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Some properties of copper-nickel at different % of Ni

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ABSTRACT

The decomposition of the elastic constant tensor into its irreducible parts is given. The norm concept of elastic constant tensor and its irreducible parts and their ratios are used to study the anisotropy of copper – nickel at different % of Ni, and the relationship of their structural properties and other properties with their anisotropy are given.

Key words: Properties, Decomposition, Norm, Anisotropy, İsotropy, and Elastic Constant Tensor.

INTRODUCTION

ELASTIC CONSTANT TENSOR DECOMPOSITION

The constitutive relation characterizing linear anisotropic solids is the generalized Hook's law [1]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}\,,\ \varepsilon_{ij} = S_{ijkl}\sigma_{kl} \tag{1}$$

Where σ_{ij} and ε_{kl} are the symmetric second rank stress and strain tensors, respectively C_{ijkl} is the fourth-rank elastic stiffness tensor (here after we call it elastic constant tensor) and S_{ijkl} is the elastic compliance tensor.

There are three index symmetry restrictions on these tensors. These conditions are:

$$C_{ijkl} = C_{jikl}\,,\ C_{ijkl} = C_{ijlk}\,,\ C_{ijkl} = C_{klij} \tag{2}$$

Which the first equality comes from the symmetry of stress tensor, the second one from the symmetry of strain tensor, and the third one is due to the presence of a deformation potential. In general, a fourth-rank tensor has 81 elements. The index symmetry conditions (2) reduce this number to 21. Consequently, for most asymmetric materials (triclinic symmetry) the elastic constant tensor has 21 independent components.

Elastic compliance tensor S_{ijkl} possesses the same symmetry properties as the elastic constant tensor C_{ijkl} and their connection is given by [2,3,4,5,6,7]:

$$C_{ijkl}S_{klmn} = \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right) \tag{3}$$

Where δ_{ij} is the Kronecker delta. The Einstein summation convention over repeated indices is used and indices run from 1 to 3 unless otherwise stated.

By applying the symmetry conditions (2) to the decomposition results obtained for a general fourth-rank tensor, the following reduction spectrum for the elastic constant tensor is obtained. It contains two scalars, two deviators, and one-nonor parts:

$$C_{ijkl} = C_{ijkl}^{(0;1)} + C_{ijkl}^{(0;2)} + C_{ijkl}^{(2;1)} + C_{ijkl}^{(2;2)} + C_{ijkl}^{(4;1)} \quad (4)$$

Where:

$$C_{ijkl}^{(0;1)} = \frac{1}{9}\delta_{ij}\delta_{kl}C_{ppqq}, \quad (5)$$

$$C_{ijkl}^{(0;2)} = \frac{1}{90}\left(3\delta_{ik}\delta_{jl} + 3\delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)\left(3C_{pqpq} - C_{ppqq}\right) \quad (6)$$

$$C_{ijkl}^{(2;1)} = \frac{1}{5}\left(\delta_{ik}C_{jplp} + \delta_{jk}C_{iplp} + \delta_{il}C_{jpkp} + \delta_{jl}C_{ipkp}\right) - \frac{2}{15}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)C_{pqpq} \quad (7)$$

$$C_{ijkl}^{(2;2)} = \frac{1}{7}\delta_{ij}\left(5C_{klpp} - 4C_{kplp}\right) + \frac{1}{7}\delta_{kl}\left(5C_{ijpp} - 4C_{ipjp}\right)$$
$$- \frac{2}{35}\delta_{ik}\left(5C_{jlpp} - 4C_{jplp}\right) - \frac{2}{35}\delta_{jl}\left(5C_{ikpp} - 4C_{ipkp}\right)$$
$$- \frac{2}{35}\delta_{il}\left(5C_{jkpp} - 4C_{iplp}\right) - \frac{2}{35}\delta_{jk}\left(5C_{ilpp} - 4C_{iplp}\right) + \frac{2}{105}\left(2\delta_{jk}\delta_{il} + 2\delta_{ik}\delta_{jl} - 5\delta_{ij}\delta_{kl}\right)$$
$$\left(5C_{ppqq} - 4C_{pqpq}\right) \quad (8)$$

$$C_{ijkl}^{(4;1)} = \frac{1}{3}\left(C_{ijkl} + C_{ikjl} + C_{iljk}\right) - \frac{1}{21}\left[\delta_{ij}\left(C_{klpp} + 2C_{kplp}\right) + \delta_{ik}\left(C_{jlpp} + 2C_{jplp}\right)\right.$$
$$+ \delta_{il}\left(C_{jkpp} + 2C_{jpkp}\right) + \delta_{jk}\left(C_{ilpp} + 2C_{iplp}\right) + \delta_{jl}\left(C_{ikpp} + 2C_{ipkp}\right)$$
$$\left. + \delta_{kl}\left(C_{ijpp} + 2C_{ipjp}\right)\right] + \frac{1}{105}\left[\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\left(C_{ppqq} + 2C_{pqpq}\right)\right] \quad (9)$$

