

Characterization of rotatory thermohaline instability in porous medium: Darcy Brinkman Model

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ABSTRACT

The present paper prescribes upper bounds for oscillatory motions of neutral or growing amplitude in rotatory thermohaline configurations of Veronis and Stern types in porous medium (Darcy-Brinkman model) in such a way that also result in sufficient conditions of stability for an initially bottom-heavy as well as initially top-heavy configuration.

Key words: Thermohaline instability, oscillatory motions, initially bottom-heavy configuration, initially top-heavy configuration, Porous medium.

INTRODUCTION

The hydrodynamic instability that manifests under appropriate conditions in a static horizontal initially homogeneous viscous and Boussinesq liquid layer of infinite horizontal extension and finite vertical depth which is kept under the simultaneous action of a uniform vertical temperature gradient and a gravitationally opposite uniform vertical concentration gradient in the force field of gravity is known as thermohaline instability. Thermohaline instability problem has been extensively investigated in the recent past on account of its interesting complexities as a double diffusive phenomenon (heat and salt) as well as its direct relevance in many problems of practical interest in the fields of oceanography, astrophysics, limnology and chemical engineering etc. For a broad view of the subject one may be referred to Turner [1] and Brandt and Fernando [2]. Two fundamental configurations have been studied in the context of thermohaline convection problem, one by Veronis [3], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing and another by Stern [4] wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing. The main results derived by Veronis and Stern for their respective problems are that instability might occur in the configuration through a stationary pattern of motions or oscillatory motions provided the destabilizing temperature gradient or the concentration gradient is sufficiently large but compatible with the conditions that the total density field is gravitationally stable. Thus thermohaline configurations of the Veronis and the Stern type can be classified into the following two classes:

1. The first class, in which thermohaline instability manifests itself when the total density field is initially bottom heavy, and
2. The second class, in which thermohaline instability manifests itself when the total density field is initially top heavy.

Banerjee et al. [5] derived a characterization theorem for thermohaline convection of the Veronis type that disallow the existence of oscillatory motions of neutral or growing amplitude in an initially bottom heavy configuration for the certain parameter regime.

The problem of thermohaline instability in porous media has attracted considerable interest during the past few decades because of its wide range of applications including the ground water contamination, disposal of waste

material, food processing, prediction of ground water movement in aquifers, the energy extraction process from the geothermal reservoirs, assessing the effectiveness of fibrous insulations etc.((Nield and Bejan [6]), (Straughan [7])).The thermohaline instability problem in porous media has been extensively investigated and the growing volume of work devoted to this area is well documented by Nield and Bezan [6] and Vafai [8]. Prakash and Vinod [9] have proved the nonexistence of nonoscillatory motions in thermohaline convection of Stern type in porous medium. In the early researches most of the researchers have studied double diffusive convection in porous medium by considering the Darcy flow model which is relevant to densely packed, low permeability porous medium. However, experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters indicate that most of the experimental data do not agree with the theoretical predictions based on the Darcy flow model. Hence, non-Darcy effects on double diffusive convection in porous media have received a great deal of attention in recent years. Poulikakos [10] has used the Brinkman extended Darcy flow model for the problem to investigate the linear stability analysis. Recently, Givler and Altobelli [11] have demonstrated that for high permeability porous media the effective viscosity is about ten times the fluid viscosity. Therefore, the effect of viscosities on the stability analysis is of practical interest. Thus in the present paper the Brinkman extended Darcy model has been used to investigate the thermohaline convection in porous medium.

The work of Banerjee et al. [5] have been extended to rotatory thermohaline instability problem in a porous medium by Prakash and Gupta [12] in the form of characterization theorems for rotatory thermohaline convection of Veronis type and Stern type that disapprove the existence of oscillatory motions of growing amplitude in initially bottom-heavy configurations of the two types respectively and left open the possibility for the derivation of the analogous theorems for an initially top-heavy configuration of the Veronis type and an initially top-heavy configuration of the Stern type.

