

Overflow and Idle Probabilities of Voice Multiplexing System

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Abstract

This letter proposes a voice multiplexing system model based on the framework of Stochastic Fluid Model (SFM) with finite capacity buffer. Using the sample path truncation method, we first obtain a truncated stochastic process of the system model, and then the transition rate matrix is derived based on the probability matrix of First Passage Time (FPT). The exact expressions of the overflow and the idle probabilities of the packet voice multiplexing system are derived by the Matrix-Analytic Methodology. Finally, numerical example is performed to illustrate our theoretical results. As an application, given the maximum tolerable overflow probability, the two optimal system parameter settings are given.

Keywords: Voice multiplexing system; Overflow probability; Idle probability; Stochastic Fluid Model (SFM)

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Introduction

With the development of the Internet, real-time voice transmission, such as Tencent meetings and DingTalk, has become an important part of people's real life. In such a packet voice switching system based on Asynchronous Transfer Mode (ATM), the packet voice signals are packaged into fixed-size voice packets and transmitted using a store-and-forward method. Telephone technology usually requires a bandwidth of more than 64 k bit/s, while the bandwidth required for the packet voice is usually less than 10 k bit/s, so packet voice technology is widely used in data communication systems and computer network systems. In the communication system, multiplexing technology combines multiple signals on a physical channel for transmission, which greatly improves the utilization of communication lines, and also saves the costs of the cable installation and maintenance.

Multiple voice sources share a link through the multiplexer in the form of statistical multiplexing. The voice source generates voice data stream and enters a multiplexing buffer, it is necessary for the packet signal to wait in the buffer for the processing of the system, thus the configuration of the system affects the quality of service (QoS) of the system, such as the call delay, speech intelligibility and so on.

The capacity of the multiplexing buffer is one of the important parts of the voice multiplexing system design [1]. Due to the randomness of the communication, the burst-scale queue phenomenon is always occurred in the voice multiplexing system [2], and this phenomenon is mainly related to the multiplexing

buffer of the system. More specifically, if the buffer capacity is too large, it will affect the transmission delay of the voice stream and increase the cost of the system; if the buffer capacity is too small, the voice stream overflow possibility will increase, which will result in the problems such as the loss of voice information, the poor voice quality and so on. Obviously, the size of buffer makes an important impact on the QoS of voice transmission.

Starting from the pioneering work [3] of Anick, Mitra and Sondhi in 1982, the Stochastic Fluid Model (SFM) has been widely used in various research fields. The stochastic fluid model can be defined as an input and output system, that is, SFM is modeled as a continuous fluid change process that enters and leaves the storage device (i.e. buffer) at a random changing rate. Anick et al. gave a stationary analytical solution of the SFM system [3]. In 2008, the authors of [4] analyzed the voice multiplexing system based on the Markov Modulated Poisson Process (MMPP) model, and estimated the broadband demand on the multiplexing node. In 2010, [5] proposed two independent finite state birth-death processes to study the occupancy distribution of the infinite capacity buffer, and gave a steady-state occupancy distribution of the buffer. In 2013, [6] calculated the steady-state probability when the buffer is empty based on the M/PH/1 fluid queue of the infinite buffer. In 2015, [7] combined the traditional fluid model and the fluid model based on Markov-renewal method to study the voice traffic queue phenomenon in the packet voice multiplexing system, and proposed a new method to calculation the mean buffer Occupancy. The literature [5-8] studied various

performances of packet voice multiplexing systems by SFM modeling.

The main solution of the SFM system in the above literature is based on the spectral decomposition method [3-5]. This method gives a stationary analytical solution of the SFM system in an exponential form of the eigenvalue. However, the numerical calculation of this method is unstable when the eigenvalue is close to zero. Moreover, in practical applications, there may be a large number of system states in the system model, which will lead to various calculation difficulties and numerical algorithm problems [9]. On the other hand, in order to derive the performance metrics (such as overflow probability, etc.) of the system, this system is often approximated by the system model with the infinite capacity [6,7]. Obviously, this assumption is not unrealistic in practical problems, and this system model limits the accuracy of the performance in the practical application scenarios [8].