These parts are orthonormal to each other. Using Voigt's notation [1] for C_{ijkl}, can be expressed in 6 by 6 reduced matrix notation, where the matrix coefficients $c_{\mu\lambda}$ are connected with the tensor components C_{ijkl} by the recalculation rules:

$$c_{\mu\lambda} = C_{ijkl}; \quad (ij \leftrightarrow \mu = 1,....,6, kl \leftrightarrow \lambda = 1,....,6)$$

That is:

$$11 \leftrightarrow 1, 22 \leftrightarrow 2, 33 \leftrightarrow 3, 23 = 32 \leftrightarrow 4\ 31 = 13 \leftrightarrow 5, 12 = 21 \leftrightarrow 6.$$

II. THE NORM CONCEPT

Generalizing the concept of the modulus of a vector, norm of a Cartesian tensor (or the modulus of a tensor) is defined as the square root of the contracted product over all indices with itself:

$$N = \|T\| = \{T_{ijkl.........}.T_{ijkl.........}\}^{1/2}$$

Denoting rank-n Cartesian $T_{ijkl.........}$, by T_n, the square of the norm is expressed as [7]:

$$N^2 = \|T\|^2 = \sum_{j,q} \left\| T^{(j;q)} \right\|^2 = \sum_{(n)} T_{(n)} T_{(n)} = \sum_{(n),j,q} T_{(n)}^{(j;q)} T_{(n)}^{(j,q)}$$

This definition is consistent with the reduction of the tensor in tensor in Cartesian formulation when all the irreducible parts are embedded in the original rank-n tensor space. Since the norm of a Cartesian tensor is an invariant quantity, we suggest the following:

<u>Rule1.</u> The norm of a Cartesian tensor may be used as a criterion for representing and comparing the overall effect of a certain property of the same or different symmetry. The larger the norm value, the more effective the property is. It is known that the anisotropy of the materials, i.e., the symmetry group of the material and the anisotropy of the measured property depicted in the same materials may be quite different. Obviously, the property, tensor must show, at least, the symmetry of the material. For example, a property, which is measured in a material, can almost be isotropic but the material symmetry group itself may have very few symmetry elements. We know that, for isotropic materials, the elastic compliance tensor has two irreducible parts, i.e., two scalar parts, so the norm of the elastic compliance tensor for isotropic materials depends only on the norm of the scalar parts, i.e. $N = N_s$, Hence, the ratio $\frac{N_s}{N} = 1$ for isotropic materials. For anisotropic materials, the elastic constant tensor additionally contains two deviator parts and one nonor part, so we can define $\frac{N_d}{N}$ for the deviator irreducible parts and $\frac{N_n}{N}$ for nonor parts. Generalizing this to irreducible tensors up to rank four, we can define the following norm ratios: $\frac{N_s}{N}$ for scalar parts, $\frac{N_v}{N}$ for vector parts, $\frac{N_d}{N}$ for deviator parts, $\frac{N_{sc}}{N}$ for septor parts, and $\frac{N_n}{N}$ for nonor parts. Norm ratios of different irreducible parts represent the anisotropy of that particular irreducible part they can also be used to assess the anisotropy degree of a material property as a whole, we suggest the following two more rules:

<u>Rule 2.</u> When N_s is dominating among norms of irreducible parts: the closer the norm ratio $\frac{N_s}{N}$ is to one, the closer the material property is isotropic.

<u>Rule3.</u> When N_s is not dominating or not present, norms of the other irreducible parts can be used as a criterion. But in this case the situation is reverse; the larger the norm ratio value we have, the more anisotropic the material property is.