Moreover, when compliment of the sufficient condition contained in the characterization theorems of Prakash and Gupta [12] holds good, oscillatory motions of growing amplitude can exist, and thus it is important to derive bounds for the complex growth rate of such motions when both the boundaries are not dynamically free, so that exact solutions in the closed form are not obtained. Thus present communication, which prescribes upper bounds for the oscillatory motions of neutral or growing amplitude in rotatory thermohaline configurations of Veronis and Stern types in porous medium in such a manner that also results in sufficient conditions for stability for an initially top heavy or initially bottom configuration, may be regarded as a further step in this scheme of extended investigations.

1 Mathematical formulation and analysis

An infinite horizontal porous layer filled with a viscous fluid is statically confined between two horizontal boundaries $z = 0$ and $z = d$ maintained at constant temperatures T_0 and T_1 and solute concentrations S_0 and S_1 at the lower and upper boundaries respectively, where $T_1 < T_0$ and $S_1 < S_0$ (as shown in Fig.1). The layer is rotating about its vertical axis with constant angular velocity $\vec{\omega}$. It is further assumed that the saturating fluid and the porous layer are incompressible and that the porous medium is a constant porosity medium. Darcy-Brinkman model has been used to investigate the present problem. Let the origin be taken on the lower boundary $z = 0$ with z-axis perpendicular to it.

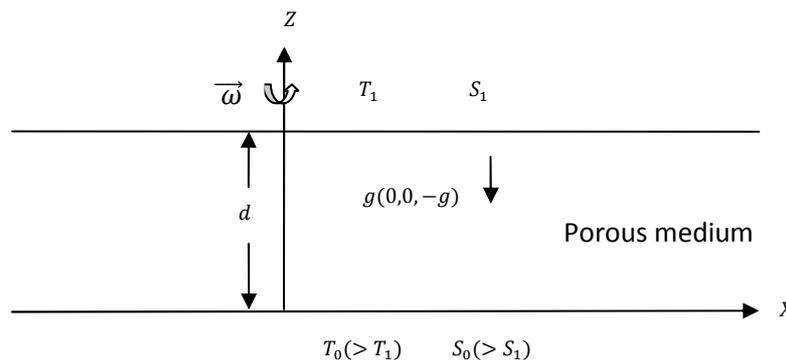


Fig.1 Physical Configuration

The governing hydrodynamic equations in the non dimensional form are given by (Prakash and Gupta [12]):

$$\Lambda(D^2 - a^2)^2 w - \left(\frac{\sigma}{\epsilon} + D_a^{-1}\right) (D^2 - a^2)w = Ra^2 \theta - R_s a^2 \phi + T_a^{1/2} D \zeta, \tag{1}$$

$$(D^2 - a^2 - E\sigma P_r) \theta = -w, \tag{2}$$

$$\left(D^2 - a^2 - \frac{E'\sigma P_r}{\tau}\right) \phi = -\frac{1}{\tau} w, \tag{3}$$

$$\left(\Lambda(D^2 - a^2) - \frac{\sigma}{\epsilon} - D_a^{-1}\right) \zeta = -T_a^{1/2} Dw, \tag{4}$$

with boundary condition

$$w = \theta = \phi = Dw = \zeta = 0 \quad \text{at } z = 0 \text{ and } z = 1 \text{ (both boundaries are rigid)} \tag{5}$$

