

Numerical study of MHD free convection heat and mass transfer from vertical surfaces in porous media considering Soret and Dufour effects

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ABSTRACT

In the present approach, a two dimensional steady MHD free convection flow of heat and mass transfer from a vertical surface in porous media has been analyzed numerically consider with Soret and Dufour effects. The governing non linear partial differential equations have been transformed by a similar transformation in to a system of ordinary differential equations, which are solved numerically by using implicit finite difference scheme. The dimensionless velocity, temperature and concentration profiles are displayed graphically showing the effects for the different values of the Lewis number, soret number and Magnetic number.

Keywords: free convection, porous medium, MHD, Dufour, Soret and finite difference method.

Nomenclature

| | |
|------------|--|
| S_r | Soret number |
| T | Temperature |
| x, y | Cartesian co-ordinates along and normal to the surface, respectively |
| α_m | Thermal diffusivity |
| β_T | Coefficient of thermal expansion |
| β_C | Coefficient of concentration expansion |
| ϕ | Dimensionless concentration |
| η | Similarity variable |
| ν | Kinematic viscosity |
| θ | Dimensionless temperature |
| ρ | Density |
| ψ | Stream function |

Subscripts

| | |
|----------|-----------------------|
| w | Condition at wall |
| ∞ | Condition at infinity |

Superscript

| | |
|---|--|
| ' | Differentiation with respect to η |
|---|--|

INTRODUCTION

Several studies have been found to analyze the influence of the combined heat and mass transfer process by natural convection in a thermal and/or mass stratified porous medium, owing to its wide applications, such as development of advanced technologies for nuclear waste management, hot dike complexes in volcanic regions for heating of ground water, separation process in chemical engineering, etc. Here stratified porous medium means that the ambient concentration of dissolved constituent and/or ambient temperature is not uniform and varies as a linear function of vertical distance from the origin.

When heat and mass transfer occur simultaneously in a moving fluid, the relation between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradient and this is the Soret or thermal-diffusion effect. These effects are considered as second-order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier's and Fick's laws, but they may become significant in areas such as geosciences or hydrology.

The study of magneto hydrodynamics(MHD) flows have stimulated extensive attention due to its significant applications in three different subject areas, such as a astrophysical, geophysical and engineering problems. Free convection in electrically conducting fluids through an external magnetic field has been a subject of considerable research interest of a large number of scholars for a long time due to its diverse applications in the fields such as nuclear reactors, geothermal engineering, liquid metals and plasma flows, among the others. Fluid flow control under magnetic force is also applicable in magneto hydrodynamics generators and a host of magnetic devices used in industries. Steady and transient free convection Coupled heat and mass transfer by natural convection in a fluid-saturated porous medium has attracted considerable attention in the last years, due to many important engineering and geophysical applications. Recent books by Nield and Bejan [1] and Ingham and Pop [2, 3] present a comprehensive account of the available information in the field. The MHD free-convection and Mass Transfer flow with Hall current, viscous dissipation, Joule heating and Thermal diffusion is studied by Singh, A.K [4].N.Kishan et all [5] studied the MHD free convection heat and mass transfer in a doubly stratified Darcy porous medium considering soret and Dufour effect with viscous dissipation.Effect of doubly stratification on free convection in Darcian porous medium have been studied by Murthy et al [6]. G.Vidyasagar et all [7] studied the Mass Transfer effects on radiative MHD flow over a non isothermal stretching sheet and embedded in a porous medium. P.Srinivasulu and N.Bhaskar Reddy [8] studied the Thermo-diffusion and Diffusion-thermo effects on MHD boundary layer flow past an exponential stretching sheet with thermal radiation and viscous dissipation.

Lakshmi Narayana and Murthy [9] investigated the effects of Soret and Dufour on free convection heat and mass transfer from a vertical surface in a doubly stratified Darcy porous medium. They have neglected effect of MHD, (Viscous dissipation). Partha et al[10] studied the effect of magnetic field and double dispersion on free convection heat and mass transport considering the Soret and Dufour effects in a non – Darcy porous medium. V.Srihari babu and G.V.Ramana reddy [11] studied the Mass transfer effects on MHD flow from a vertical surface with ohmic heating and viscous dissipation. Satya sagar Saxena and G.K.Dubey [12] studied the MHD free convection heat and mass transfer flow of viscoelastic fluid embedded in a porous medium of variable permeability with radiation effect and heatsource in slip flow regime. Satyasagar Saxena and G.K.Dubey [13] studied the Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion.