The square of the norm of the elastic stiffness tensor (elastic constant tensor) C_{mn} is:

$$\|N\|^2 = \sum_{mn} \left(C_{mn}^{(0;1)}\right)^2 + \sum_{mn} \left(C_{mn}^{(0;2)}\right)^2 + 2\sum_{mn} \left(C_{mn}^{(0;1)}.C_{mn}^{(0;2)}\right) + \sum_{mn} \left(C_{mn}^{(2;1)}\right)^2 + \sum_{mn} \left(C_{mn}^{(2;2)}\right)^2 + 2\sum_{mn} \left(C_{mn}^{(2;1)}.C_{mn}^{(2;2)}\right) + \sum_{mn} \left(C_{mn}^{(4;1)}\right)^2 \quad (10)$$

Let us consider the irreducible decompositions of the elastic stiffness tensor (elastic constant tensor) in the following crystals:

Table 1, Elastic Constants (GPa), [8]

Element, Cubic System	c_{11}	c_{12}	c_{44}
Copper, Cu	169	122	75.3
Nickel, Ni, Zero field.	247	153	122
Saturation field.	249	152	124

Table 2, Elastic Constants (GPa) [8]

Alloy, Cubic System Copper Nickel, Cu-Ni,At % Ni	c_{11}	c_{12}	c_{44}
0	168.1	121.4	75.1
2.34	169.3	121.8	76.3
3.02	169.4	121.8	76.7
4.49	170.1	121.9	77.3
6.04	171.1	122.4	78.1
9.73	172.3	122.6	79.1
0 (Non magnetic)	168.3	121.2	75.7
31.1 (Non magnetic)	189.1	131.9	89.7
53.8 (Non magnetic)	208.6	142.8	100.9
65.5 (Unmagnetized)	216.8	146.3	106.1
77.2 (Magnetized)	227.0	150.9	112.5
82.2 (Magnetized)	232.7	151.3	115.9
92.7 (Magnetized)	244.7	153.9	121.7
100 (Magnetized)	252.8	155.1	125.0

By using table1 and table 2, and the decomposition of the elastic constant tensor, we have calculated the norms and the norm ratios as is shown in table 3 and in table 4.

Table 3, the norms and norm ratios

Element	N_s	N_d	N_n	N	$\frac{N_s}{N}$	$\frac{N_d}{N}$	$\frac{N_n}{N}$
Copper, Cu	450.929	0	94.951	460.818	0.9785	0	0.2061
Nickel, Ni, Zero field.	631.595	0	137.477	646.385	0.9771	0	0.2127
Saturation field.	634.501	0	138.394	649.419	0.9770	0	0.2131

Table 4, the norms and norm ratios

Alloy, Cubic System Copper Nickel, Cu-Ni, At % Ni	N_s	N_d	N_n	N	$\frac{N_s}{N}$	$\frac{N_d}{N}$	$\frac{N_n}{N}$
0	448.767	0	94.859	458.683	0.9784	0	0.2068
2.34	451.775	0	96.326	461.930	0.9780	0	0.2085
3.02	451.595	0	96.601	461.812	0.9779	0	0.2092
4.49	453.689	0	97.517	464.051	0.9777	0	0.2101
6.04	456.302	0	98.525	466.818	0.9775	0	0.2111
9.73	458.856	0	99.442	469.507	0.9773	0	0.2118
0 (Non magnetic)	449.173	0	95.592	459.232	0.9781	0	0.2082
31.1 (Non magnetic)	501.962	0	111.998	514.305	0.9760	0	0.2178
53.8 (Non magnetic)	551.346	0	124.646	565.260	0.9754	0	0.2205
65.5 (Unmagnetized)	570.965	0	129.870	585.549	0.9751	0	0.2218
77.2 (Magnetized)	595.733	0	136.469	611.164	0.9747	0	0.2233
82.2 (Magnetized)	606.269	0	137.844	621.742	0.9751	0	0.2217
92.7 (Magnetized)	629.848	0	139.860	645.189	0.9762	0	0.2168
100 (Magnetized)	644.426	0	139.585	659.370	0.9773	0	0.2117

CONCLUSION

We can conclude from table 3, by considering the ratio $\frac{N_s}{N}$ that Copper, (first ionization energy is 745KJ/mole) is more isotropic than Nickel, (first ionization energy is737KJ/mole), and by considering the value of N which is more high in the case of Nickel, so can say that Nickel elastically is more stronger than Copper, and Nickel with saturation field is more anisotropic and elastically is more stronger than Nickel with zero field..

And we can conclude from table 4 by considering the ratio $\frac{N_s}{N}$ that in the Alloy Cu-Ni as the percentage of Ni increases the anisotropy of the alloy increases, and by considering the value of $\boldsymbol{N}$ which is increasing as the percentage of Ni increases, so can say that the alloy becomes elastically more strongest.

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