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Effect of slip on peristaltic pumping of a hyperbolic tangent fluid in an inclined asymmetric channel

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ABSTRACT

The present paper deals with the study of the effect of slip on peristaltic flow of a Hyperbolic Tangent fluid in an inclined asymmetric channel under long wavelength and low Reynolds number assumptions. The non linear governing equations are solved using the regular perturbation method. Analysis has been carried out in the presence of velocity and slip conditions. Expressions for stream function, pressure gradient and pressure rise coefficients are derived. The effect of various parameters on the pumping phenomenon is discussed with the help of graphs.

Keywords: Hyperbolic Tangent fluid, inclined asymmetric channel, peristaltic pumping.

INTRODUCTION

Peristalsis is an important mechanism for pumping biological and industrial fluids, which is generated by progressive wave of contraction or expansion moving on the wall of the tube. Due to peristaltic motion, the movement of organ walls can propel food, liquid and can also mix the contents within each organ. From industrial point of view the peristaltic flows play an important role in sanitary fluid transport, transport of corrosive fluids, a toxic liquid transport in the nuclear industry etc. After the first investigation of Latham [1], the initial mathematical models for the peristaltic flow in an infinitely long symmetric channel or tube have been investigated by Shapiro et al. [2].Many of the contributors to the area of peristaltic pumping have either followed Shapiro or Fung. Most of the studies on peristaltic flow deal with Newtonian fluids. The complex rheology of biological fluids has motivated investigations involving different non-Newtonian fluids. There are many engineering processes as well in which peristaltic pumps are used to handle a wide range of fluids particularly in chemical and pharmaceutical industries. It is also used in sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluids, where the contact of the fluid with the machinery parts is prohibited. Because most of the physiological fluids behave like a non-Newtonian fluid, therefore, some interesting studies dealing with the flows of on-Newtonian fluids are given in [3 – 13].

Nadeem and Akram [14] discussed peristaltic transport of a Hyperbolic Tangent fluid in an asymmetric channel. Mekheimer [15] studied the peristaltic transport of MHD flow in an inclined planar channel. Hayat et al. [8] extended the idea of Ealshahed et al. [16] for partial slip condition. Srinivas et al. [17] studied the Peristaltic transport in an asymmetric channel with heat transfer. .

Hence, several non-Newtonian models are being proposed by various researchers to investigate the flow behavior in physiological system of a living body. Among them Hyperbolic Tangent model is expected to explain most of the features of a physiological fluid.The governing equations of hyperbolic tangent fluid model for peristaltic fluid flow in a two dimensional inclined asymmetric channel has been modeled in the present paper. The governing equations are reduced using long wave length approximation and then the reduced problem has been solved by using regular perturbation method. The expression for pressure rise is computed numerically using Mathematica software. At the end, the graphical results are presented to discuss the physical behavior of various parameters of interest.

HYPERBOLIC TANGENT FLUID MODEL

Consider an incompressible fluid whose balance law of mass and linear momentum are given by

$$\operatorname{div} V = 0 \tag{1}$$

$$\rho \frac{dV}{dt} = div\, S + \rho f \tag{2}$$

where ρ is the density, *V* is the velocity vector, S is the Cauchy stress tensor, f represents the specific body force and d/d*t* represents the material time derivative. The constitutive equation for hyperbolic tangent fluid is given by

$$S = -PI + \tau \tag{3}$$

$$\tau = -\left[\eta_\infty + (\eta_0 + \eta_\infty)\tanh\left(\Gamma \bar{\dot{\gamma}}\right)^n\right]\bar{\dot{\gamma}}, \tag{4}$$

in which $-PI$ is the spherical part of the stress due to constraint of incompressibility, τ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, *n* is the power law index, and $\bar{\dot{\gamma}}$ is defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi}$$

$$\Pi = \frac{1}{2} trac\left(gradV + (gradV)^T\right)^2. \tag{5}$$

Here Π is the second invariant strain tensor. We consider constitution (4), the case for which $\eta_\infty = 0$ and $\Gamma\ \bar{\dot{\gamma}} < 1$. The component of extra stress tensor, therefore, can be written as

$$\bar{\tau} = -\eta_0\left[\left(\Gamma\bar{\dot{\gamma}}\right)^n\right]\bar{\dot{\gamma}} = -\eta_0\left[\left(1 + \Gamma\bar{\dot{\gamma}} - 1\right)^n\right]\dot{\gamma}$$

$$= -\eta_0\left[1 + n\left(\Gamma\bar{\dot{\gamma}} - 1\right)\right]\bar{\dot{\gamma}}. \tag{6}$$

MATHEMATICAL FORMULATION

We consider an incompressible Hyperbolic Tangent fluid in an inclined asymmetric channel of width $d_1 + d_2$. The angle of inclination is α. A sinusoidal wave propagating with constant speed c on the channel walls induces the flow. The geometry of the wall surface is defined as

$$Y = H_1 = \bar{d}_1 + \bar{a}_1 \cos\left[\frac{2\pi}{\lambda}\left(\bar{X} - c\bar{t}\right)\right] \tag{7}$$

$$Y = H_2 = -\bar{d}_2 - \bar{b}_1 \cos\left[\frac{2\pi}{\lambda}\left(\bar{X} - c\bar{t}\right) + \phi\right]$$

where $\bar{a}_1$ and $\bar{b}_1$ are the amplitudes of the waves, λ is the wave length, $\bar{d}_1 + \bar{d}_2$ is the width of the channel, c is the velocity of propagation, $\bar{t}$ is the time, and $\bar{X}$ is the direction of wave propagation. The phase difference ϕ varies in the range $0 \le \phi \le \pi$ in which ϕ =0 corresponds to a symmetric channel with waves out of phase and ϕ = π, the waves are in phase, further, $\bar{a}_1, \bar{b}_1, \bar{d}_1, \bar{d}_2$, and ϕ satisfies the condition

$$\bar{a}_1^2 + \bar{b}_1^2 + 2\,\bar{a}_1\,\bar{b}_1 \cos\phi \le \left(\bar{d}_1 + \bar{d}_2\right)^2.$$

