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The Comparison between Gumbel and Exponentiated Gumbel Distributions and their Applications in Hydrological Process

Anita Abdollahi Nanvapisheh*

Department of Statistics, Islamic Azad University, Tehran north branch, Tehran, Iran

***Corresponding author:** Anita Abdollahi Nanvapisheh, Department of Statistics, Islamic Azad University, Tehran north branch, Tehran, Iran, E-mail: anita.abdollahi@yahoo.com

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Abstract

The Exponentiated Gumbel (EG) distribution has been proposed to capture some aspects of the data that the Gumbel distribution fails to specify. It has an increasing hazard rate. The Exponentiated gumbel distribution has applications in hydrology, meteorology, climatology, insurance, finance and geology, among many others. In this paper Firstly, the mathematical and statistical characteristics of the gumbel and Exponentiated gumbel distribution are presented, then the applications of this distributions are studied using the real data set. Its first moment about origin and moments about mean have been obtained and expressions for skewness, kurtosis has been given. Estimation of its parameter has been discussed using the method of maximum likelihood. In the end, two applications of the gumbel and exponentiated gumbel distribution have been discussed with two real lifetime data sets. The results also confirmed the suitability of the Exponentiated gumbel distribution for real data collection.

Keywords: Gumbel distribution; Exponentiated gumbel (EG); Parameter estimation; Hydrological data

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Introduction

Gumbel presents a new model the (Gumbel model) as an extension of the exponential distribution. It is well known that this distribution presents a hazard function that is increasing and decreasing depending on the parameter values [1]. Moreover, another feature of the distribution is that it can be used to fitting extreme data sets. The Gumbel distribution and extensions have being applied to different areas of scientific knowledge such as hydrology, meteorology, climatology, insurance, finance and geology, among many others. Gumbel uses the model to fit extreme values of random data. The book by Kotz and Nadarajah deals with the Gumbel distribution with a view of applying it to data sets raging from wind speed to flooding data [2]. The cdf of Gumbel distribution is Another exponentiated.

The purpose of this paper is the comparison between gumbel and exponentiated gumbel distributions in hydrological models. We show that the exponentiated gumbel distributions in the same conditions work better than other distributions for modeling environmental data. The exponentiated gumbel distribution and its properties is introduced in Section2. The moments and incomplete moments of the exponentiated gumbel distribution are given in Section 3. Section 4 deals with the Maximum likelihood (ML) estimation of the unknown parameters. We provide two real data applications in Section 5. The paper ends with some concluding remarks.

Materials and Methods

The Exponentiated gumbel distribution and its properties: A random variable X has the EG distribution with two parameters if its cumulative distribution function (cdf). The pdfs of the EG distribution are plotted in Figure 1 for some selected values of parameters [3]. It can be seen that the pdf of this distribution is decreasing depending on the parameter values.

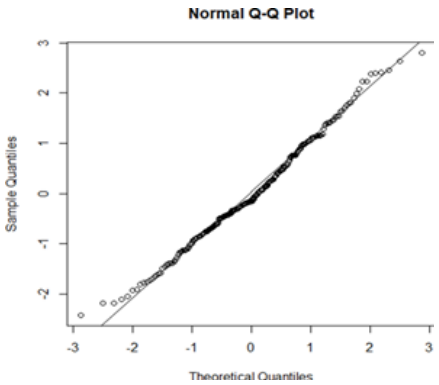

Figure 1: Normal Q-Q plot and a boxplot of the snowfall in inches.

The moments and incomplete moments: Moments are necessary and important in any statistical analysis, especially in applications. It can

be used to study the most important features and characteristics of a distribution [4]. In this section, we present complete and incomplete moments of the EG distribution. But first, we present an expansion for f(x) in order to obtain expressions for the moments. Using the exponential expansion, we have: So, the moment generating functions of this distribution using [5, 6] is obtained.

Maximum likelihood estimation: Let 1 be a random sample of size n from the EG distribution. Then, the likelihood function the maximum likelihood estimates of the parameters may be obtained by maximizing the log-likelihood function with respect to the parameters. To this end, we take the derivatives of the log-likelihood function with respect to the parameters and then equate the results with zero [7]. Therefore, the maximum likelihood estimates of denoted by respectively, are obtained by solving the following nonlinear equations simultaneously (, 4).

Under the regularity conditions, the asymptotic inference for the vector of parameters, based on normal approximation can be used. When the sample size n is large enough, is asymptotically a two-variety normal random vector with mean and the variance-covariance matrix that equates to the inverse of the expected Fisher information matrix [8]. This asymmetric behavior holds if we replace matrix, then they can be replaced by their respective maximum likelihood estimates [9]. Using the normal approximation, we can obtain approximate (asymptotic) confidence intervals for the parameters.

Applications: In this section, we present applications of EG distribution using real data. These applications demonstrate the flexibility of this distribution compared to the other models for the real data set. We compare the fit of the EG distribution with those of some other lifetime distributions which are Weibull Lomax, Exponential Lomax and Rayleigh Lomax distributions. All the computations presented in this section were done using the MATLAB and R software.

The first data set analyzed in this application is related to the snow accumulation in inches in the Raleigh-Durham airport, North Carolina [10]. The data set involve 63 observations and are listed next:

The second data set analyzed corresponds to 264 observations of the maximum of monthly wind speed (mph) in Palm Beach, Florida (USA) for the months January, 1914 to December, 2005. Data is available for downloading at the site http://www.ncdc.noaa.gov. For the sake of completeness are presented in the following:

Result and Discussion

Anderson-Darling (ADR) test statistics, the Akaike information criterion (AIC) and the Bayesian criterion (BIC) in order to compare the fits. The computed MLEs, ADR and the values of AIC and BIC for both data sets given. These criteria are widely utilized to check how closely a specified cdf fits the empirical distribution of a given data set. It is well-known that the smaller values of AIC, BIC and ADR test statistic mean a better fit to the data. Here, it is observed from that the E-G model outperforms all the other considered models in the sense of the considered criteria. For the sake of visual comparison, the estimated pdfs of the considered distributions as well as the empirical histograms of the data sets. It is obvious from the figure, that the exponentiated gumbel distribution provides the best fit for real data set in comparison with the other considered distributions.

Conclusion

In this paper, the gumbel and the exponentiated gumbel was introduced and some of its math properties were discussed. This distribution has two parameters and its cdf and hrf have simple forms. The hrf of the exponentiated gumbel distribution can be increasing shaped depending on the values of the parameters. Generally, we can say that the Exponentiated gumbel distribution provides a more flexible model for fitting a wide range of real data sets in comparison with some other distributions and thus it can be an appropriate alternative distribution for some other existing models, in modeling real data that may appear in many areas like engineering, survival analysis, hydrology, economics and so on.

Several properties of this distribution, like sequential statistics, sequential statistical moments, and the asymptotic distribution of extreme values, stochastic orderings and distribution of the ratio of two random variables have not been presented in this paper. In addition, there exist some inferential topics related to the exponentiated gumbel distribution like Bayesian estimation of the parameters, estimation based on censored samples, estimation by means of other methods like the diagonally weighted least-squares method, prediction of the future observations from this distribution and so on. We hope to work on some of the mentioned topics and report our findings in future.

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