P.R Sharma and Singh [14] studied the effect of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Recently, Adrian postelnicu [15] studied the heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a chemical reaction by taking account the Dufour and Soret effects.

The objective of this paper is to study simultaneous heat and mass transfer by natural convection from a vertical surface embedded in a fluid saturated Darcian porous medium under the influence of magnetic field including Soret and Dufour effect.

PROBLEM FORMULATION

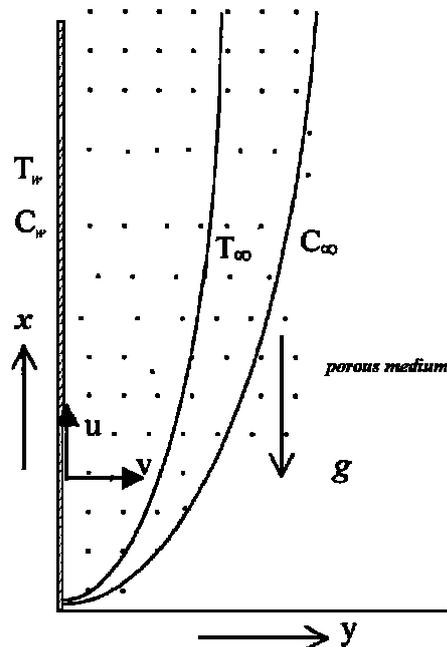


Fig. 1 Flow model and physical coordinate system

Consider the natural convection in a porous medium saturated with a Newtonian fluid on a vertical flat plate. The x -coordinate is measured along the surface and the y -coordinate normal to it (see Fig. 1). The temperature of the ambient medium is T_w and the wall temperature is T_w . The flow along the vertical flat plate contains a species A slightly soluble in the fluid B, the concentration at the plate surface is C_w and the solubility of A in B far away from the plate is C_∞ .

Several assumptions are used throughout the present paper: (a) the fluid and the porous medium are in local thermodynamic equilibrium; (b) the flow is laminar, steady-state and two-dimensional; (c) the porous medium is isotropic and homogeneous; (d) the properties of the fluid and porous medium are constant; (e) the Boussinesq approximation is valid and the boundary layer approximation is applicable; (f) the concentration of dissolved A is small enough. In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u = \frac{gK}{\nu} [\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] - \frac{\sigma}{\rho} (\beta_0^2 u) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where all quantities are defined in the list of symbols.

The boundary conditions of the problem are

$$y \rightarrow \infty : u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty$$

$$y = 0 : v = 0, T = T_w, C = C_w \quad (5a)$$

$$y \rightarrow \infty : u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad (5b)$$

Where T_w, T_∞, C_w and C_∞ have constant values.

Equations 1, 2, 3, 4, 5 are now nondimensionalized using the following quantities:

$$\psi = \alpha_m Ra_x^{1/2} f(\eta), \quad \theta = (T - T_\infty) / (T_w - T_\infty),$$

$$\phi = (C - C_\infty) / (C_w - C_\infty), \quad \eta = \frac{y}{x} Ra_x^{1/2}, \quad (6)$$

Where the stream function ψ is defined in the usual way

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (7)$$

and $Ra_x = gK\beta(T_w - T_\infty)x / (\nu\alpha_m)$ is the local Rayleigh number. The governing equations become

$$f'(1+M) = \theta + N\phi \quad (8)$$

$$\theta'' - f\theta' + D_f\phi'' = 0 \quad (9)$$

$$\frac{1}{Le}\phi'' + f\phi' + S_r\theta'' = 0 \quad (10)$$

Where Le, D_f, S_r and N are Lewis, Dufour, Soret numbers and sustentation parameter, respectively

$$Le = \frac{\alpha_m}{D_m}, D_f = \frac{D_m k_T (C_w - C_\infty)}{C_s C_p \alpha_m (T_w - T_\infty)}, S_r = \frac{D_m k_T (T_w - T_\infty)}{C_s C_p \alpha_m (C_w - C_\infty)}, N = \frac{\beta_c (C_w - C_\infty)}{\beta_T (T_w - T_\infty)} \quad (11a)$$