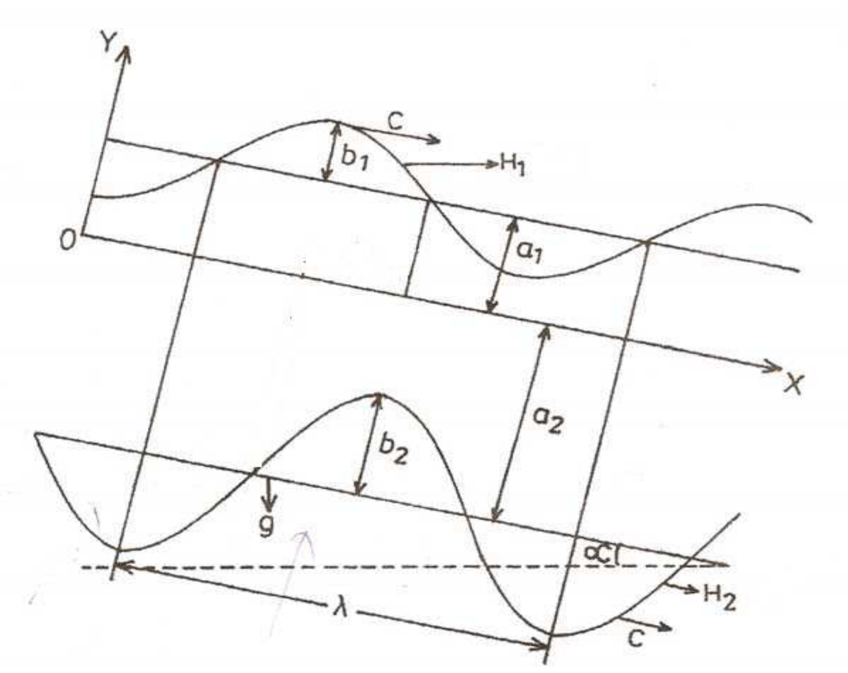


Fig.1. Physical Model

The equations governing the flow are given by

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0, \tag{8}$$

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} + \eta \sin\alpha, \tag{9}$$

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Y}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} - \eta \cos\alpha, \tag{10}$$

We introduce a wave frame $(\bar{x}, \bar{y})$ moving with velocity c away from the fixed frame $(\bar{X}, \bar{Y})$ by the transformation

$$\bar{x}=\bar{X}-c\bar{t}\quad,\ \bar{y}=\bar{Y}\quad,\ \bar{u}=\bar{U}-c,\ \ \bar{v}=\bar{V}\ ,\ and\ \bar{p}(x)=\bar{P}(\bar{X},t). \tag{11}$$

We use the following non-dimensional quantities

$$x=\frac{\bar{x}}{\lambda},\quad y=\frac{\bar{y}}{d_1},\quad u=\frac{\bar{u}}{c},\quad t=\frac{c}{\lambda}\bar{t}\ ,h_1=\frac{\bar{h}_1}{\bar{d}_1},\quad h_2=\frac{\bar{h}_2}{\bar{d}_1},\quad \tau_{xx}=\frac{\lambda}{\eta_0 c}\bar{\tau}_{\bar{x}\bar{x}},\quad \tau_{xy}=\frac{\bar{d}_1}{\eta_0 c}\bar{\tau}_{\bar{x}\bar{y}},$$
$$\tau_{yy}=\frac{\bar{d}_1}{\eta_0 c}\bar{\tau}_{\bar{y}\bar{y}},\quad \delta=\frac{\bar{d}_1}{\lambda},\quad \mathrm{Re}=\frac{\rho c\bar{d}_1}{\eta_0},\quad We=\frac{\Gamma c}{d_1},\quad P=\frac{\bar{d}_1^2}{c\lambda\eta_0}\bar{p},\ \dot{\gamma}=\frac{\bar{\dot{\gamma}}\bar{d}_1}{c}. \tag{12}$$

Using the above non-dimensional quantities in Eqs. (8) – (10) can be written as(after dropping bars)

$$u=\frac{\partial\Psi}{\partial y},v=-\frac{\partial\Psi}{\partial x} \tag{13}$$

$$\delta\,\mathrm{Re}\left[\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Psi}{\partial y}\right]=-\frac{\partial P}{\partial x}-\delta^2\frac{\partial\tau_{xx}}{\partial x}-\frac{\partial\tau_{xy}}{\partial y}+\eta\sin\alpha, \tag{14}$$

$$\delta^3\,\mathrm{Re}\left[\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Psi}{\partial x}\right]=-\frac{\partial P}{\partial y}-\delta^2\frac{\partial\tau_{xy}}{\partial x}-\delta\frac{\partial\tau_{yy}}{\partial y}-\eta\cos\alpha, \tag{15}$$

where $\tau_{xx}=-2[1+n(We\dot{\gamma}-1)]\frac{\partial^2\Psi}{\partial x\partial y}$, $\tau_{xy}=-1[1+n(We\dot{\gamma}-1)]\left(\frac{\partial^2\Psi}{\partial y^2}-\delta^2\frac{\partial^2\Psi}{\partial x^2}\right)$,

$$\tau_{yy}=2\delta[1+n(We\dot{\gamma}-1)]\frac{\partial^2\Psi}{\partial x\partial y},\dot{\gamma}=\left[2\delta^2\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\Psi}{\partial y^2}-\delta^2\frac{\partial^2\Psi}{\partial x^2}\right)^2+2\delta^2\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2\right]^{\frac{1}{2}}$$

Here δ is wave number, Re is Reynolds number and *We is* Wiessenberg number.

Under the assumptions of long wavelength $\delta \leq 1$ and low Reynolds number, and neglecting the terms of order δ and higher, Eqs. (14) and (15) take the form

$$\frac{\partial P}{\partial x}=\frac{\partial}{\partial y}\left[1+n\left(We\frac{\partial^2\Psi}{\partial y^2}-1\right)\right]\frac{\partial^2\Psi}{\partial y^2}+\eta\sin\alpha \tag{16}$$

$$\frac{\partial P}{\partial y}=0 \tag{17}$$

Elimination of pressure from Eq.(16) and Eq.(17) yield

$$\frac{\partial^2}{\partial y^2}\left[1+n\left(We\frac{\partial^2\Psi}{\partial y^2}-1\right)\right]\frac{\partial^2\Psi}{\partial y^2}=0 \tag{18}$$

The dimensionless mean flow Θ is defined as

$$\Theta = F + 1 + d \quad (19)$$

In which

$$F = \int_{h_2(x)}^{h_1(x)} \frac{\partial \Psi}{\partial y} dy = \Psi(h_1(x)) - \Psi(h_2(x)) \quad (20)$$

where

$$h_1(x) = 1 + a\cos 2\pi x \text{ and } h_2(x) = -d - b\cos(2\pi x + \phi) \quad (21)$$

The boundary conditions in terms of stream function Ψ are defined as:

$$\Psi = \frac{F}{2} \;;\; \frac{\partial \Psi}{\partial y} + \beta \frac{\partial^2 \Psi}{\partial y^2} = -1 \quad at \quad y = h_1(x)$$
$$\Psi = -\frac{F}{2} \;;\; \frac{\partial \Psi}{\partial y} - \beta \frac{\partial^2 \Psi}{\partial y^2} = -1 \quad at \quad y = h_2(x) \quad (22)$$

where F is the flux, $\beta = \frac{\sqrt{k}}{\alpha d_1} = \frac{\sqrt{Da}}{\alpha}$ is the permeability parameter including slip, Da is the Darcy number , α is the slip parameter and k is the permeability.

NUMERICAL SOLUTION

We used the perturbation technique to find the numerical solution of Eq.(18), which is non linear.

we expand Ψ , P and F as

$$\Psi = \Psi_0 + We\,\Psi_1 + O(We^2), \quad (23)$$

$$P = P_0 + We\,P_1 + O(We^2), \quad (24)$$

$$F = F_0 + We\,F_1 + O(We^2), \quad (25)$$

By using Eqs. (23) – (25), the problem is resolved into zeoth and first order systems as mentioned below.