Motivated by these facts, in this paper, we analyze the packet voice multiplexing system based on the theoretical framework of the SFM. Compared to the literature mentioned above, and the main contributions of this paper are as follows:

- This paper proposes finite capacity SFM to describe the packet voice multiplexing system, That is, instead of infinite buffer, the multiplexing buffer in the packet voice multiplexing system is directly set to a finite capacity, when the voice traffic entering the buffer is larger than the capacity of the buffer, voice stream overflow will be occurred. Obviously, the finite capacity buffer is more suitable for practical application scenarios, and the obtained analysis performance is more accurate.
- To analyze the packet voice multiplexing system under the finite capacity SFM framework, we use the method introduced in [10] in this paper. Firstly, the system path is truncated, and then based on which, Matrix-Analytic Methodology is used to derive the exact expressions of the overflow and idle probabilities. This method has stronger stability and faster convergence speed than spectral decomposition method. Based on the above performance metrics, we investigate the characteristics of overflow and idle probabilities of the buffer, we also discusses the parameter setting in packet voice multiplexing system, that is, the size of the buffer and the number of users that can be accommodated are analyzed numerically.

Model and Problem Formulation

Throughout this paper, we use the following notations. For a matrix \mathbf{A} , denote its (i,j) -th element by A_{ij} and let \mathbf{A}_{ij} be the block matrix of \mathbf{A} . Assume that $\mathbf{0}$, \mathbf{I} and $\mathbf{1}$ represent zero matrix, unit matrix and unit vector with an appropriate dimension, respectively.

Subsection

Similar to literature [4,9] we consider a packet voice multiplexing system as shown in Figure 1. There are N mutually independent voice sources, which are indexed by $\{v_1, v_2, \dots, v_N\}$. Each voice source has two states: active (that is, voice call phase) and silent (that is, voice pause), and each voice source alternates between

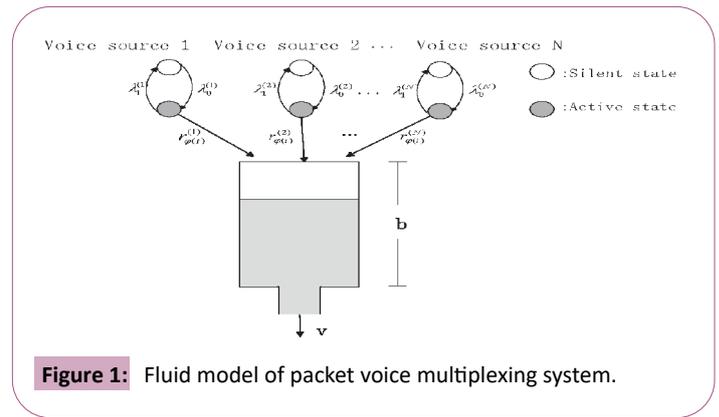


Figure 1: Fluid model of packet voice multiplexing system.

the two states. The voice stream generated by each voice source in the active period enters the data buffer at a constant rate. We assume that the capacity, which is denoted by b , of the buffer in the system is finite. When the voice stream in the buffer reaches the upper bound b , the newly entered voice stream will overflow and lead to the loss of voice information; when the buffer is empty and no new voice stream enters, then the system is idle.

Consider the voice source $v_k, k=1,2,\dots,N$, denote its state space by $S^{(k)}=\{0,1\}$, where 0 represents the silent state and 1 represents the active state. Suppose that the sojourn time of voice source v_k in state $i, i=0,1$, is the exponential distributed with mean $1/\lambda_i^{(k)}$. Thus, the evolution process of the voice source v_k , which is denoted by $\varphi^{(k)}=\{\varphi^{(k)}(t), t \geq 0\}$, becomes a continuous time Markov chain (CTMC) on state space $S^{(k)}$ with transition rate matrix:

$$\mathbf{M}^{(k)} = \begin{bmatrix} -\lambda_0^{(k)} & \lambda_0^{(k)} \\ \lambda_1^{(k)} & -\lambda_1^{(k)} \end{bmatrix}, k=1,2,\dots,N.$$

The voice stream generated by v_k is injected into the buffer with time varying input rate. Specifically, at time t , the voice stream input rate is denoted by $r_{\varphi^{(k)}}^{(k)}$. When the state of the voice source v_k is 0, there is no voice stream enters multiplexing buffer, and the input rate is $r_0^{(k)}=0$. When the state of v_k is 1, then v_k input the voice stream into the buffer at rate $r_1^{(k)} > 0$. We denote $\mathbf{R}^{(k)} = \text{diag}[r_0^{(k)}, r_1^{(k)}]$, and $\mathbf{R}^{(k)}$ is referred to as input rate matrix of v_k .

The voice stream in the multiplexing buffer is transmitted to the network system at a constant rate, which is denoted by ν , then the net input rate of v_k is $r_{\varphi^{(k)}}^{(k)} - \nu \mathbf{I}$. We let $\mathbf{D}^{(k)} = \mathbf{R}^{(k)} - \nu \mathbf{I}$, and we call $\mathbf{D}^{(k)}$ the net input matrix of $v_k, k=1,2,\dots,N$.

