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Some Operations and Properties of Neutrosophic Cubic Soft Set

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Abstract

In this paper we define some operations such as P-union, P-intersection, R-union, R-intersection for neutrosophic cubic soft sets (NCSSs). We prove some theorems on neutrosophic cubic soft sets. We also discuss various approaches of Internal Neutrosophic Cubic Soft Sets (INCSSs) and external neutrosophic cubic soft sets (ENCSSs). We also investigate some of their properties.

Keywords: Neutrosophic cubic soft set; Neutrosophic soft set; Cubic set; Internal neutrosophic Cubic soft set; External neutrosophic cubic soft set

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Introduction

Neutrosophic set [1] grounded by Smarandache in 1998, is the generalization of fuzzy set [2] introduced by Zadeh in 1965 and intuitionistic fuzzy set [3] by Atanassov in 1983. In 1999, Molodstov [4] introduced the soft set theory to overcome the inadequate of existing theory related to uncertainties. Soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory [2], rough set theory [5], probability theory for dealing with uncertainty. The concept of soft set theory penetrates in many directions such as fuzzy soft set [6-9], intuitionistic fuzzy soft set [10-13], interval valued intuitionistic fuzzy soft set [14], neutrosophic soft set [15-18], interval neutrosophic set [19,20]. In 2012, Jun et al. [21] introduced cubic set combining fuzzy set and interval valued fuzzy set. Jun et al. [21] also defined internal cubic set, external cubic set, P-union, R-union, P-intersection and R-intersection of cubic sets, and investigated several related properties. Cubic set theory is applied to CI-algebras [22], B-algebras [23], BCK/BCI-algebras [24,25], KU-Algebras ([26,27], and semi-groups [28]. Using fuzzy set and interval-valued fuzzy set Abdullah et al. [29] proposed the notion of cubic soft set [29] and defined internal cubic soft set, external cubic soft set, P-union, R-union, P-intersection and R-intersection of cubic soft sets, and investigated several related properties. Ali et al. [30] studied generalized cubic soft sets and their applications to algebraic structures. Wang et al. [31] introduced the concept of interval neutrosophic set. In 2016, Ali et al. [32] presented the concept of neutrosophic cubic set by combining the concept of neutrosophic set and interval neutrosophic set. Ali et al. [32] mentioned that neutrosophic cubic set is basically the generalization of cubic set. Ali et al. [32] also defined some new type of internal neutrosophic cubic set (INCSs) and external neutrosophic cubic set (ENCSs) namely, $\frac{1}{3}INCS(or\frac{2}{3}ENCS),\frac{2}{3}INCS(or\frac{1}{3}ENCS)$. Ali et al. [32] also presented a numerical problem for pattern recognition. Jun et al. [33] also studied neutrosophic cubic set and proved some properties. In 2016, Chinnadurai et al. [34] introduced the neutrosophic cubic soft sets and proved some properties.

In this paper we discuss some new operations and new approach of internal and external neutrosophic cubic soft sets, and P-union, R-union, P-intersection, R-intersection. We also prove some theorems related to neutrosophic cubic soft sets.

Rest of the paper is presented as follows. Section 2 presents some basic definition of neutrosophic sets, interval-valued neutrosophic sets, soft sets, cubic set, neutrosophic cubic sets and their basic operation. Section 3 is devoted to presents some new theorems related to neutrosophic cubic soft sets. Section 4 presents conclusions and future scope of research.

Preliminaries

In this section, we recall some well-established definitions and properties which are related to the present study.

Definition 1: Neutrosophic set [1]

Let U be the space of points with generic element in U denoted by u. A neutrosophic set λ in U is defined as λ={<u, t^{λ} (u), i^{λ} (u), f^{λ} (u)>:u∈U} , where t^{λ} (u):U →]$^{-}$0, 1^{+}[,i^{λ} (u):U →]$^{-}$0, 1^{+}[, and f^{λ} (u):U →]$^{-}$0, 1^{+}[and $^{-}0 \le t^{\lambda}(u) + i^{\lambda}(u) + f^{\lambda}(u) \le 3^{+}$.