Theorem 1:- If $(\sigma, w, \theta, \phi, \zeta)$, $(\sigma = \sigma_r + i\sigma_i)$, $\sigma_r \geq 0$, $\sigma_i \neq 0$ is a non-trivial solution of equations (1)-(4), together with boundary condition (5) and $R > 0$, $R_s > 0$ and $T_a \geq 0$, then

$$|\sigma| < \frac{\lambda R_s}{E P_r (4\pi^2(\Lambda + \delta) + D_a^{-1})} \sqrt{\Omega^2 - 1},$$

where $\Omega = \frac{\lambda R_s}{\left(\frac{27}{4}\pi^4(\Lambda + \delta) + 4\pi^2 D_a^{-1}\right)}$, $\delta = \min\left(\frac{\tau}{\epsilon E' P_r}, \Lambda\right)$ and $\lambda = \frac{R}{R_s}$

Proof: Multiply equation (1) by w^* throughout, integrating the resulting equation over vertical range of z and utilizing equations (2)-(4), we get

$$\begin{aligned} &\Lambda \int_0^1 w^* (D^2 - a^2)^2 w \, dz - \left(\frac{\sigma}{\epsilon} + D_a^{-1}\right) \int_0^1 w^* (D^2 - a^2) w \, dz + \\ &R a^2 \int_0^1 \theta (D^2 - a^2 - E P_r \sigma^*) \theta^* \, dz - R_s a^2 \tau \int_0^1 \phi \left(D^2 - a^2 - \frac{E' \sigma^* P_r}{\tau}\right) \phi^* \, dz - \int_0^1 \zeta \left(\Lambda(D^2 - a^2) - \frac{\sigma^*}{\epsilon} - D_a^{-1}\right) \zeta^* \, dz = 0. \end{aligned} \tag{6}$$

Integrate various terms of equation (6) for an appropriate number of times and using boundary conditions (5), we get

$$\begin{aligned} &\Lambda \int_0^1 (|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) \, dz + \left(\frac{\sigma}{\epsilon} + D_a^{-1}\right) \int_0^1 (|Dw|^2 + a^2 |w|^2) \, dz - R a^2 \int_0^1 (|D\theta|^2 + a^2 |\theta|^2) \, dz + \\ &R_s a^2 \tau \int_0^1 (|D\phi|^2 + a^2 |\phi|^2) \, dz - R a^2 E \sigma^* P_r \int_0^1 |\theta|^2 \, dz + R_s a^2 E' \sigma^* P_r \int_0^1 |\phi|^2 \, dz + \Lambda \int_0^1 (|D\zeta|^2 + a^2 |\zeta|^2) \, dz + \left(\frac{\sigma^*}{\epsilon} + D_a^{-1}\right) \int_0^1 |\zeta|^2 \, dz = 0. \end{aligned} \tag{7}$$

Equating real and imaginary part of (7) equal to zero and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we have

$$\begin{aligned} &\Lambda \int_0^1 (|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) \, dz + \left(\frac{\sigma_r}{\epsilon} + D_a^{-1}\right) \int_0^1 (|Dw|^2 + a^2 |w|^2) \, dz - R a^2 \int_0^1 (|D\theta|^2 + a^2 |\theta|^2) \, dz - \\ &R a^2 E P_r \sigma_r \int_0^1 |\theta|^2 \, dz + R_s a^2 \tau \int_0^1 (|D\phi|^2 + a^2 |\phi|^2) \, dz + R_s a^2 E' P_r \sigma_r \int_0^1 |\phi|^2 \, dz + \Lambda \int_0^1 (|D\zeta|^2 + a^2 |\zeta|^2) \, dz + \left(\frac{\sigma_r}{\epsilon} + D_a^{-1}\right) \int_0^1 |\zeta|^2 \, dz = 0 \end{aligned} \tag{8}$$

and

$$\frac{1}{\epsilon} \int_0^1 (|Dw|^2 + a^2 |w|^2) \, dz + R a^2 E P_r \int_0^1 |\theta|^2 \, dz - R_s a^2 E' P_r \int_0^1 |\phi|^2 \, dz - \frac{1}{\epsilon} \int_0^1 |\zeta|^2 \, dz = 0. \tag{9}$$