Now, define superposition process:

$$\hat{\varphi} = \{\hat{\varphi}(t), t \geq 0\},$$

Where $\hat{\varphi}(t) = [\varphi^{(1)}(t), \varphi^{(2)}(t), \dots, \varphi^{(N)}(t)]$, and refer to $\hat{\varphi}$ as the background process. In the packet voice multiplexing system, since N voice sources are independent of each other, then φ is CTMC on state space:

$$\mathbf{M} = \mathbf{M}^{(1)} \oplus \mathbf{M}^{(2)} \oplus \dots \oplus \mathbf{M}^{(N)},$$

With the transition rate matrix:

$$\mathbf{M} = \mathbf{M}^{(1)} \oplus \mathbf{M}^{(2)} \oplus \dots \oplus \mathbf{M}^{(N)},$$

where \oplus is Kronecker sum [11].

For the convenience of presentation, the states in S are numbered

in sequence, and the state space is rewritten as $S = \{1, 2, \dots, 2^N\}$. Obviously, $\hat{\varphi}$ characterizes the evolution process of all voice sources in the system. Similarly, at time t , the input rate and the net input rate of $\hat{\varphi}$ are denoted by $r_{\hat{\varphi}(t)}$ and $d_{\hat{\varphi}(t)}$, respectively. Then $r_{\hat{\varphi}(t)} = \sum_{k=1}^N r_{\hat{\varphi}(t)}^{(k)}$ and $d_{\hat{\varphi}(t)} = r_{\hat{\varphi}(t)} - \nu$. Thus, we can write the input rate and the net input rate matrices of $\hat{\varphi}$ is:

$$\mathbf{R} = \mathbf{R}^{(1)} \oplus \mathbf{R}^{(2)} \oplus \dots \oplus \mathbf{R}^{(N)}$$

net input rate matrix is:

$$\mathbf{D} = \mathbf{R} - \nu \mathbf{I}.$$

Let $\hat{X}(t)$ represent the amount (level) of voice stream in the multiplexing buffer at time t , we denote $\hat{X} = \{\hat{X}(t), t \geq 0\}$ by the evolution process of the voice stream in the buffer, then $\hat{X}(t)$ satisfies:

$$\frac{d\hat{X}(t)}{dt} = \begin{cases} \max\{0, r_{\hat{\varphi}(t)} - \nu\} & \hat{X}(t) = 0; \\ r_{\hat{\varphi}(t)} - \nu & 0 < \hat{X}(t) < b; \\ \min\{0, r_{\hat{\varphi}(t)} - \nu\} & \hat{X}(t) = b. \end{cases} \quad (1)$$

It can be seen from (1) that when $\hat{X}(t) = 0$, and the rate $r_{\hat{\varphi}(t)}$ is less than ν , the buffer continues to be empty; when $\hat{X}(t) = b$, and the rate $r_{\hat{\varphi}(t)}$ is greater than ν , the voice stream in the buffer will remain; when $\hat{X}(t)$ is between $[0, b]$, the voice stream in the buffer changes at rate $r_{\hat{\varphi}(t)} - \nu$. Without loss of generality, for any state $i \in S$, we assume $r_{\hat{\varphi}(t)} \neq \nu$.

Let

$$(\hat{X}, \hat{\varphi}) = \{\hat{X}(t), \hat{\varphi}(t) : t \geq 0\}$$

and it is easy to see that $(\hat{X}, \hat{\varphi})$ is a SFM model and it is describe the packet voice multiplexing system model under consideration in this paper. In SFM $(\hat{X}, \hat{\varphi})$, the level process \hat{X} describes the change process of the voice stream level in the multiplexing buffer. Because the multiplexing buffer is finite, when the voice stream reaches the level b , overflow will occur; when the voice stream level is reduced to 0, the multiplexing buffer is empty. The background process $\hat{\varphi}$ describes the change of the voice source state, which affects the input rate of the voice multiplexing system, and controls the change of the voice stream level \hat{X} in the multiplexing buffer.

In the packet voice multiplexing system, the multiplexing buffer size and the number of voice sources make an important impact on the performance metrics of the system. The main goal of this paper is to derive the exact expressions of overflow probability and idle probability of the packet voice multiplexing system $(\hat{X}, \hat{\varphi})$, and investigate numerically the setting of the optimal buffer size and the optimal number of users that can be accommodated given the maximum tolerable overflow probability.

Results and Discussion

Model analysis

Before our analysis, we first give the following definitions.