We notice that N is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal-driven flows. Further, in order to get similarity solutions, the constant dimensionless chemical reaction parameter

$$\gamma = \frac{K_1}{\alpha_m} \cdot \frac{x^2}{Ra_x} \quad (11b)$$

Was introduced in Eq. 10. we notice to this end that primes denote with respect to η . The transformed boundary conditions are

$$f(0) = 0, \theta(0) = 1, \phi(0) = 1 \quad (12a)$$

$$\theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \quad (12b)$$

The parameters of engineering interest for the present problem are the local Nusselt number and local Sherwood number, which are given by the expressions

$$Nu_x / Ra_x^{1/2} = -\theta'(0), \quad Sh_x / Ra_x^{1/2} = -\phi'(0). \quad (13)$$

MATHEMATICAL SOLUTION

The set of non-linear ordinary differential equations (8) - (10) with boundary conditions (12) have been solved numerically, by using Crank Nickels implicit finite difference method. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criteria of 10^{-5} in all cases. The value of η_∞ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. In order to see the effect of step size $\Delta\eta$ we ran the code for our model with two different step sizes $\Delta\eta = 0.01$, $\Delta\eta = 0.001$ and each case we found very good agreement between them.

RESULTS AND DISSCUSSION

Numerical Calculations were carried out for different values of D_f, S_r, M and N . For the Purpose of discussing the effect of various parameters on the flow behavior some numerical calculations have been carried out for non dimensional velocity profiles f' , temperature profiles θ , and concentration profiles ϕ .

In fig1, the velocity profile D_f, S_r, M presented for fixed values of D_f, S_r, M . The non dimensional velocity f' increases with the increasing of N . In fig.2, we plotted velocity profile f' for different values of D_f, S_r, M, N . From Fig.2 we observed that the velocity profile increases with the increasing of Lewis number Le . From fig.3 we observed that the velocity profile f' increases with the increases magnetic parameter M .

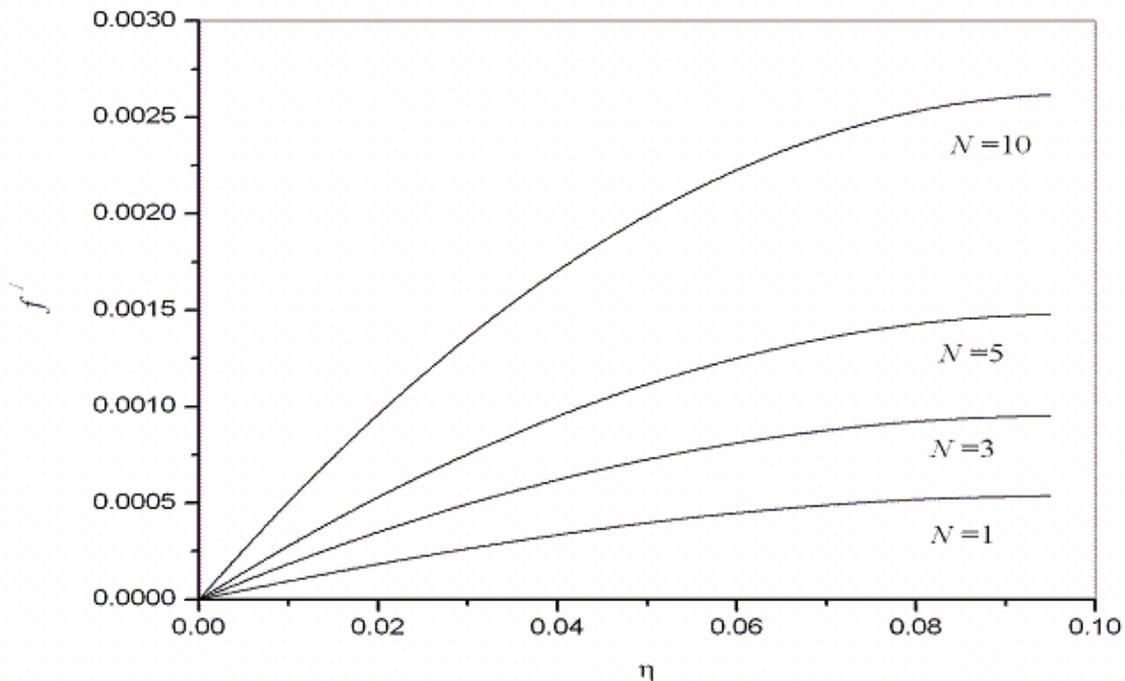


Fig.1: Velocity profile for different values of N with $S_r = 0.001, D_f = 10.0, M = 0$