Definition 2: Interval value neutrosophic set [31]

Let U be the space of points with generic element in U denoted by u. An interval neutrosophic set A in U is characterized by truth-membership function t_A, the indeterminacy function i_A and falsity membership function f_A. For each u ∈ U, t_A (u), i_A (u), f_A (u) ⊆ [0, 1] and A is defined as

A={<u, [t_A^{-} (u), t_A^{+} (u)], [i_A^{-} (u), i_A^{+} (u)], [f_A^{-} (u), f_A^{+} (u)]:u∈U}.

Definition 3: Neutrosophic cubic set [32]

Let U be the space of points with generic element in U denoted by u ∈ U. A neutrosophic cubic set in U defined as $\ddot{N}$ ={< u, A (u), λ (u) >: u ∈ U} in which A (u) is the interval valued neutrosophic set and λ(u) is the neutrosophic set in U. A neutrosophic cubic set in U denoted by $\ddot{N}$ = <A, λ>. We use $C\ddot{N}(U)$ as a notation which implies that collection of all neutrosophic cubic sets in U.

Definition 4: Soft set [4]

Let U be the initial universe set and E be the set of parameters. Then soft set F_K over U is defined by F_K ={< u, F (e)>: e ∈ K, F (e) ∈ P (U)}

Where F: K ⟶ P (U), P (U) is the power set of U and K ⊂ E.

Definition 5: Neutrosophic cubic soft set [34]

A soft set $\tilde{F}_K$ is said to be neutrosophic cubic soft set iff $\tilde{F}$ is the mapping from K to the set of all neutrosophic cubic sets in U (i.e., $C\ddot{N}(U)$).

i.e. $\tilde{F}$: K ⟶ $C\ddot{N}(U)$, where K is any subset of parameter set E and U is the initial universe set.

Neutrosophic cubic soft set is defined by

$\tilde{F}_K = \{(u, < A(e_i), \lambda(e_i) > /u, e_i \in K, u \in U\}$,

Where, A(e_i) is the interval valued neutrosophic soft set and λ(e_i) is the neutrosophic soft set.

Definition: Internal neutrosophic cubic soft set (INCSS)

A neutrosophic cubic soft set $\tilde{F}_K$ is said to be INCSS if for all e_i ∈ K E

$T^{-}_{A(e_i)}(u) \le T_{\lambda(e_i)}(u) \le T^{+}_{A(e_i)}(u)$,

$F^{-}_{A(e_i)}(u) \le F_{\lambda(e_i)}(u) \le F^{+}_{A(e_i)}(u)$,

$F^{-}_{A(e_i)}(u) \le F_{\lambda(e_i)}(u) \le F^{+}_{A(e_i)}(u)$, for all $u \in U$.

Definition: External neutrosophic cubic soft set (ENCSS)

A neutrosophic cubic soft set $\tilde{F}_K$ is said to be ENCSS if for all e_i ∈ K E

$T_{\lambda(e_i)}(u) \notin (T^{-}_{A(e_i)}(u), T^{+}_{A(e_i)}(u)$,

$I_{\lambda(e_i)}(u) \notin (I^{-}_{A(e_i)}(u), I^{+}_{A(e_i)}(u)$,

$F_{\lambda(e_i)}(u) \notin (F^{-}_{A(e_i)}(u), F^{+}_{A(e_i)}(u))$, for all u∈U.