Zeroth order system (System of order We^0):

$$\frac{\partial^4 \Psi_0}{\partial y^4} = 0 \quad (26)$$

$$\frac{\partial P_0}{\partial x} = (1-n)\frac{\partial^3 \Psi_0}{\partial y^3} + \eta \sin\alpha \quad (27)$$

$$\Psi_0=\frac{F_0}{2}\quad;\quad \frac{\partial\Psi_0}{\partial y}+\beta\frac{\partial^2\Psi_0}{\partial y^2}=-1\quad at\quad y=h_1(x)$$
$$\Psi_0=-\frac{F_0}{2};\quad \frac{\partial\Psi_0}{\partial y}-\beta\frac{\partial^2\Psi_0}{\partial y^2}=-1\quad at\quad y=h_2(x)\tag{28}$$

First order system (System of order We^1):

$$\frac{\partial^4\Psi_1}{\partial y^4}=\frac{n}{(n-1)}\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^2\tag{29}$$

$$\frac{\partial P_1}{\partial x}=(1-n)\frac{\partial^3\Psi_1}{\partial y^3}+n\frac{\partial}{\partial y}\left(\frac{\partial^2\Psi_0}{\partial y^2}\right)^2\tag{30}$$

$$\Psi_1=\frac{F_1}{2}\quad;\quad \frac{\partial\Psi_1}{\partial y}+\beta\frac{\partial^2\Psi_1}{\partial y^2}=-1\quad at\quad y=h_1(x)$$
$$\Psi_1=-\frac{F_1}{2}\quad;\quad \frac{\partial\Psi_1}{\partial y}-\beta\frac{\partial^2\Psi_1}{\partial y^2}=-1\quad at\quad y=h_2(x)\tag{31}$$

Zeroth order solution

On solving Eqs (26) with the boundary conditions given by Eqs (28), we get the solution to the zeroth order problem as

$$\Psi_0=A_1\frac{y^3}{3!}+A_2\frac{y^2}{2!}+A_3y+A_4,\tag{32}$$
$$A_1=\frac{-12(F_0+h_1-h_2)}{(h_1-h_2)^3+6\beta(h_1-h_2)^2},\qquad A_2=\frac{6(F_0+h_1-h_2)(h_1+h_2)}{(h_1-h_2)^3+6\beta(h_1-h_2)^2},$$
$$A_3=\frac{6F_0(\beta(h_1-h_2)-h_1h_2)-(h_1-h_2)(h_1^2+h_2^2-h_1h_2)}{(h_1-h_2)^3+6\beta(h_1-h_2)^2},$$
$$A_4=\frac{F_0}{2}-\frac{\left[F_0\left(h_1^3-3h_1^2h_2+6\beta h_1(h_1-h_2)\right)\right]+h_1h_2\left[(h_1-h_2)^2+3h_1(h_1-h_2)\right]}{(h_1-h_2)^3+6\beta(h_1-h_2)^2},$$

The axial pressure gradient at zeroth order is

$$\frac{\partial P_0}{\partial x}=\frac{12(n-1)(F_0+h_1-h_2)}{(h_1-h_2)^3+6\beta(h_1-h_2)^2}+\eta\sin\alpha\tag{33}$$

For one wavelength the integration of (33) gives

$$\Delta P_{\lambda_0}=\int_0^1\frac{\partial P_0}{\partial x}dx\tag{34}$$

Substituting the zeroth-order solution given by Eq.(32) into (29),the solution of the resulting first order problem satisfying the boundary conditions take the following form:

$$\frac{\partial^4 \Psi_1}{\partial y^4} = 2A_1^2 \frac{n}{n-1} \tag{35}$$

$$\Psi_1 = A_5 \frac{y^3}{3!} + A_6 \frac{y^2}{2!} + A_7 y + A_8 + A y^4 \tag{36}$$

where

$$A_5 = \frac{-(12F_1 - A_{12}(h_1 - h_2))}{(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2},\ A_6 = \frac{6F_1(h_1 + h_2) + A_{13}}{(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2},\ A_7 = \frac{6F_1(\beta(h_1 - h_2) - h_1 h_2 + A_{14})}{(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2},$$

$$A_8 = \frac{F_1}{2} + \frac{F_1[-h_1^3 + 3h_1^2 h_2 - 6\beta h_1(h_1 - h_2)] + A_{15}}{(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2},\ A_{11} = \frac{A[4(h_1^3 - h_2^3) + 6\beta(h_1^2 + h_2^2)]}{(h_1 - h_2 + 2\beta)},$$

$$A_{12} = A[(h_1^2 + h_2^2)(h_1 + h_2) - 4h_1^3 - 6\beta h_1^2] - \frac{A_{11}}{2}(h_1 - h_2 - 2\beta),$$

$$A_{13} = -6A_{12}(h_1^2 - h_2^2) - A_{11}(h_1 - h_2)^2(h_1 - h_2 + 6\beta),$$

$$A_{14} = \frac{-6A_{12}(h_1 - h_2)(h_1 + 2\beta h_1) - A_{13}(h_1 + \beta) - 6\beta A h_1^2(h_1 - h_2)(h_1 - h_2 + 6\beta)}{(h_1 - h_2)^2(h_1 - h_2 + 6\beta)},$$

$$A_{15} = -2A_{12}(h_1 - h_2)h_1^3 - A_{14}h_1 - Ah_1^4(h_1 - h_2)^2(h_1 - h_2 + 6\beta),$$

The axial pressure gradient at first order is

$$\frac{\partial P_1}{\partial x} = \frac{\frac{2n}{(n-1)}[-72(F_0 + h_1 - h_2)^2(h_1 + h_2) + [12F_1 - A_{12}(h_1 - h_2)]][(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2]}{[(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2]^2}, \tag{37}$$

For one wavelength, the integration of Eq.(37) yields

$$\Delta P_{\lambda_1} = \int_0^1 \frac{\partial P_1}{\partial x} dx \tag{38}$$

Summarizing the perturbation results for small parameter *We*, the expression for stream functions and pressure gradient can be written as:

$$\Psi = \frac{-12(F + h_1 - h_2)}{(h_1 - h_2)^3 + 6\beta(h_1 - h_2)^2}$$

$$(-2y^3+3(h_1+h_2)\,y^2)+(\frac{6F\ (\beta(h_1-h_2)-h_1h_2)+h_1-h_2-(h_1-h_2)(h_1^{\ 2}+h_2^{\ 2}-h_1h_2)}{(h_1-h_2)^3+6\beta(h_1-h_2)^2})y+\frac{F}{2}-$$
$$(\frac{F(h_1^{\ 3}-3h_1^{\ 3}h_2+6\beta h_1(h_1-h_2))+h_1h_2((h_1-h_2)^2+3h_1(h_1-h_2))}{(h_1-h_2)^3+6\beta(h_1-h_2)^2})+We(A_5\frac{y^3}{3!}+A_6\frac{y^2}{2!}+A_7y+A_8+Ay^4)$$