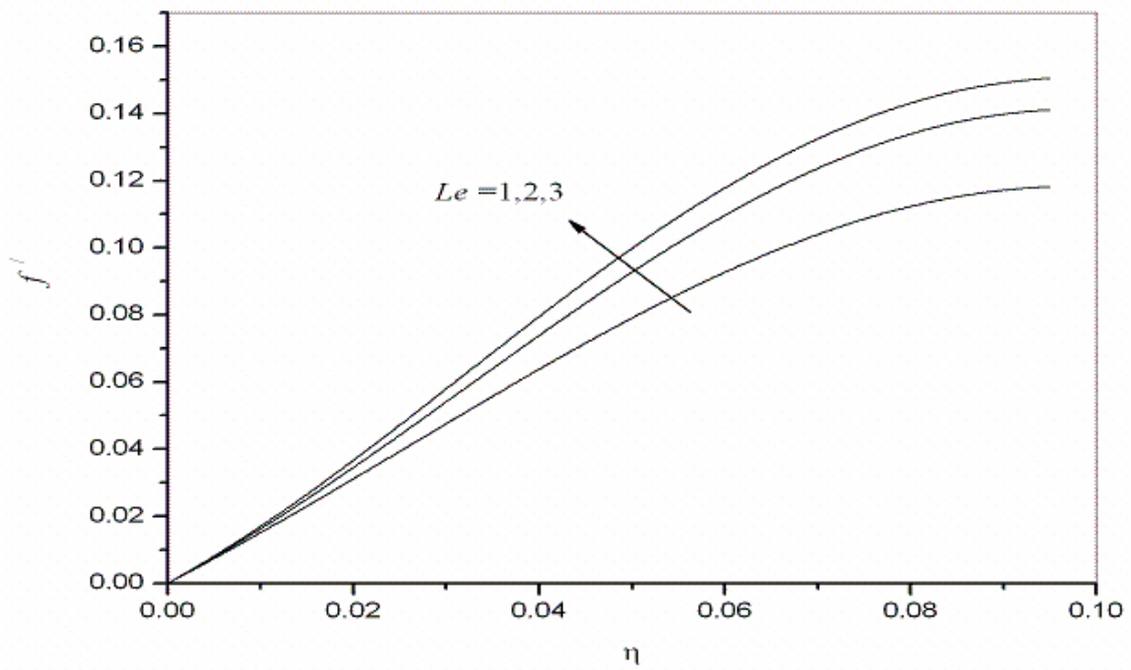


Fig.2: Velocity profile for different Le with $S_r = 0.001, D_f = 10.0, M = 0, N = 1$.

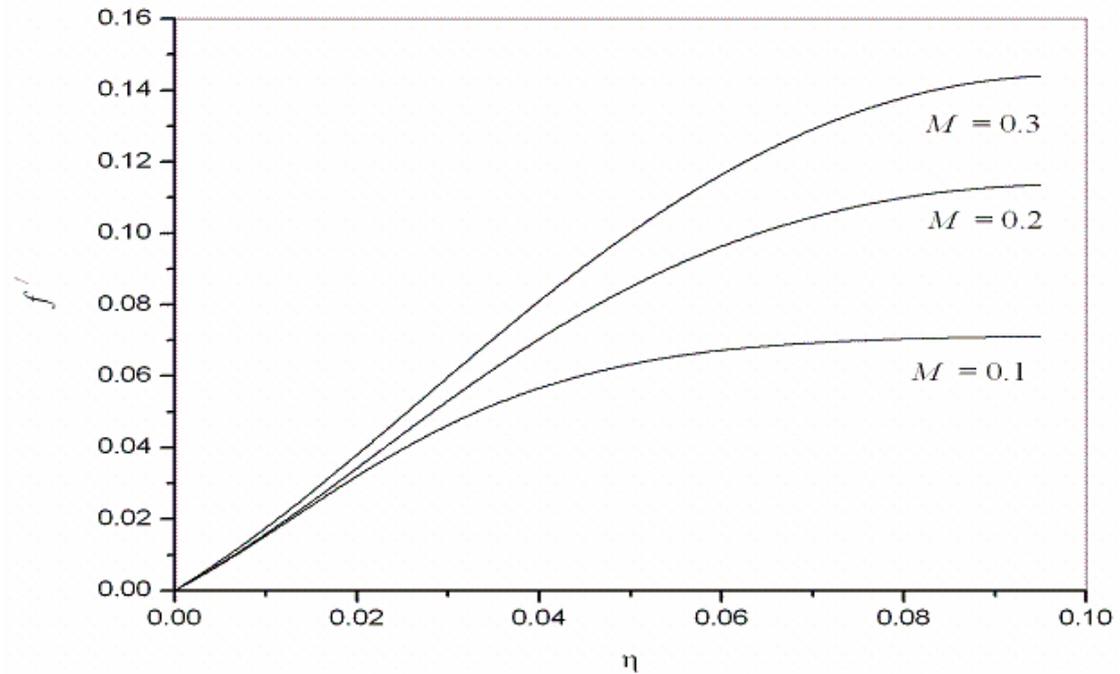


Fig.3: Velocity profile for different Magnetic parameters with $S_r = 0.001, D_f = 10.0$.