Some theorem related to these topics

Theorem 1

Let $\tilde{F}_K$ be a neutrosophic cubic soft set in U which is not an ENCSS. Then there exists at least one e_i ∈ K ⊆ E for which there exists some u∈U such that

$T_{\lambda(e_i)}(u) \in (T^{-}_{A(e_i)}(u), T^{+}_{A(e_i)}(u)$, $I_{\lambda(e_i)}(u) \in (I^{-}_{A(e_i)}(u), I^{+}_{A(e_i)}(u)$,

$F_{\lambda(e_i)}(u) \in (F^{-}_{A(e_i)}(u), F^{+}_{A(e_i)}(u)$

Proof

From the definition of ENCSSs, we have

$T_{\lambda(e_i)}(u) \notin (T^{-}_{A(e_i)}(u), T^{+}_{A(e_i)}(u)$,

$I_{\lambda(e_i)}(u) \notin (I^{-}_{A(e_i)}(u), I^{+}_{A(e_i)}(u)$,

$F_{\lambda(e_i)}(u) \notin (F^{-}_{A(e_i)}(u), F^{+}_{A(e_i)}(u))$, for all u∈U, corresponding to each e_i ∈ K ⊆ E.

But given that $\tilde{F}_K$ is not an ENCSS, so at least one e_i ∈ K ⊆ E.

There exists some u∈U such that $T_{\lambda(e_i)}(u) \in (T^{-}_{A(e_i)}(u), T^{+}_{A(e_i)}(u)$, $I_{\lambda(e_i)}(u) \in (I^{-}_{A(e_i)}(u), I^{+}_{A(e_i)}(u)$, $F_{\lambda(e_i)}(u) \in (F^{-}_{A(e_i)}(u), F^{+}_{A(e_i)}(u)$.

Hence the proof is complete.

Theorem 2

Let $\tilde{F}_K$ ={< u, $\tilde{F}$(e)>: e $\tilde{F}$(e) K, $\tilde{F}(e) \in C\ddot{N}(U)$} be a NCSS in U. If $\tilde{F}_K$ is both an INCSS and ENCSS in U for all $u \in U$, corresponding to each e_i ∈ K, then, $T_{\lambda(e_i)}(u) \in \tilde{U}(T_{A(e_i)}) \cup \tilde{L}(T_{A(e_i)})$, $I_{\lambda(e_i)}(u) \in \tilde{U}(I_{A(e_i)}) \cup \tilde{L}(I_{A(e_i)})$, $F_{\lambda(e_i)}(u) \in \tilde{U}(F_{A(e_i)}) \cup \tilde{L}(F_{A(e_i)})$,where $\tilde{U}(T_{A(e_i)})$ = { $T^{+}_{A(e_i)}(u) : u \in U$} , $\tilde{L}(T_{A(e_i)})$ = { $T^{-}_{A(e_i)}(u) : u \in U$} , $\tilde{U}(I_{A(e_i)})$ = {$I^{+}_{A(e_i)}(u) : u \in U$}, $\tilde{L}(I_{A(e_i)})$={$I^{-}_{A(e_i)}(u) : u \in U$}, $\tilde{U}(F_{A(e_i)})$ = {$F^{+}_{A(e_i)}(u) : u \in U$}, $\tilde{L}(F_{A(e_i)})$ = {$F^{-}_{A(e_i)}(u) : u \in U$}.

Proof

Suppose $\tilde{F}_K$ be both an INCSS and ENCSS corresponding to each e_i ∈ K and for all u∈U. We have $T_{\lambda(e_i)}(u) \in (T^{-}_{A(e_i)}(u), T^{+}_{A(e_i)}(u)$, $I_{\lambda(e_i)}(u) \in (I^{-}_{A(e_i)}(u), I^{+}_{A(e_i)}(u)$, $F_{\lambda(e_i)}(u) \in (F^{-}_{A(e_i)}(u), F^{+}_{A(e_i)}(u)$.

Again by definition of ENCSS corresponding to each e_i ∈K and for all u∈U, we have

$T_{\lambda(e_i)}(u) \notin (T^{-}_{A(e_i)}(u), T^{+}_{A(e_i)}(u)$, $I_{\lambda(e_i)}(u) \notin (I^{-}_{A(e_i)}(u), I^{+}_{A(e_i)}(u)$,

$F_{\lambda(e_i)}(u) \notin (F^{-}_{A(e_i)}(u), F^{+}_{A(e_i)}(u)$.