In SFM $(\hat{X}, \hat{\varphi})$, we define the stationary probability density and boundary probability

$$\hat{\mathbf{p}}^{(b)} = [\hat{p}_i^{(b)}, i \in S], \hat{\mathbf{p}}^{(0)} = [\hat{p}_i^{(0)}, i \in S], \hat{\mathbf{p}}^{(0)} = [\hat{p}_i^{(0)}, i \in S],$$

where:

$$\hat{f}_i(x) = \lim_{t \rightarrow \infty} \frac{\partial}{\partial t} \mathbb{P}[\hat{X}(t) \leq x, \hat{\varphi}(t) = i],$$

$$\hat{p}_i^{(b)} = \lim_{t \rightarrow \infty} \mathbb{P}[\hat{X}(t) = b, \hat{\varphi}(t) = i],$$

$$\hat{p}_i^{(0)} = \lim_{t \rightarrow \infty} \mathbb{P}[\hat{X}(t) = 0, \hat{\varphi}(t) = i].$$

According to the backward equation, the stationary probability (density) of SFM $(\hat{X}, \hat{\varphi})$ satisfies:

$$\begin{cases} \hat{f}'(x) \mathbf{D} = \hat{f}(x) \mathbf{M}, \\ \hat{\mathbf{p}}^{(0)} \mathbf{D} = \hat{\mathbf{p}}^{(0)} \mathbf{M}, \\ \hat{\mathbf{p}}^{(b)} \mathbf{D} = \hat{\mathbf{p}}^{(b)} \mathbf{M}, \end{cases} \quad (2)$$

where $0 \leq x \leq b$. Let

$$\mathbf{f}(x) = \hat{f}(x) |\mathbf{D}|, \quad (3)$$

$$\mathbf{p}^{(0)} = \hat{\mathbf{p}}^{(0)} |\mathbf{D}|, \quad (4)$$

$$\mathbf{p}^{(b)} = \hat{\mathbf{p}}^{(b)} |\mathbf{D}|. \quad (5)$$

where $|\mathbf{D}| = [|\mathbf{D}_{ij}|]$, Then (2) can be rewritten as:

$$\begin{cases} \mathbf{f}'(x) \mathbf{I}_* = \mathbf{f}(x) \mathbf{Q}, \\ \mathbf{p}^{(0)} \mathbf{I}_* = \mathbf{p}^{(0)} \mathbf{Q}, \\ \mathbf{p}^{(b)} \mathbf{I}_* = \mathbf{p}^{(b)} \mathbf{Q}. \end{cases} \quad (6)$$

where $\mathbf{I}_* = [|\mathbf{D}_{ij}| / |\mathbf{D}_{ij}|]$, $\mathbf{Q} = |\mathbf{D}|^{-1} \mathbf{M}$.

It can be seen from (6) that the SFM $(\hat{X}, \hat{\varphi})$ is transformed into a new SFM with net input rate matrix \mathbf{I}_* (that is, the net input rate is ± 1), and the transition rate matrix \mathbf{Q} . We denote this new SFM by (X, φ) . Now, we mainly study SFM (X, φ) , and the performance metrics of the original SFM $(\hat{X}, \hat{\varphi})$ can be obtained based on (3-5).

According to the sign of the net input rate, the state space S is divided into two disjoint subsets: $S = S_1 \cup S_2$, where:

$$S_1 = \{i \in S | r_i - \nu > 0\};$$

$$S_2 = \{i \in S | r_i - \nu < 0\}.$$

Based on the division of the state, the matrices \mathbf{Q} and \mathbf{I}_* can be written in the following block form accordingly:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix}; \mathbf{I}_* = \begin{bmatrix} \mathbf{I}_1 & \\ & -\mathbf{I}_2 \end{bmatrix}.$$

Similarly, $\mathbf{p}^{(0)}$ and $\mathbf{p}^{(b)}$ can be written in block form: $\mathbf{p}^{(0)} = [\mathbf{p}_1^{(0)}, \mathbf{p}_2^{(0)}]$, $\mathbf{p}^{(b)} = [\mathbf{p}_1^{(b)}, \mathbf{p}_2^{(b)}]$.

When the voice stream in the buffer reaches the upper bound b , and the net input rate is negative at this time, the voice stream in the buffer cannot be maintained at the upper bound, therefore, $\mathbf{p}_2^{(b)} = \mathbf{0}$, and we call $\mathbf{p}_1^{(b)} = \mathbf{0}$ the overflow probability; Similarly, when the multiplexing buffer is empty and the net input rate is positive, the voice stream in the buffer cannot be empty, therefore, $\mathbf{p}_1^{(0)} = \mathbf{0}$, and we call $\mathbf{p}_2^{(0)}$ the idle probability.