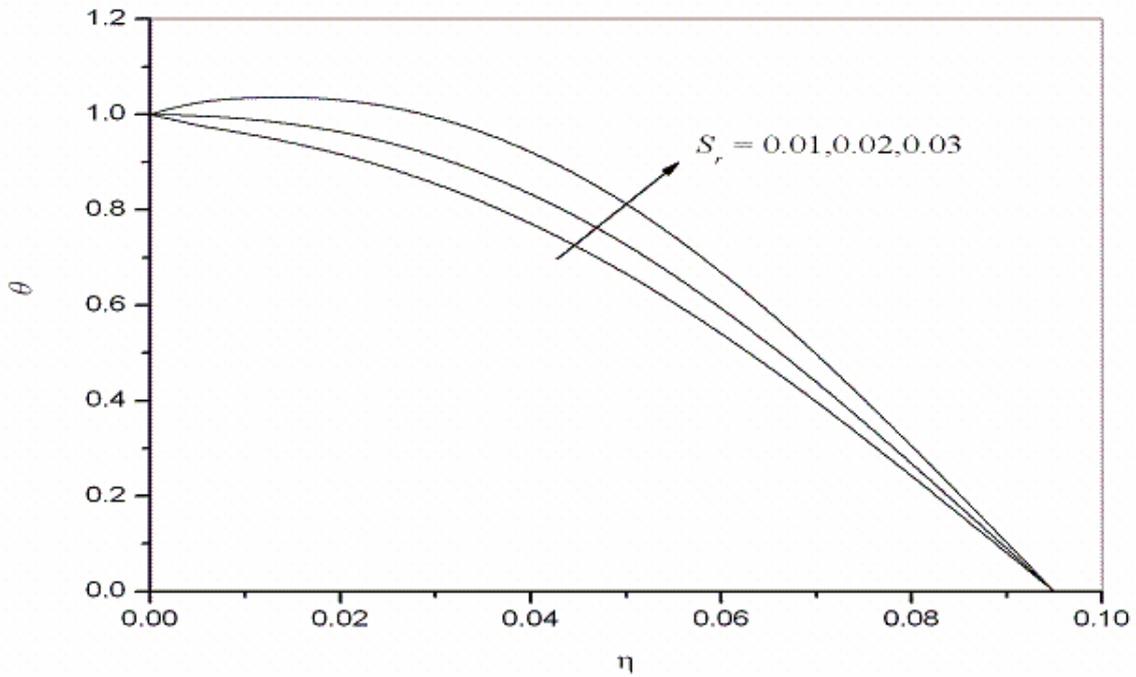


Fig.4: Temperature profiles for different solet parameters with $D_f = 10.0, M = 0, N = 1$.