(39)

$$\frac{\partial P}{\partial x}=\frac{\partial P_0}{\partial x}+We\frac{\partial P_1}{\partial x} \tag{40}$$

The non- dimensional pressure rise over one wavelength for the velocity is

$$\Delta P_\lambda=\int_0^1\frac{\partial P}{\partial x}dx \tag{41}$$

The frictional forces are given by

$$F_{\lambda 1}=\int_0^1-h_1(\frac{\partial p}{\partial x})dx \quad \text{at} \quad y=h_1 \quad ,F_{\lambda 2}=\int_0^1-h_2(\frac{\partial p}{\partial x})dx \quad \text{at} \quad y=h_2 \tag{42}$$

EXACT NUMERICAL SOLUTION FOR THE LARGE WEISSENBERG NUMBER REGIME

Let us present the solution to Eq.(18) in the large (We > 0) regime

Let $\phi=\frac{\partial^2\Psi}{\partial y^2}$,

From Eq.(18),we get

$$\frac{\partial^2}{\partial y^2}((1-n)\phi+nWe\phi^2)=0, \tag{43}$$

And upon integrating twice we obtain

$$(1-n)\phi+nWe\phi^2=k_1(x)y+k_2(x), \tag{44}$$

Where k_1 (x) and k_2 (x) are real valued functions of x but do not depending on y . Solving this algebraic equation for ϕ , we obtain

$$\phi=\frac{\partial^2\Psi}{\partial y^2}=\frac{(n-1)\pm\sqrt{(n-1)^2+4nWe(K_1(x)y+K_2(x))}}{2nWe} \tag{45}$$

$$\frac{\partial u}{\partial y}=\frac{(n-1)\pm\sqrt{(n-1)^2+4nWe(K_1(x)y+K_2(x))}}{2nWe} \tag{46}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{K_1(x)}{\sqrt{(n-1)^2+4nWe(K_1(x)y+K_2(x))}} \tag{47}$$

Physically , we expect $\partial u/\partial y$ to change sign and for $\frac{\partial^2 u}{\partial y^2}$ < 0. In order for this to occur, we must take the positive root in and consider only k_1 (x) <0.

In order to get real valued solutions, integrating Eq. (43) twice gives

$$\therefore \Psi = K_3(x)y + K_4(x) + \frac{(n-1)y^2}{4nWe} + \frac{\left[(n-1)^2+4nWe(K_1(x)y+K_2(x))\right]^{\frac{5}{2}}}{120We^3n^2K_1^2(x)} \tag{48}$$

The explicit formulas for the kj's (j= 1, 2, 3, 4) may be obtained from the four boundary conditions Eq. (22) and we find k_3 (x) and k_4 (x) explicity in terms of k_1 (x) and k_2 (x):

$$K_3(x)=\frac{F}{(h_1-h_2)}-\frac{(n-1)(h_1+h_2)}{4nWe}+\frac{\left[(n-1)^2+4nWe(K_1h_2+K_2)\right]^{\frac{5}{2}}-\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{5}{2}}}{120We^3n^2K_1^2(x)} \tag{49}$$

$$K_4(x)=-\frac{1}{2}\left[K_3(x)(h_1+h_2)+\frac{(n-1)(h_1^2+h_2^2)}{4nWe}+\frac{\left[(n-1)^2+4nWe(K_1h_2+K_2)\right]^{\frac{5}{2}}-\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{5}{2}}}{120We^3n^2K_1^2(x)}\right] \tag{50}$$

The relations for k_1 (x) and k_2 (x) are given in implicit form by substituting the above into

$$\frac{F}{(h_1-h_2)}-\frac{(n-1)(h_1+h_2)}{4nWe}+\frac{\left[(n-1)^2+4nWe(K_1h_2+K_2)\right]^{\frac{5}{2}}-\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{5}{2}}}{120We^3n^2K_1^2}$$
$$+\frac{\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{3}{2}}}{12We^2nK_1}+\frac{\beta\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{1}{2}}}{2We}=\frac{(1-n)(h_1+\beta)}{2nWe}-1 \tag{51}$$

$$\frac{F}{(h_1-h_2)}-\frac{(n-1)(h_1+h_2)}{4nWe}+\frac{\left[(n-1)^2+4nWe(K_1h_2+K_2)\right]^{\frac{5}{2}}-\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{5}{2}}}{120We^3n^2K_1^2}$$
$$+\frac{\left[(n-1)^2+4nWe(K_1h_1+K_2)\right]^{\frac{3}{2}}}{12We^2nK_1}-\frac{\beta\left[(n-1)^2+4nWe(K_1h_2+K_2)\right]^{\frac{1}{2}}}{2We}=\frac{(1-n)(h_2-\beta)}{2nWe}-1 \tag{52}$$

And these may be solved numerically for the kj's (j= 1, 2, 3, 4) given specific h_1 (x), h_2 (x) and F . Thus, we have an exact relation () for the stream function Ψ, and we may determine y, independent coefficients kj's (j= 1, 2, 3, 4) numerically from the relations . the velocity u is then given by

$$u = \frac{\partial \Psi}{\partial y} = K_3(x) + \frac{(n-1)y}{2nWe} + \frac{\left[(n-1)^2 + 4nWe(K_1(x)y + K_2(x))\right]^{\frac{3}{2}}}{12We^2 n\ K_1(x)} \quad (53)$$