To analyze the overflow probability and idle probability of the voice multiplexing system model, we define First Passage Time (FPT) as follows.

$$\begin{aligned} \tau_a^y &= \inf\{t \geq 0, X(t) = y, X(0) = a\}, \\ {}^c\tau_a^y &= \inf\{t \geq 0, \\ &X(t) = y, X(0) = a, X(s) \neq c, 0 \leq s \leq t\}. \end{aligned}$$

We are interested in the two first passage times ${}^0\tau_0^b$ and ${}^b\tau_b^0$. That is, ${}^0\tau_0^b$ is the first time epoch that the level process reaches the buffer upper bound b with initial level 0 before the buffer is empty; and ${}^b\tau_b^0$ is the first time epoch that the level process starts from the buffer upper bound level 0 , and first reaches the level 0 without returning level b .

We define the following probability matrix of FPT as $\mathcal{F}(\tau_a^b) = ([\mathcal{F}(\tau_a^b)]_{ij})$, and denote its (i, j) -th elements by:

$$[\mathcal{F}(\tau_a^b)]_{ij} = \mathbb{P}[\varphi(\tau_a^b) = j \mid X(0) = a, \varphi(0) = i].$$

Obviously, $[\mathcal{F}(\tau_a^b)]_{ij}$ represents the probability that the SFM (X, φ) starts from state (a, i) , and arrives at (b, j) for the first time.

Similarly, we define matrix $\mathcal{F}({}^c\tau_a^b) = ([\mathcal{F}({}^c\tau_a^b)]_{ij})$, and denote its (i, j) -th elements by:

$$[\mathcal{F}({}^c\tau_a^b)]_{ij} = \mathbb{P}[\varphi({}^c\tau_a^b) = j \mid X(0) = a, \varphi(0) = i].$$

In order to simplify the calculation, the level-reverse process is denoted by:

$$\{X^{(r)}, \varphi^{(r)}\} = \{X^{(r)}(t), \varphi^{(r)}(t) : t \geq 0\}$$

which is considered by [12,13]. The level-reverse SFM $\{X^{(r)}, \varphi^{(r)}\}$ has the same transition rate matrix of the SFM $\{X, \varphi\}$, but the net input rate of the state is exactly the opposite. That is, in $\{X^{(r)}(t), \varphi^{(r)}(t) : t \geq 0\}$, let the net input rate be $d_i^{(r)} = -d_i, i \in \mathcal{S}$.

For the convenience of description, a quantum B in (X, φ) is correspondingly denoted by $B^{(r)}$ in the level-reverse process $(X^{(r)}, \varphi^{(r)})$.

According to the division of state, $\mathcal{F}(\tau_x^0)$ has the following matrix block form:

$$\mathcal{F}(\tau_x^0) = \begin{bmatrix} \mathcal{F}_{11}(\tau_x^0) & \mathcal{F}_{12}(\tau_x^0) \\ \mathcal{F}_{21}(\tau_x^0) & \mathcal{F}_{22}(\tau_x^0) \end{bmatrix}.$$

Because the level process X can only reach the level 0 for the first time in the state set \mathcal{S}_2 , therefore:

$$\mathcal{F}(\tau_x^0) = \begin{bmatrix} \mathbf{0} & \mathcal{F}_{12}(\tau_x^0) \\ \mathbf{0} & \mathcal{F}_{22}(\tau_x^0) \end{bmatrix}.$$

The following lemma gives the expressions of $\mathcal{F}_{12}(\tau_x^0)$ and $\mathcal{F}_{22}(\tau_x^0)$.

Lemma 1 [14] For the SFM (X, φ) with $b = +\infty$, $\mathcal{F}_{12}(\tau_x^0)$ and $\mathcal{F}_{22}(\tau_x^0)$ have the following expressions:

$$\mathcal{F}_{12}(\tau_x^0) = \Phi e^{[Q_{22} + Q_{21}\Phi]x}, \quad (7)$$

$$\mathcal{F}_{22}(\tau_x^0) = e^{[Q_{22} + Q_{21}\Phi]x}. \quad (8)$$

Matrix Φ satisfies the following Riccati equation [12].

$$\Phi Q_{21} \Phi + Q_{11} \Phi + \Phi Q_{22} + Q_{12} = \mathbf{0} \quad (9)$$

The literature [15-17] gives an effective algorithm for solving Φ in the (9). It can be seen from the above expression that after Φ and $\hat{\Phi}^{(r)}$ are given, all the quantities are available.

