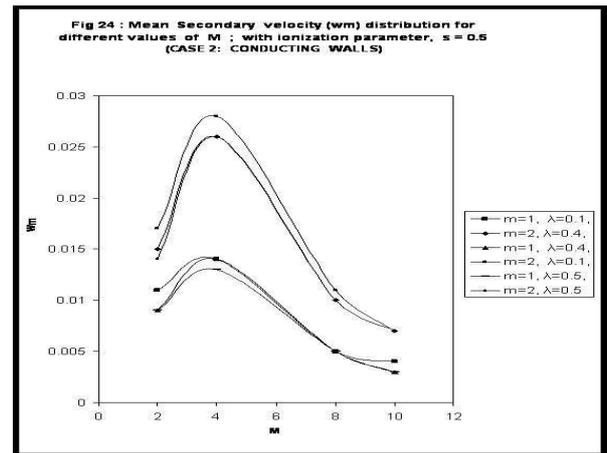
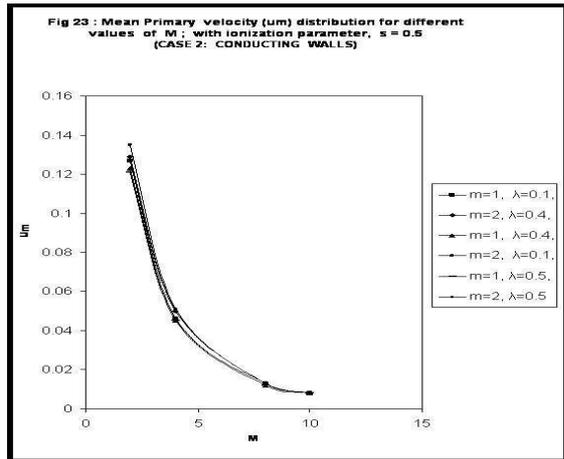


The mean velocities u_m and w_m in case of non-conducting plates are plotted against the Hartmann number M in figures 19 and 20 respectively for set of fixed values of Hall parameter m , and porous parameter λ . It is seen that, as M increases, u_m and w_m decrease.

The mean velocities u_m and w_m in case of conducting plates and when the ionization parameter $s = 0$ are plotted against the Hartmann number M in figures 21 and 22 respectively for set of fixed values of Hall parameter m , and porous parameter λ . And it is seen that as M increases, u_m and w_m decrease.

The mean velocities u_m and w_m in case of conducting plates and when the ionization parameter $s = 0.5$ are plotted against the Hartmann number M in figures 23 and 24 respectively for set of fixed values of Hall parameter m , and porous parameter λ . From these figures, it is observed that as M increases, u_m and w_m decrease.





APPENDICES

$$a_1 = \frac{-M^2}{1+m^2}, \quad a_2 = \frac{mM^2}{1+m^2}, \quad a_3 = a_1 + i a_2, \quad a_4 = \left(\frac{i+m}{1+m^2} \right) s^{m-1}, \quad a_5 = \frac{-\lambda - \sqrt{\lambda^2 - 4a_3}}{2},$$

$$a_6 = \frac{-\lambda + \sqrt{\lambda^2 - 4a_3}}{2}, \quad a_7 = \frac{\sinh(a_6)}{\sinh(a_5 - a_6)}, \quad a_8 = \frac{\sinh(a_5)}{\sinh(a_5 - a_6)}, \quad a_9 = a_7 a_{12}, \quad a_{10} = -a_8 a_{12},$$

$$a_{11} = \frac{a_7 e^{a_5} - a_8 e^{a_6}}{a_5 - a_6} + \frac{a_7}{a_5} + \frac{a_8}{a_6} + 1, \quad a_{12} = \frac{a_4}{a_3}, \quad a_{13} = \frac{-(im+1)}{a_{11}M^2 + (1+im)a_3}, \quad b_1 = \frac{-\lambda + \sqrt{\lambda^2 - 4(a_3 - i\omega)}}{2},$$

$$b_2 = \frac{-\lambda - \sqrt{\lambda^2 - 4(a_3 - i\omega)}}{2}, \quad b_3 = \frac{-e^{-b_2}}{2 \sinh(b_2 - b_1)}, \quad b_4 = \frac{e^{-b_1}}{2 \sinh(b_2 - b_1)}, \quad b_5 = \frac{b_3(e^{b_1} - 1)}{b_1} + \frac{b_4(e^{b_2} - 1)}{b_2}.$$

CONCLUSION

An unsteady hydro-magnetic flow of an ionized gas between two parallel porous plates, in presence of Hall currents, when one of the plates is set into uniform accelerated motion is considered. It is observed that, an increase in M decreases the primary velocity distribution. While, the secondary velocity distribution increases as Hartmann number increases and then decreases the same near the center of the channel when this number is greater than 4, but this distribution increases at nearer to both the plates as M increases. The primary velocity distribution decreases as the porous parameter λ increases. The secondary velocity distribution decreases at the center of the channel as λ increases, but the same is increasing at the lower plate, where as it decreases at the upper plate. An increase in Hall parameter enhances the primary velocity distribution, but the secondary velocity distribution first increases and then decreases the same when m > 4.

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