The axial pressure gradient is given by

$$\frac{\partial P}{\partial x} = k_1(x) < 0, \quad (54)$$

The non-dimensional pressure rise over one wavelength is given by

$$\Delta P_\lambda = \int_0^1 k_1(x)dx < 0, \quad (55)$$

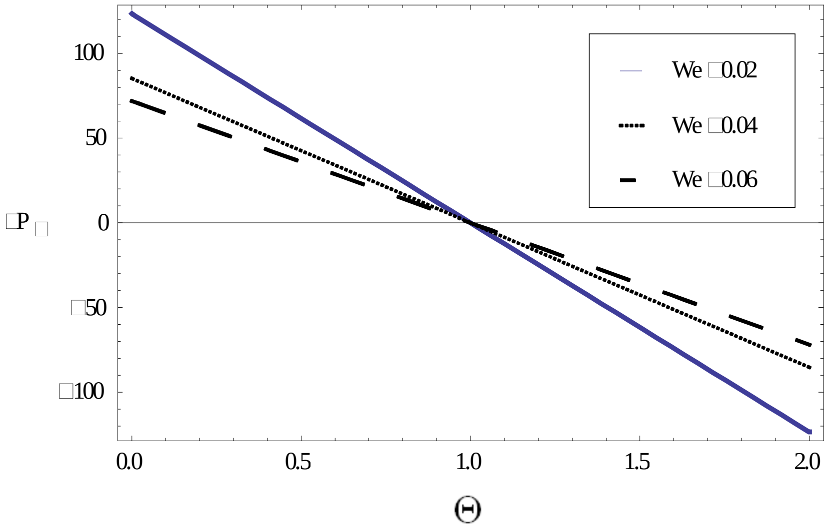


Fig.2. Variation of ΔP_λ with Θ for different values of *We*

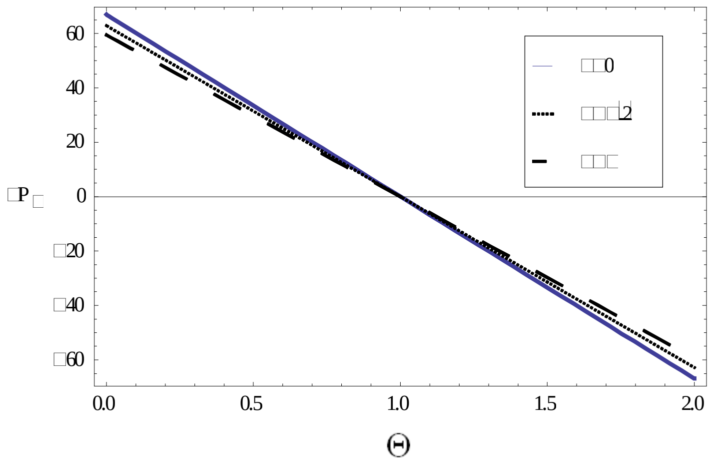


Fig.3. Variation of ΔP_λ with Θ for different values of ϕ

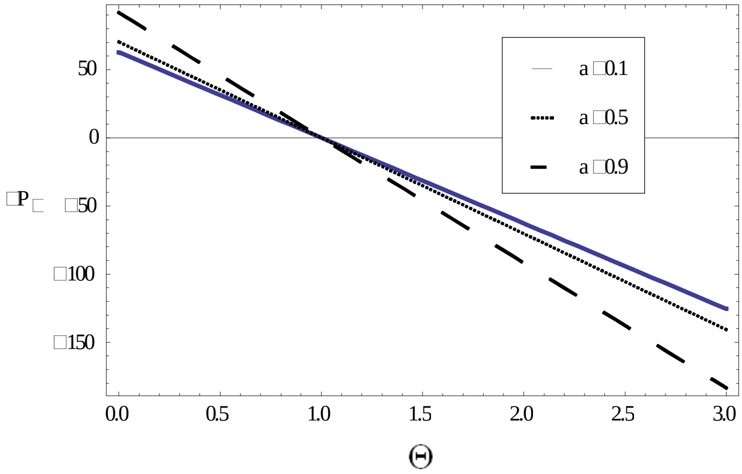


Fig.4. Variation of ΔP_{λ} with Θ for different values of a

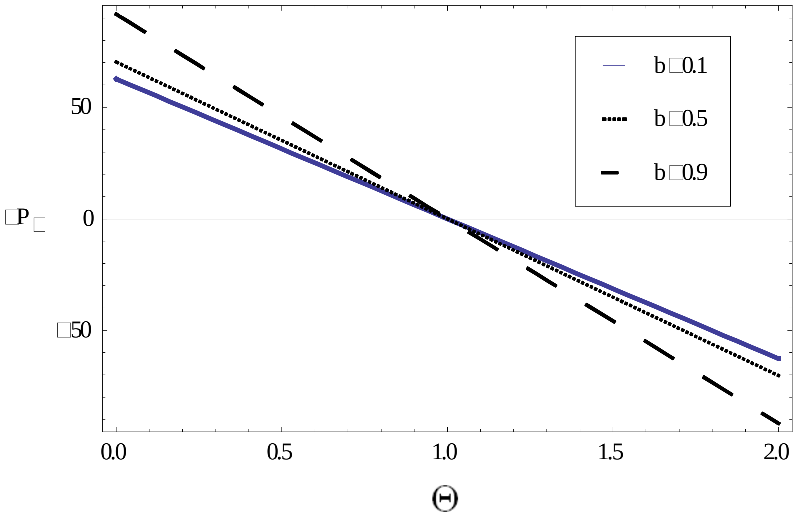