Similarly, we have the following conclusions for the block matrices $\mathcal{F}_{12}({}^0\tau_0^b)$ and $\mathcal{F}_{11}({}^0\tau_0^b)$.

Lemma 2 [14] For the SFM (X, φ) with $b = +\infty$, the following equation holds:

$$\begin{aligned} &[\mathcal{F}_{12}({}^0\tau_0^b), \mathcal{F}_{11}({}^0\tau_0^b)] \\ &= [\Phi, \mathcal{F}_{22}^{(r)}(\tau_\theta^0)] \begin{bmatrix} \mathbf{I} & \mathcal{F}_{12}^{(r)}(\tau_\theta^0) \\ \mathcal{F}_{12}(\tau_\theta^0) & \mathbf{I} \end{bmatrix}^{-1} \end{aligned}$$

Based on the above two lemmas, we begin to derive the stationary boundary probability of the SFM (X, φ) .

As shown in **Figure 2**, we truncate (X, φ) and only remain the path sample of the system on the boundary level 0 and the boundary level b . The state space of the truncated process can be written as

$$\mathcal{S}^{tr} = \{(0, i) \cup (b, k), i \in \mathcal{S}_2, k \in \mathcal{S}_1\}.$$

We consider the transition rate of the truncated process in the interval $[t, t + \Delta]$, let Δ be small enough. Let $i, j \in \mathcal{S}_1$. At the time t , we assume the state of the truncated process is (b, i) , then two cases will be occurred when the truncation process transits to state (b, j) from state (b, i) :

The truncated process transits directly from state (b, i) to state (b, j) with probability $Q_{ik}\Delta + o(\Delta)$;

The truncation process transits from state (b, i) to state (b, k) , where $k \in \mathcal{S}_2$ with probability $Q_{ik}\Delta + o(\Delta)$, and then the process starts from state (b, k) to state (b, j) , while during the transition, the sample path cannot hits the level 0 . From lemma 2, the probability of this event is $\mathcal{F}_{21}^{(r)}({}^0\tau_b^b)$.

Thus, the transition rate matrix from (b, \mathcal{S}_1) to (b, \mathcal{S}_1) has the following form: $Q_{11} + Q_{12}\mathcal{F}_{21}^{(r)}({}^0\tau_b^b)$.

Similarly, we can construct the transition rate of truncation process in the following block form:

$$W = \begin{bmatrix} Q_{11} + Q_{12}\mathcal{F}_{21}^{(r)}({}^0\tau_b^b) & Q_{12}\mathcal{F}_{22}^{(r)}({}^b\tau_b^0) \\ Q_{21}\mathcal{F}_{11}({}^0\tau_0^b) & Q_{22} + Q_{21}\mathcal{F}_{12}({}^b\tau_0^0) \end{bmatrix}. \quad (10)$$

At the same time, we notice that when the fluid queue is at the middle level, the time for upward transition is equal to the time for downward transition [10]. Therefore:

$$(\mathbf{p}_1^{(b)} - \xi_1)\mathbf{1} = (\mathbf{p}_2^{(0)} - \xi_2)\mathbf{1},$$

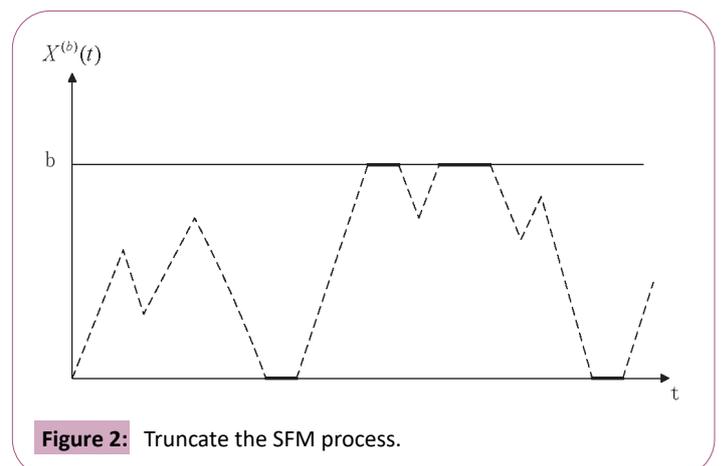


Figure 2: Truncate the SFM process.

where ξ_1, ξ_2 are the stationary distributions of background processes, that is, we have $[\xi_1, \xi_2]Q = 0$ and $[\xi_1, \xi_2]1 = 1$.