Since $\tilde{F}_K$ is both an ENCSS and INCSS, so only possibility is that, $T_{\lambda(e_i)}(u) = T^{-}_{A(e_i)}(u) \, or \, T^{+}_{A(e_i)}(u))$, $I_{\lambda(e_i)}(u) = I^{-}_{A(e_i)}(u) \, or \, I^{+}_{A(e_i)}(u))$, $F_{\lambda(e_i)}(u) = F^{-}_{A(e_i)}(u) \, or \, F^{+}_{A(e_i)}(u)$ for all u∈U.

Hence proved.

Definition

Let $\tilde{F}_{K_1}$ and $\tilde{G}_{K_2}$ be two neutrosophic cubic soft sets in U and K_1, K_2 be any two subsets of K. Then, we define the following:

1. $\tilde{F}_{K_1} = \tilde{G}_{K_2}$ if $K_1 = K_2$ and $\tilde{F}(e_i) = \tilde{G}(e_i)\ \forall\ e_i \in K$
$\Rightarrow A(e_i) = B(e_i)$ and $\lambda_1(e_i) = \lambda_2(e_i)\ \forall u \in U$
2. If $\tilde{F}_{K_1}$ and $\tilde{G}_{K_2}$ are two NCSSs then we define P- order as $\tilde{F}_{K_1} \subseteq_P \tilde{G}_{K_2}$ iff the following conditions are satisfied:

i. $K_1 \subseteq K_2$, and

ii. $\tilde{F}(e_i) \subseteq_P \tilde{G}(e_i)$ for all $e_i \in K_1$ iff

$A(e_i) \subseteq B(e_i)$ and $\lambda_1(e_i) \subseteq \lambda_2(e_i)\ \forall u \in U$ corresponding to each $e_i \in K_1$, where, $A(e_i) \subseteq B(e_i) \Rightarrow T^-_{A(e_i)}(u) \le T^-_{B(e_i)}(u)$
$T^+_{A(e_i)}(u) \le T^+_{B(e_i)}(u)$, $I^-_{A(e_i)}(u) \ge I^-_{B(e_i)}(u)$,
$I^+_{A(e_i)}(u) \ge I^+_{B(e_i)}(u)$, $F^-_{A(e_i)}(u) \ge F^-_{B(e_i)}(u)$,
$F^+_{A(e_i)}(u) \ge F^+_{B(e_i)}(u)$ for all $u \in U$, and $T_{\lambda_{1(e_i)}}(u) \le T_{\lambda_{2(e_i)}}(u)$,
$I_{\lambda_{1(e_i)}}(u) \ge I_{\lambda_{2(e_i)}}(u)$, $F_{\lambda_{1(e_i)}}(u) \ge F_{\lambda_{2(e_i)}}(u)$.

3. If $\tilde{F}_{K_1}$ and $\tilde{G}_{K_2}$ are two neutrosophic cubic soft sets, then we define the R-order as $\tilde{F}_{K_1} \subseteq_R \tilde{G}_{K_2}$ iff the following conditions are satisfied:

i. $K_1 \subseteq K_2$ and

ii. $\tilde{F}(e_i) \subseteq_R \tilde{G}(e_i)$ for all $e_i \in K_1$ iff $A(e_i) \subseteq_R B(e_i)$ and $\lambda_1(e_i) \ge \lambda_2(e_i)\ \forall u \in U$ corresponding to each $e_i \in K_1$.