Multiplying equation (9) by σ_r and adding resulting equation to equation (8), we get

$$\begin{aligned} &\Lambda \int_0^1 (|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) \, dz + \left(\frac{2\sigma_r}{\epsilon} + D_a^{-1}\right) \int_0^1 (|Dw|^2 + a^2 |w|^2) \, dz - \\ &R a^2 \int_0^1 (|D\theta|^2 + a^2 |\theta|^2) \, dz + R_s a^2 \tau \int_0^1 (|D\phi|^2 + a^2 |\phi|^2) \, dz + \Lambda \int_0^1 (|D\zeta|^2 + a^2 |\zeta|^2) \, dz + D_a^{-1} \int_0^1 |\zeta|^2 \, dz = 0. \end{aligned} \tag{10}$$

Also, since w, θ, ϕ, ζ vanish at $z = 0$ and $z = 1$, the Rayleigh-Ritz inequality (Schultz [13]) gives

$$\int_0^1 |Dw|^2 \, dz \geq \pi^2 \int_0^1 |w|^2 \, dz, \tag{11}$$

$$\int_0^1 |D\theta|^2 \, dz \geq \pi^2 \int_0^1 |\theta|^2 \, dz, \tag{12}$$

$$\int_0^1 |D\phi|^2 \, dz \geq \pi^2 \int_0^1 |\phi|^2 \, dz, \tag{13}$$

$$\int_0^1 |D\zeta|^2 \, dz \geq \pi^2 \int_0^1 |\zeta|^2 \, dz, \tag{14}$$

Furthermore, utilizing the Schwartz inequality, we have

$$\left(\int_0^1 |w|^2 \, dz\right)^{\frac{1}{2}} \left(\int_0^1 |D^2 w|^2 \, dz\right)^{\frac{1}{2}} \geq \left| -\int_0^1 w^* D^2 w \, dz \right| = \int_0^1 |Dw|^2 \, dz \geq \pi^2 \int_0^1 |w|^2 \, dz.$$

Consequently, $\int_0^1 |D^2 w|^2 \, dz \geq \pi^4 \int_0^1 |w|^2 \, dz$, (15)

and thus

$$\int_0^1 |(D^2 - a^2)w|^2 dz = \int_0^1 (|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) dz \geq (\pi^2 + a^2)^2 \int_0^1 |w|^2 dz. \tag{16}$$

Further,

$$\begin{aligned} \int_0^1 |w|^2 dz &= \int_0^1 w w^* dz = \int_0^1 (D^2 - a^2 - E\sigma P_r)\theta (D^2 - a^2 - E\sigma^* P_r)\theta^* dz \\ &= \int_0^1 |(D^2 - a^2)\theta|^2 dz + 2E P_r \sigma_r \int_0^1 (|D\theta|^2 + a^2 |\theta|^2) dz + E^2 |\sigma|^2 P_r^2 \int_0^1 |\theta|^2 dz. \end{aligned} \tag{17}$$

Since $\sigma_r \geq 0$, it follows from inequality (17) that

$$\int_0^1 |w|^2 dz \geq \int_0^1 |(D^2 - a^2)\theta|^2 dz + E^2 |\sigma|^2 P_r^2 \int_0^1 |\theta|^2 dz \tag{18}$$

and

$$\int_0^1 |w|^2 dz \geq \int_0^1 (D^2 - a^2)\theta|^2 dz. \tag{19}$$

Also, emulating the derivation of inequality (15), we have

$$\int_0^1 (D^2 - a^2)\theta|^2 dz = \int_0^1 (|D^2 \theta|^2 + 2a^2 |D\theta|^2 + a^4 |\theta|^2) dz \geq (\pi^2 + a^2)^2 \int_0^1 |\theta|^2 dz. \tag{20}$$

Combining inequality (18) and (20), we obtain

$$\int_0^1 |w|^2 dz \geq \{(\pi^2 + a^2)^2 + E^2 |\sigma|^2 P_r^2\} \int_0^1 |\theta|^2 dz. \tag{21}$$