Fig.5. Variation of ΔP_{λ} with Θ for different values of b

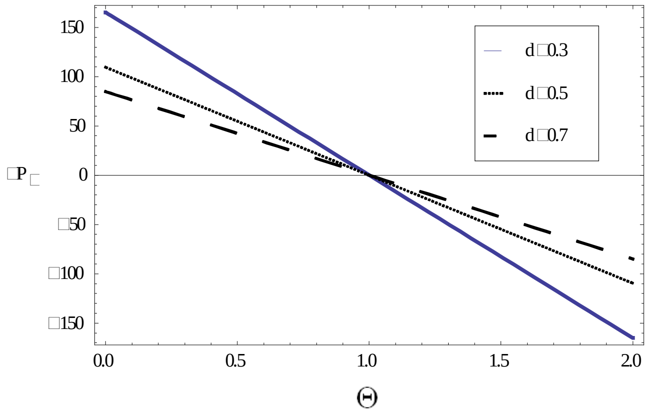


Fig.6. Variation of ΔP_{λ} with Θ for different values of d

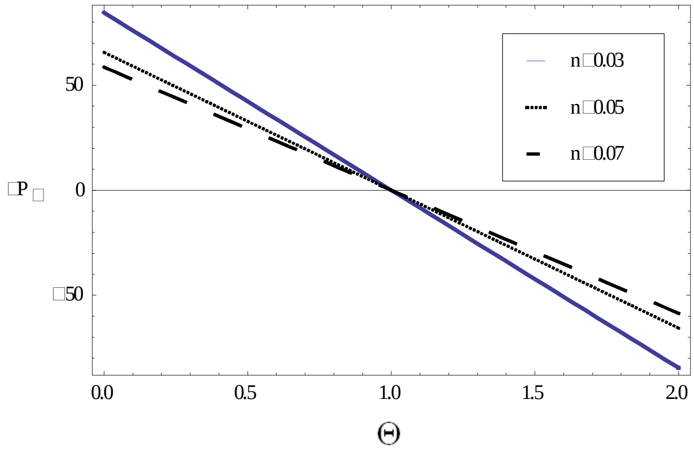


Fig.7. Variation of ΔP_{λ} with Θ for different values of n

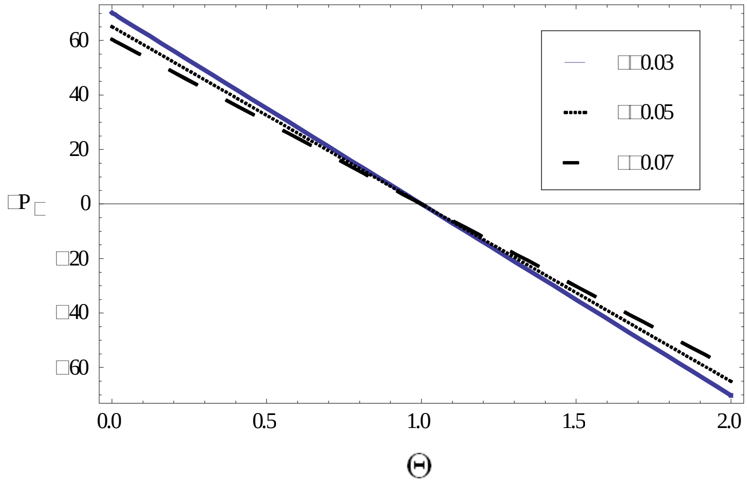


Fig.8. Variation of ΔP_{λ} with Θ for different values of β

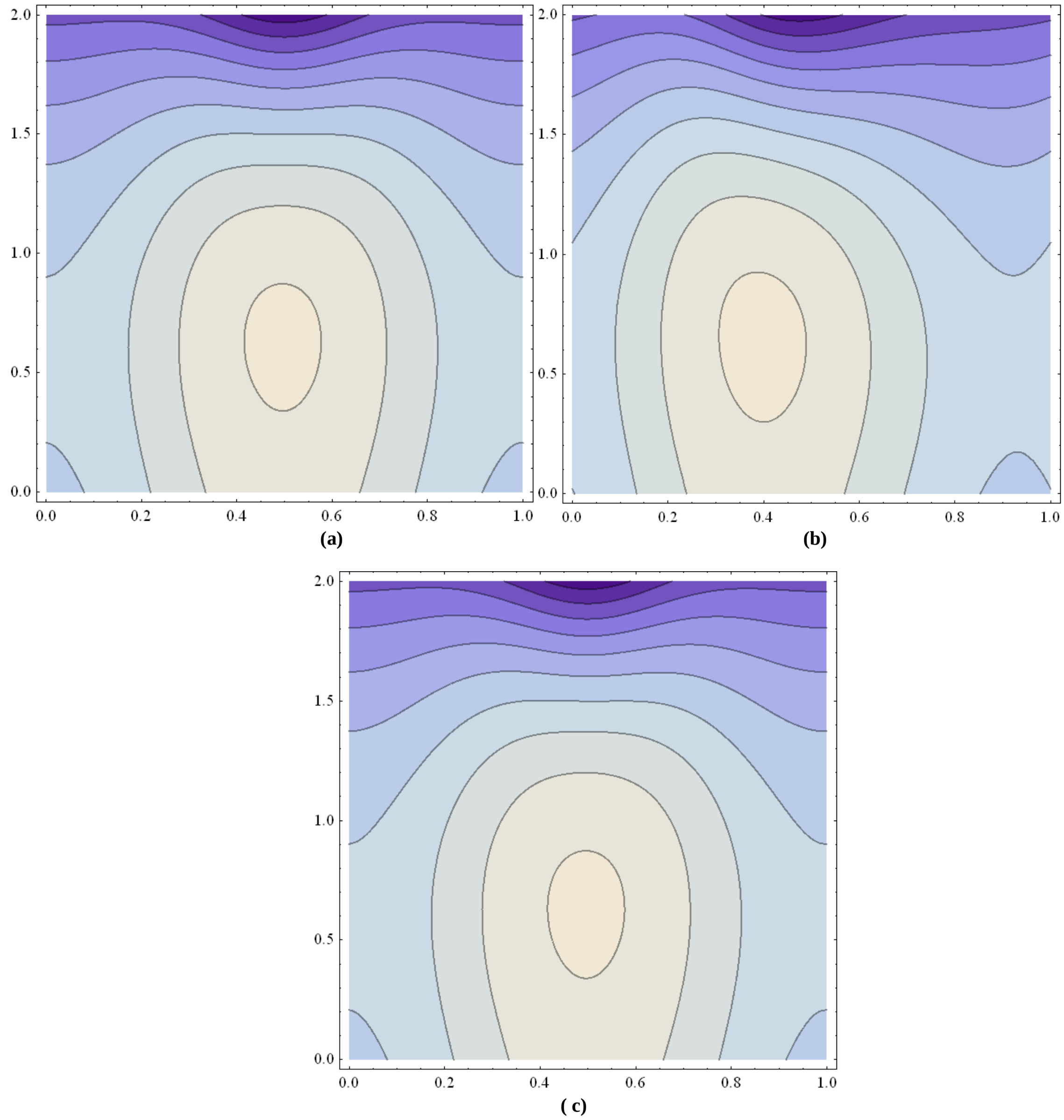
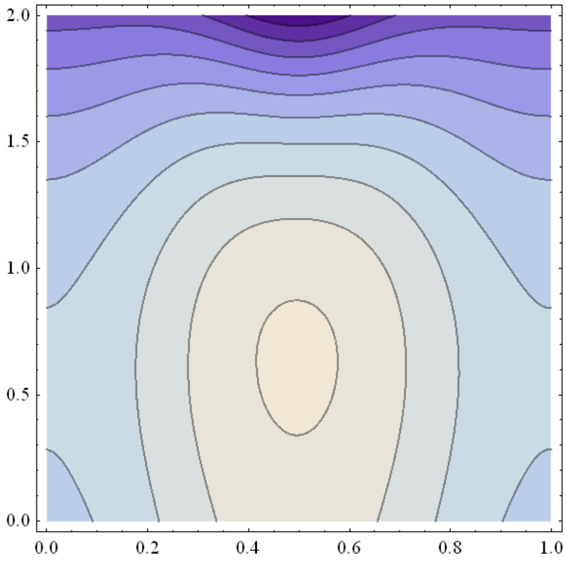
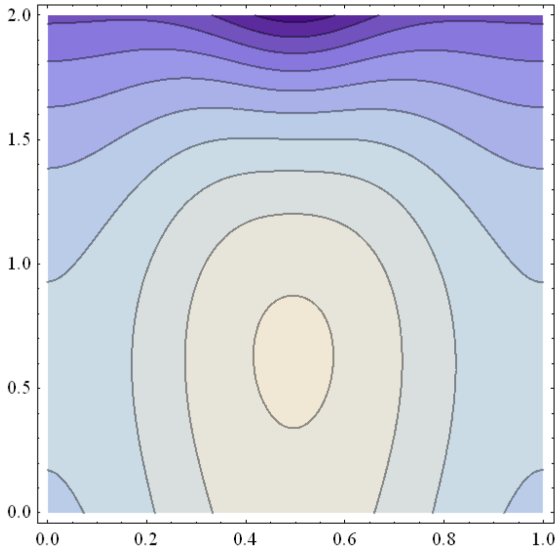


Fig. 9. Stream lines for three different values of *Q*. (a) for *Q* = 0.25, (b) for *Q* = 0.27, (c) for *Q* = 0.29