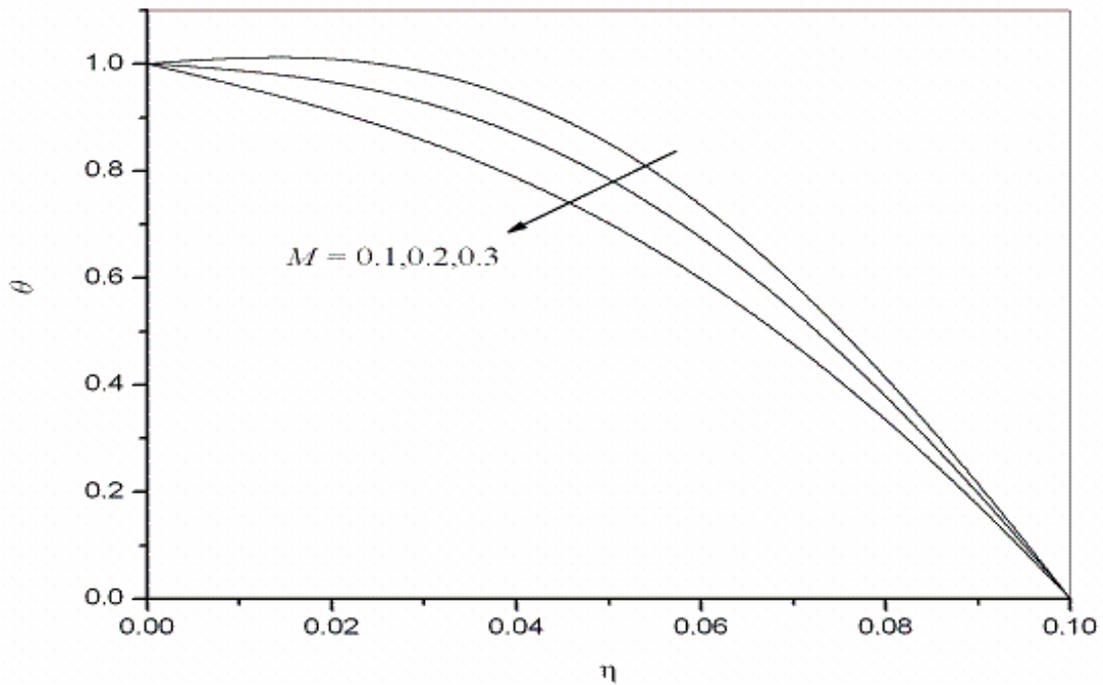


Fig.5: Temperature profiles for different magnetic parameters wit $S_r = 0.001, D_f = 10.0$

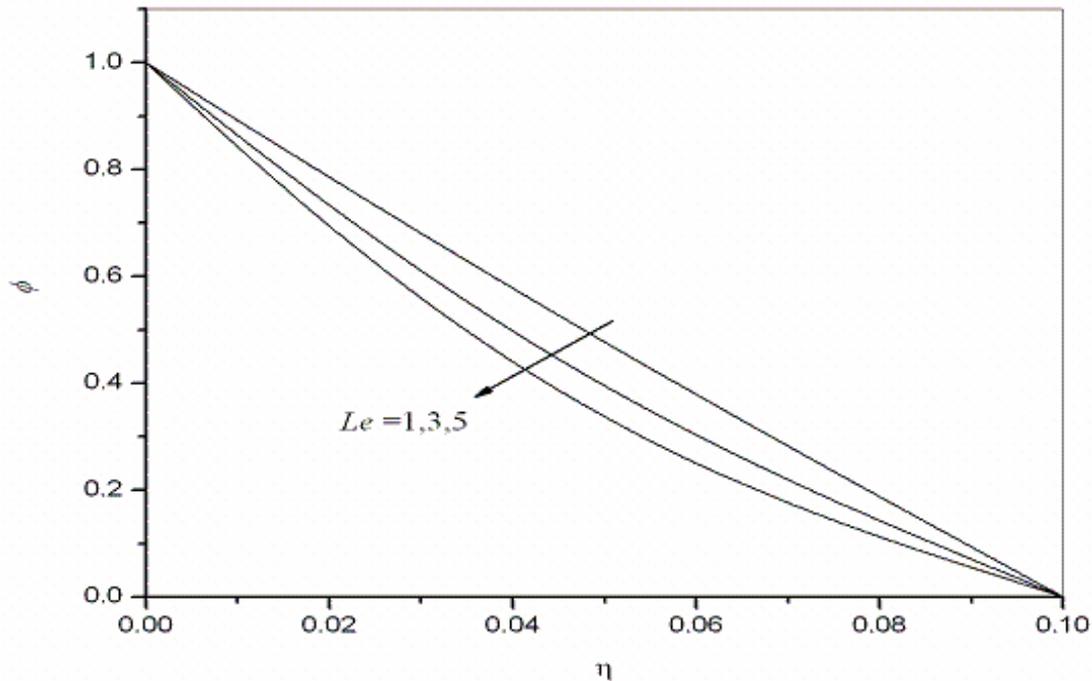


Fig.6: Concentration profiles for different Le with $D_f = 10.0, M = 0, S_r = 0.001$

From figures 4-5 show that the temperature profiles for different parameters Le and M . Fig.4 shows that the Temperature profile increases with the increase of Soret number S_r . From fig 5 we observed that the temperature profile increases with increase of magnetic parameter M . In fig 6, the concentration profile plotted for different Lewis number Le . From fig 6 we observed that the concentration profile decreases with the increase of Lewis number Le .

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