Further,

$$\begin{aligned} \int_0^1 |w|^2 dz &= \left(\int_0^1 |w|^2 dz\right)^{\frac{1}{2}} \left(\int_0^1 |w|^2 dz\right)^{\frac{1}{2}} \\ &\geq \{(\pi^2 + a^2)^2 + E^2 |\sigma|^2 P_r^2\}^{\frac{1}{2}} \left(\int_0^1 |\theta|^2 dz\right)^{\frac{1}{2}} \left(\int_0^1 |(D^2 - a^2)\theta|^2 dz\right)^{\frac{1}{2}} \\ &\geq (\pi^2 + a^2) \left\{1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2 + a^2)^2}\right\}^{\frac{1}{2}} \left| \int_0^1 \theta^* (D^2 - a^2)\theta dz \right| \text{ (Using Schwartz inequality)} \\ &= (\pi^2 + a^2) \left\{1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2 + a^2)^2}\right\}^{\frac{1}{2}} \int_0^1 (|D\theta|^2 + a^2 |\theta|^2) dz. \end{aligned} \tag{22}$$

Using inequalities (11),(13),(14),(16),(22) in equation (10) and utilizing the fact $\sigma_r \geq 0$, we have

$$\begin{aligned} \{\Lambda(\pi^2 + a^2)^2 + (\pi^2 + a^2)D_a^{-1}\} \int_0^1 |w|^2 dz + R_s a^2 \tau (\pi^2 + a^2) \int_0^1 |\phi|^2 dz + \{\Lambda(\pi^2 + a^2) + D_a^{-1}\} \int_0^1 |\zeta|^2 dz < \\ \frac{Ra^2}{(\pi^2 + a^2)} \left\{1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2 + a^2)^2}\right\}^{\frac{1}{2}} \int_0^1 |w|^2 dz. \end{aligned} \tag{23}$$

Equation (9) upon using inequality (11) implies

$$R_s a^2 \int_0^1 |\phi|^2 dz > \frac{(\pi^2 + a^2)}{\epsilon E' P_r} \int_0^1 |w|^2 dz - \frac{1}{\epsilon E' P_r} \int_0^1 |\zeta|^2 dz, \tag{24}$$

and

$$\int_0^1 |\zeta|^2 dz > (\pi^2 + a^2) \int_0^1 |w|^2 dz - R_s a^2 E' P_r \int_0^1 |\phi|^2 dz. \tag{25}$$

Inequality (23) coupled with each of inequality (24) and (25) yield the following inequalities

$$\begin{aligned} \left\{(\pi^2 + a^2)^2 \left(\Lambda + \frac{\tau}{\epsilon E' P_r}\right) + (\pi^2 + a^2)D_a^{-1}\right\} \int_0^1 |w|^2 dz + \left\{(\pi^2 + a^2)\Lambda + D_a^{-1} - \frac{(\pi^2 + a^2)\tau}{\epsilon E' P_r}\right\} \int_0^1 |\zeta|^2 dz < \frac{Ra^2}{(\pi^2 + a^2)} \left\{1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2 + a^2)^2}\right\}^{\frac{1}{2}} \int_0^1 |w|^2 dz \end{aligned} \tag{26}$$

and

$$\begin{aligned} 2\{\Lambda(\pi^2 + a^2)^2 + (\pi^2 + a^2)D_a^{-1}\} \int_0^1 |w|^2 dz + R_s a^2 \epsilon E' P_r \left[\frac{(\pi^2 + a^2)\tau}{\epsilon E' P_r} - \{(\pi^2 + a^2)\Lambda + D_a^{-1}\}\right] \int_0^1 |\phi|^2 dz < \\ \frac{Ra^2}{(\pi^2 + a^2)} \left\{1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2 + a^2)^2}\right\}^{\frac{1}{2}} \int_0^1 |w|^2 dz. \end{aligned} \tag{27}$$