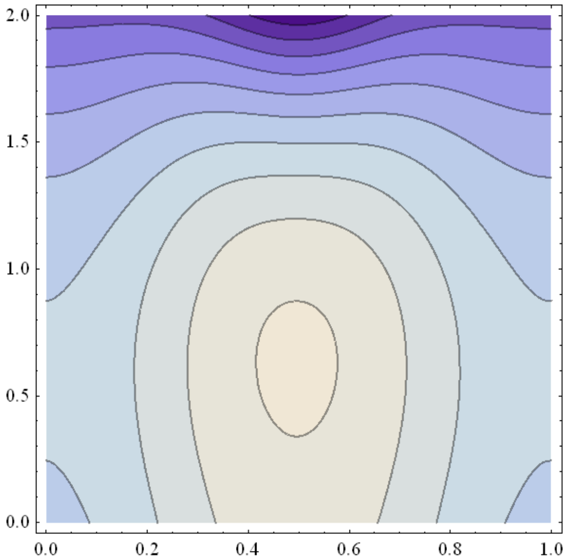
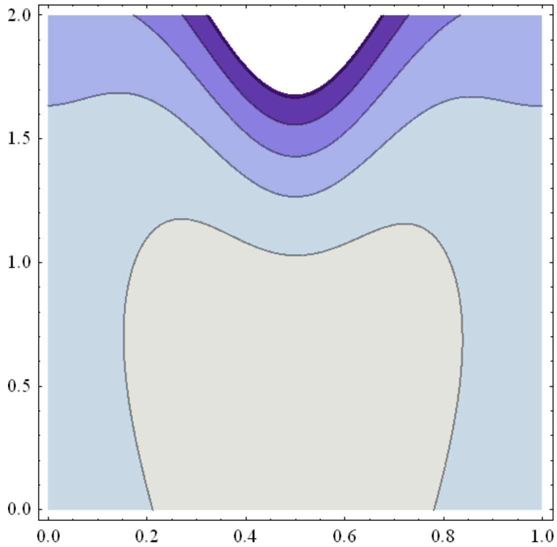
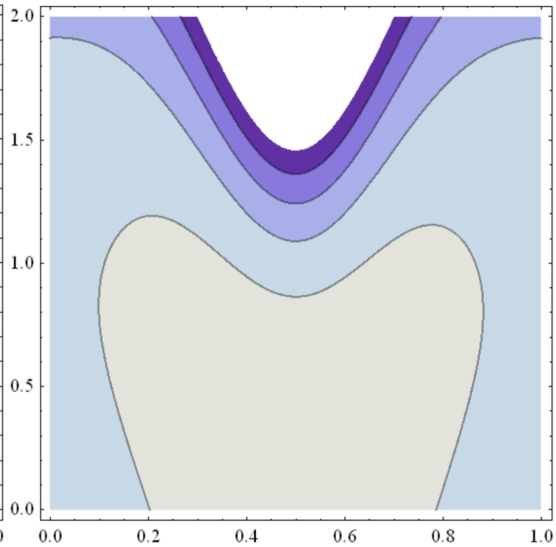


Fig. 10. Stream lines for three different values of We. (a) for We = 0.*04*, (b) for We = 0.06, (c) for We = 0.08


(a)


(b)

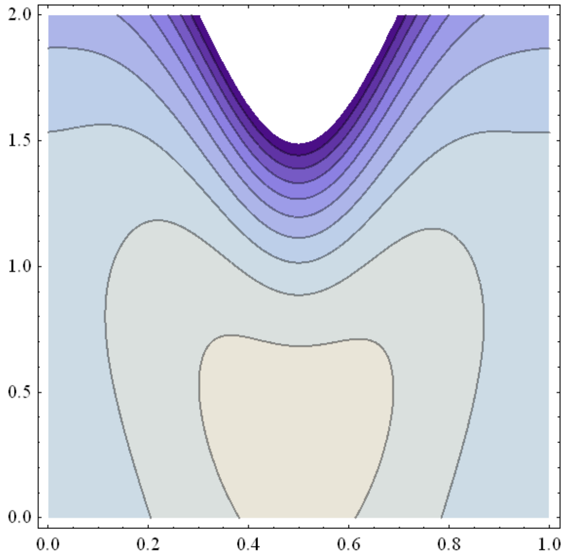

(c)

Fig. 11. Stream lines for three different values of a. (a) for a = 0.*42*, (b) for a = 0.*46*, (c) for a = 0.*48*

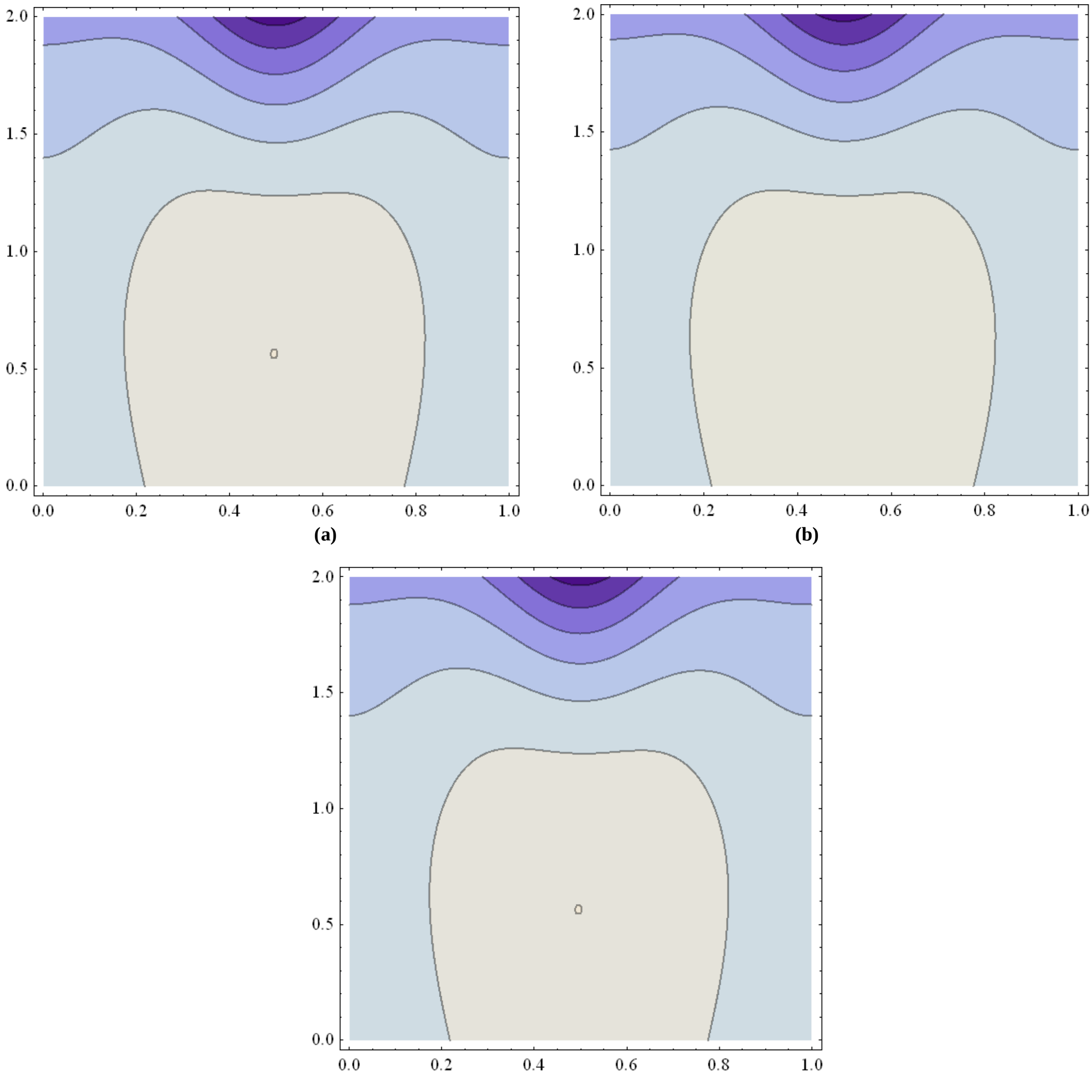


Fig. 12. Stream lines for three different values of b. (a) for b = 0.*52*, (b) for b = 0.*54*, (c) for b = 0.*56*