Now we can solve the following equation to obtain the stationary boundary probability $p_1^{(b)}$ and $p_2^{(0)}$:

$$\begin{cases} (p_1^{(b)}, p_2^{(0)})W = 0 \\ (p_1^{(b)} - \xi_1)1 - (p_2^{(0)} - \xi_2)1 = 0 \end{cases} \quad (11)$$

It can be seen from (2)-(5) that $[p_1^{(b)}, p_2^{(0)}]$ and $[p_1^{(b)}|D|^{-1}, p_2^{(0)}|D|^{-1}]$ are proportional to each other, then we have:

$$p_1^{(b)} = \alpha p_1^{(b)}|D|^{-1}, \quad (12)$$

$$p_2^{(0)} = \alpha p_2^{(0)}|D|^{-1}. \quad (13)$$

Notice that:

$$\int_0^b f_1(x) dx + p_1^{(b)} = \xi_1,$$

$$\int_0^b f_2(x) dx + p_2^{(0)} = \xi_2.$$

Thus, α can be determined by the following equation:

$$1 = \hat{p}_1^{(b)}1 + \hat{p}_2^{(0)}1 + \int_0^b \hat{f}(x) dx 1 = \alpha [\xi_1, \xi_2]|D|1.$$

From the above formula, we have:

$$\alpha = ([\xi_1, \xi_2]|D|^{-1}1)^{-1}. \quad (14)$$

Now, we obtain the overflow and idle probabilities of the buffer in the packet voice multiplexing system $(\hat{X}, \hat{\varphi})$, we summary the corresponding algorithms in Algorithm 1 (Table 1).

Numerical example

In this section, we will give numerical examples to illustrate our analytical conclusions.

Numerical setting: Consider a small experimental scenario of multi-user voice stream transmission. We assume N identical and independent users $v_k, k = 1, 2, \dots, N$, in the scenario. The voice signal is transmitted to the channel at rate $r_1^{(k)} = 8$ kbit/s. Since the voice multiplexing system adopts the principle of frequency division multiplexing, the frequency range of the voice signal is 300-400 Hz, and the signal-noise ratio of the general link

is 30 dB [18]. So by Shannon formula, the transmission rate of the voice signal to the network system is about $\nu = 31$ kbit/s. Suppose that the average sojourn time of the voice in state "active" is 3 min, and the average time in state "silent" is 2 min. Then the transition rate matrix is:

$$M^{(k)} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

the input rate matrix is $R^{(k)} = \text{diag}[0.47, 0]$, (Unit: Mbit/min), for $k = 1, 2, \dots, N$. According to the theoretical findings developed in this paper, we obtain and describe the corresponding numerical results in the following figure.

Numerical analysis

Figure 3 shows the relationship between the overflow probability and the buffer size under different number of users. The abscissa represents the size of the buffer, and the ordinate represents the overflow probability of the packet voice multiplexing system. As can be seen from Figure 3, given the same buffer size, the overflow probability increases with the increasing the number of users. It can also be noticed that given the number of uses, the overflow probability decreases when the buffer capacity increases. However, as the buffer size becomes larger and larger, the impact becomes smaller and smaller. This observation suggests that simply increasing buffer size is not a suitable way to decrease the overflow probability, and the number of users need to be under consideration.

In Figure 4 we can see that given the number of users, when

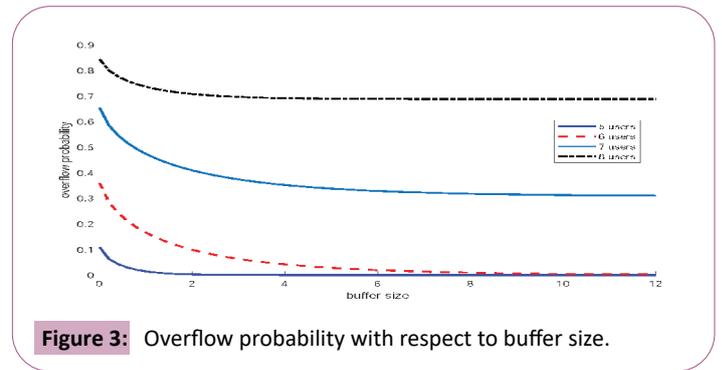


Figure 3: Overflow probability with respect to buffer size.