Now if $\delta = \min\left(\frac{\tau}{\epsilon E' P_r}, \Lambda\right)$, then depending on the value of δ exactly one of the inequalities (26)-(27) will imply that

$$\{(\pi^2 + a^2)^2 (\Lambda + \delta) + (\pi^2 + a^2)D_a^{-1}\} \int_0^1 |w|^2 dz < \frac{Ra^2}{(\pi^2 + a^2)} \left\{1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2 + a^2)^2}\right\}^{\frac{1}{2}} \int_0^1 |w|^2 dz. \tag{28}$$

Since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ is $4\pi^2$ (for $a^2 = \pi^2$) and the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ is $\frac{27\pi^4}{4}$ (for $a^2 = \frac{\pi^2}{2}$), therefore it follows from inequality (28) that

$$\left\{ \frac{27}{4} \pi^4 (\Lambda + \delta) + 4\pi^2 D_a^{-1} \right\} \left\{ 1 + \frac{E^2 |\sigma|^2 P_r^2}{(\pi^2+a^2)^2} \right\}^{\frac{1}{2}} < R = \lambda R_s. \tag{29}$$

Inequality (29) implies that

$$|\sigma| < \frac{(\pi^2+a^2)}{EP_r} \sqrt{\Omega^2 - 1}, \tag{30}$$

where $\Omega = \frac{\lambda R_s}{\left(\frac{27}{4} \pi^4 (\Lambda + \delta) + 4\pi^2 D_a^{-1}\right)}$

Further it follows from inequality (28) that

$$(\pi^2 + a^2) \left\{ \frac{(\pi^2+a^2)^2}{a^2} (\Lambda + \delta) + D_a^{-1} \right\} < R = \lambda R_s. \tag{31}$$

Since the minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ is $4\pi^2$ (for $a^2 = \pi^2$), therefore it follows from inequality (31) that

$$(\pi^2 + a^2) < \frac{\lambda R_s}{\{4\pi^2 (\Lambda + \delta) + D_a^{-1}\}}. \tag{32}$$

Combining inequalities (30) and (32), we get

$$|\sigma| < \frac{\lambda R_s}{EP_r(4\pi^2(\Lambda+\delta)+D_a^{-1})} \sqrt{\Omega^2 - 1},$$

which complete the proof of the theorem

Theorem 1, from the physical point of view of hydrodynamic stability theory, may be stated as: the complex growth rate $\sigma = \sigma_r + i\sigma_i$ of an arbitrary oscillatory perturbation ($\sigma_i \neq 0$) of neutral or growing amplitude ($\sigma_r \geq 0$) in rotatory thermohaline convection in porous medium of Veronis' type lies inside a semicircle in the right half of the $\sigma_r\sigma_i$ -plane whose center is at the origin and radius equals $\frac{\lambda R_s}{EP_r(4\pi^2(\Lambda+\delta)+D_a^{-1})} \sqrt{\Omega^2 - 1}$.

This result is uniformly valid for an initially top heavy ($\lambda > 1$) as well as an initially bottom heavy ($\lambda < 1$) configuration.

Corollary 1: If $(\sigma, w, \theta, \phi, h_z), \sigma = \sigma_r + i\sigma_i, \sigma_i \neq 0$ is a non trivial solution of equations (1) to (4), together with boundary condition (5) and $R > 0, R_s > 0, T_a > 0$ and

$$\lambda < \frac{\frac{27}{4} \pi^4 (\Lambda + \delta) + 4\pi^2 D_a^{-1}}{R_s} \text{ then } \sigma_r < 0.$$

Proof: Follows from Theorem 1

Corollary 1 implies that oscillatory motions of growing amplitude are not allowed in rotatory thermohaline convection in porous medium of Veronis' type if the initial stability parameter λ does not exceed the value $\frac{\frac{27}{4} \pi^4 (\Lambda + \delta) + 4\pi^2 D_a^{-1}}{R_s}$. Further this result is uniformly valid for an initially top heavy ($\lambda > 1$) as well as an initially bottom heavy ($\lambda < 1$) configuration.