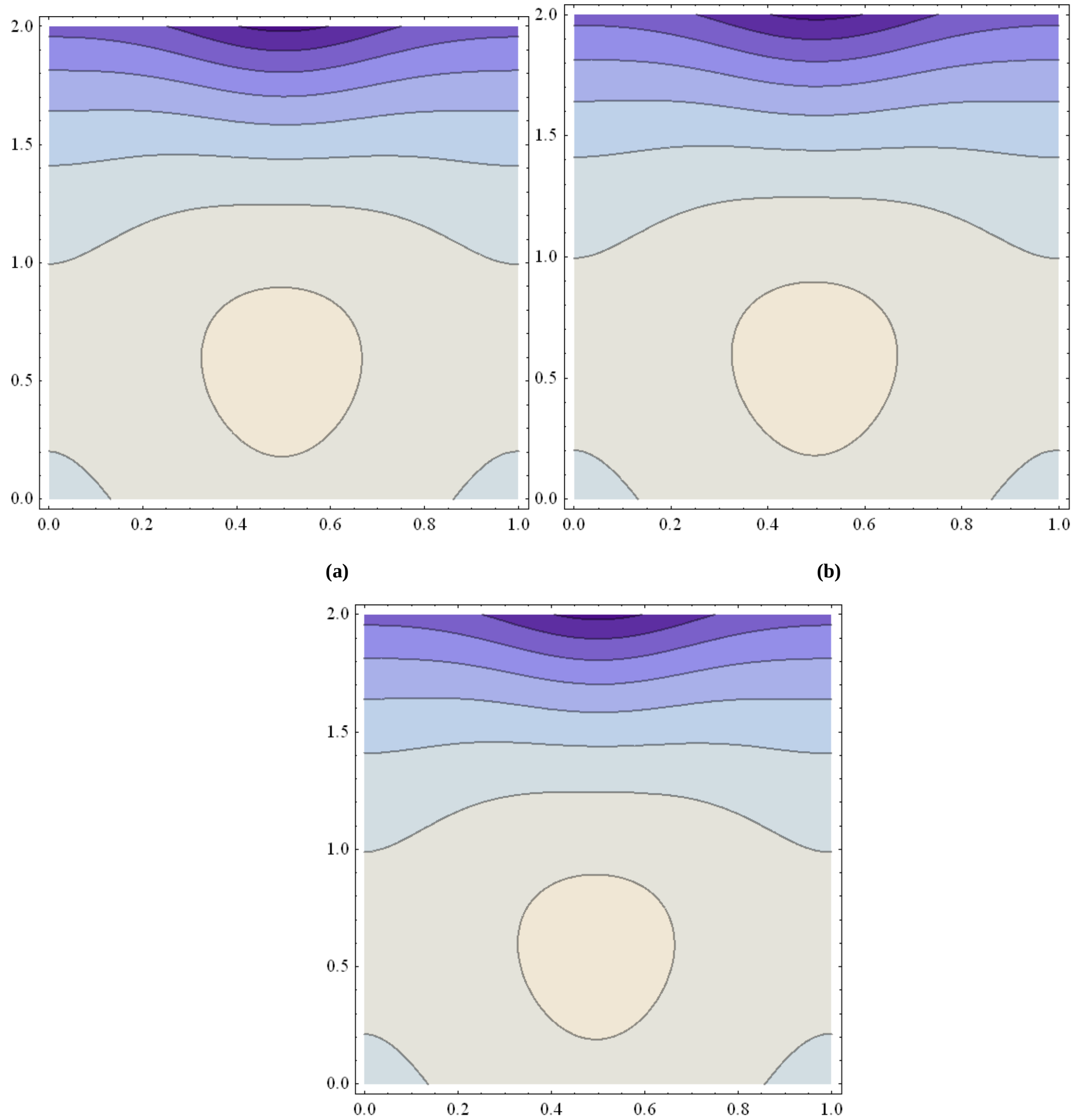


Fig. 13. Stream lines for three different values of β. (a) for β = 0.*04*, (b) for β = 0.*06*, (c) for β = 0.*08*

RESULTS AND DISCUSSION

The effect of Weissenberg parameter on the on the pressure rise is observed from Fig.2. It is observed that the pressure rise decreases with increasing values of Weisssenberg number in the pumping region $(\Delta P_\lambda > 0)$ and in the co- pumping region $(\Delta P_\lambda < 0)$.

From the Fig.3, the effect of amplitude ratio φ on pumping phenomena can studied. It is observed that for a given mean flow rate, the pressure rise decreases with an increases in φ.

From Figs. 4 and 5, the effect of wave amplitudes a and b on the pumping phenomena is studied. For a given Θ, the pressure rise decreases with the increase in *a* and *b* in all the pumping regions $(\Delta P_\lambda > 0, \Delta P_\lambda = 0, \Delta P_\lambda < 0)$.

From Fig.6, the effect of *d as* pumping phenomena is observed. It is noticed the pressure rise decreases with the increase in *d.*

The effect of permeability parameter on pressure rise is observed from Fig.8. For a fixed Θ, the pressure rise decreases with increasing permeability parameter.

TRAPPING PHENOMENA

A very interesting phenomenon in the peristaltic transport is trapping. In the wave frame, streamlines under certain circumstances swell to trap a bolus which travels as an inlet with the wave speed. Figures 9 – 13 illustrate the stream lines for different values of *Q*, *We*, a, b and β. The stream lines for different values of volume flow rate *Q* are shown in Figure 9. It is found that with the increase in volume flow rate *Q*, the size and the number of trapping bolus increases. In Figure 10 the stream lines are prepared for different value of Weissenberg number *We*. It is depicted that the size of the trapped bolus increases with the increase in *We*. It is observed from Figure 11 that the size and the number of the trapping bolus increases with the increases in amplitude of the wave *a*. It is observed from Figure 12 that the size and the number of the trapping bolus increases with the increases in amplitude of the wave *b*. It is observed from Figure 13 that the size and the number of the trapping bolus increases with the increases in amplitude of the wave β.

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