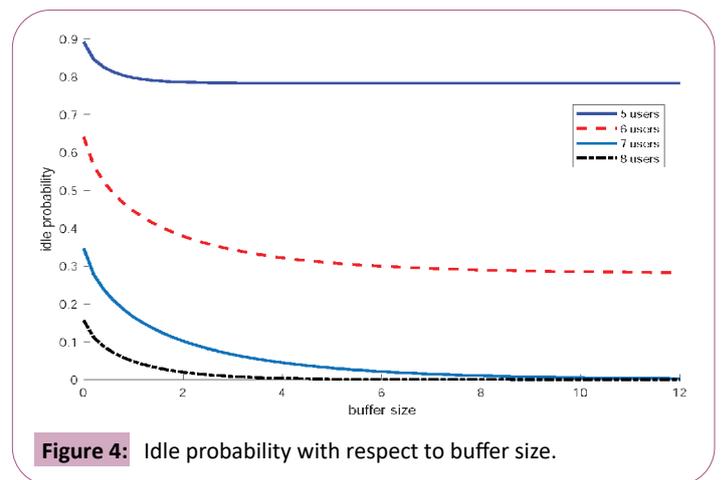


Figure 4: Idle probability with respect to buffer size.

Table 1: Overflow and idle probability algorithm of buffer in packet voice multiplexing system.

S. No	Algorithm 1 Overflow and idle probabilities algorithm of the packet voice multiplexing system
1	Step 1. Divide S into $S_1 \cup S_2$, construct block matrix Q .
2	Step 2. Use (9) to obtain Φ .
3	Step 3. Use lemma 1 to obtain $\mathcal{F}_{12}(\tau_x^0)$ and $\mathcal{F}_{22}(\tau_x^0)$.
4	Step 4. Use lemma 2 to obtain $\mathcal{F}_{12}(\tau_0^0)$ and $\mathcal{F}_{11}(\tau_0^0)$.
5	Step 5. Construct transition rate matrix W base on step 3 and step 4.
6	Step 6. Obtain $p_1^{(b)}$ and $p_2^{(0)}$ base on (11).
7	Step 7. Obtain α base on (14).
8	Step 8. Finally, obtain $\hat{p}_1^{(b)}$ and $\hat{p}_2^{(0)}$ base on (4)-(5).

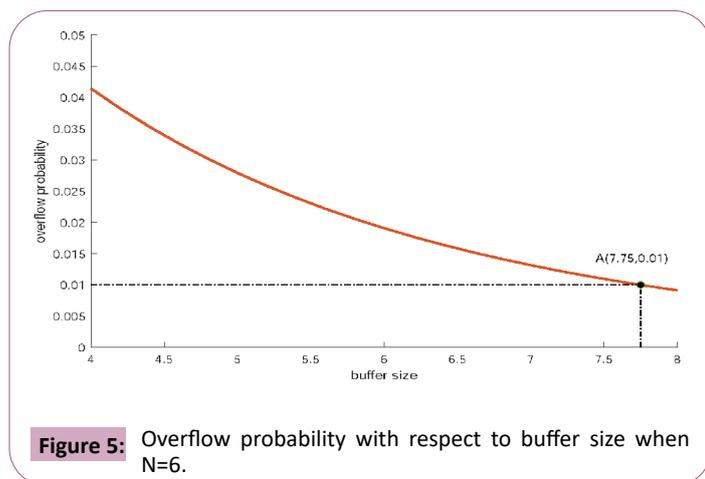


Figure 5: Overflow probability with respect to buffer size when $N=6$.

the buffer size increases, as expected, the idle probability of the system decreases. This observation can be explained by the fact that as the buffer size increases, the overflow probability becomes smaller, and then the more voice information of the system has to process, so the idle probability decreases with the increasing of the buffer size. When given the buffer size, we can see that the idle probability decreases with the increasing of the number of users. This because as the number of users increases, the more voice stream in the buffer, and then the more voice information of the system has to process, so the idle probability decreases with the increasing of the number of users.

System parameter setting

Based on the above scenario settings, as an application, in the

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following we illustrate, given the maximum tolerable overflow probability, how to design the two important parameters of the packet voice multiplexing system, i.e., the buffer size and the maximum number of users that system can accommodate (**Figure 5**). We should first determine the maximum number of users the system can accommodate, this because the overflow probability has not been significantly impacted by the changes of the buffer size (**Figure 3**).

Conclusion

This paper investigates the overflow and idle probabilities of the packet voice multiplexing system based on the framework of SFM. We truncate the system sample path, and derive the overflow and the idle probabilities by the matrix-analytic methodology. Numerical investigation of the theoretical results is performed, as an application, the optimal buffer size and the maximum number of users is investigated with the maximum tolerable overflow probability constraint. The theoretical findings in this paper are of important guiding significance for the design and management of the packet voice multiplexing systems. In future work, we can extend the system model proposed in this paper, and analyze the important performance metrics of the packet voice multiplexing system with priority.

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