Remarks: The following remarks, now deserve attention

1. If $0 < R \leq R_s \leq \frac{27}{4} \pi^4 (\Lambda + \delta) + 4\pi^2 D_a^{-1}$ and $\sigma_i \neq 0$ even then Corollary 1 implies that $\sigma_r < 0$. This result is the characterization theorem of Prakash and Gupta [12], which we see is built into our characterization Corollary 1.
2. If $0 < R \leq \frac{27}{4} \pi^4 (\Lambda + \delta) + 4\pi^2 D_a^{-1} < R_s$ and $\sigma_i \neq 0$, even then Corollary 1 implies that $\sigma_r < 0$, a new result that obviously cannot be averred from characterization theorem of Prakash and Gupta [12].

Theorem 2: If $(\sigma, w, \theta, \phi, h_z), \sigma = \sigma_r + i\sigma_i, \sigma_r \geq 0, \sigma_i \neq 0$ is a non-trivial solution of equations (1)-(4), together with boundary condition (5) and $R < 0, R_s < 0, T_a > 0$ then

$$|\sigma| < \frac{\bar{\lambda} |R|}{E' P_r (4\pi^2(\Lambda + \delta) + D_a^{-1})} \sqrt{\Omega^2 - 1}$$

where $\bar{\Omega} = \frac{\bar{\lambda}|R|}{\tau\left\{\frac{27}{4}\pi^4(\Lambda+\bar{\delta})+4\pi^2D_a^{-1}\right\}}$, $\bar{\delta} = \min\left(\frac{1}{\epsilon EP_r}, \Lambda\right)$ and $\bar{\lambda} = \frac{|R_s|}{|R|}$

Proof: Replace R with $-|R|$ and R_s with $-|R_s|$ in equation (1) and adopting a similar procedure used in proving Theorem 1, we obtain the desired result.

Corollary 2: If $(\sigma, w, \theta, \phi, h_z), \sigma = \sigma_r + i\sigma_i, \sigma_i \neq 0$ is a non trivial solution of equations (1) to (4), together with the boundary condition (5) and $R < 0, R_s < 0, T_a > 0$ and $\bar{\lambda} < \frac{\tau\left\{\frac{27}{4}\pi^4(\Lambda+\bar{\delta})+4\pi^2D_a^{-1}\right\}}{|R|}$ then $\sigma_r < 0$.

Proof: Follows from theorem 2

Corollary 2 implies that oscillatory motions of growing amplitude are not allowed in rotatory thermohaline convection in porous medium of Stern type if the initial stability parameter λ does not exceed the value $\frac{\tau\left\{\frac{27}{4}\pi^4(\Lambda+\bar{\delta})+4\pi^2D_a^{-1}\right\}}{|R|}$. Further this result is uniformly valid for an initially top heavy ($\lambda > 1$) as well as an initially bottom heavy ($\lambda < 1$) configuration. We also have

3. If $0 < |R_s| \leq |R| \leq \tau\left\{\frac{27}{4}\pi^4(\Lambda+\bar{\delta})+4\pi^2D_a^{-1}\right\}$ and $\sigma_i \neq 0$ then $\sigma_r < 0$, a more general result than that following from theorem 2 of Prakash and Gupta [12] for the present problem.

4. If $0 < |R_s| \leq \tau\left\{\frac{27}{4}\pi^4(\Lambda+\bar{\delta})+4\pi^2D_a^{-1}\right\} < |R|$ and $\sigma_i \neq 0$, then $\sigma_r < 0$, a new result that obviously cannot be averred from characterization theorem 2 of Prakash and Gupta [12].

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