Definition

Let $\tilde{G}_{K_2}$ and $\tilde{G}_{K_2}$ be two NCSSs in U and K_1, K_2 be any two subsets of parameter set K. Then we define P-union as $\tilde{F}_{K_1} \cup_P \tilde{G}_{K_2} = \tilde{H}_{K_3}$, where $K_3 \in K_1 \cup K_2$,

$\tilde{H}(e_i) = \tilde{F}(e_i)$, if $e_i \in K_1 - K_2$
$= \tilde{G}(e_i)$, if $e_i \in K_2 - K_1$
$= \tilde{F}(e_i) \vee_P \tilde{G}(e_i)$, if $e_i \in K_1 \cap K_2$

Here $\tilde{F}(e_i) = \{< u, A(e_i), \lambda_1(e_i) >: \forall u \in U\}$,
$\tilde{G}(e_i) = \{< u, B(e_i), \lambda_2(e_i) >: \forall u \in U\}$ and
$A(e_i) = \{< u, [T^-_{A(e_i)}, T^+_{A(e_i)}], [I^-_{A(e_i)}, I^+_{A(e_i)}], [F^-_{A(e_i)}, F^+_{A(e_i)}] >, \forall u \in U\}$,
$B(e_i) = \{< u, [T^-_{B(e_i)}, T^+_{B(e_i)}], [I^-_{B(e_i)}, I^+_{B(e_i)}], [F^-_{B(e_i)}, F^+_{B(e_i)}] >, \forall u \in U\}$,
$\lambda_1(e_i) = \{< u, T_{\lambda_{1(e_1)}}(u), I_{\lambda_{1(e_1)}}(u), F_{\lambda_{1(e_1)}}(u) >, \forall e_i \in K_1, \forall\ u \in U\}$'
$\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< u, r\max\{A(e_i), B(e_i)\}, r\max\{\lambda_1(e_i), \lambda_2(e_i)\}, \text{for all } u \in U \text{ and } e_i \in K_1 \cap K_2\}$.

Definition

Let $\tilde{F}_{K_1}$ and $\tilde{G}_{K_2}$ be two NCSSs in U and K_1, K_2 be any two subsets of parameter set K. The P-intersection of $\tilde{F}_{K_1}$ and $\tilde{G}_{K_2}$ is denoted by $\tilde{F}_{K_1} \cap_P \tilde{G}_{K_2} = \tilde{N}_{K_3}$ where $K_3 = K_1 \cap K_2$ and $\tilde{N}_{K_3}$ defined as $\tilde{N}_{K_3} =$
$\tilde{F}(e_i) \wedge_P G(e_i)$ if $e_i \in K_1 \cap K_2$.

Here, $\tilde{F}(e_i) \wedge_P G(e_i) = \{< u, r\min\{A(e_i), B(e_i)\}, r\min\{\lambda_1(e_i), \lambda_2(e_i)\}, \text{for all } u \in U \text{ and } e_i \in K_1 \cap K_2\}$.

Definition: Compliment

The compliment of $\tilde{F}_{K_1}$ denoted by $\tilde{F}^c_{K_1}$ is defined by
$\tilde{F}_{K_1}(e_i) = \tilde{F}(e_i) = \{< u, A(e_i), \lambda_1(e_i) >: \forall u \in U, e_i \in K_1\}$,
$\tilde{F}^c_{K_1}(e_i) = \tilde{F}^c(e_i) = \{< u, A^c_{e_i}(u), \lambda^c_{e_i}(u) >: \forall u \in U, e_i \in K_1\}$

Where,
$A^c_{e_i}(u) = \{< u, [1 - T^-_{A(e_i)}(u), 1 - T^+_{A(e_i)}(u)], [1 - I^-_{A(e_i)}(u), 1 - I^+_{A(e_i)}(u)], [1 - F^-_{A(e_i)}(u), 1 - F^+_{A(e_i)}(u)] >, \forall u \in U\}$,
$\lambda^c_{e_i}(u) = \{< u, 1 - T_{\lambda_{1(e_i)}}(u), 1 - I_{\lambda_{1(e_i)}}(u), 1 - F_{\lambda_{1(e_i)}}(u) >, \forall u \in U\}$

Some properties of P-union and P-intersection

$\tilde{F}_{K_1} \cup_P \tilde{G}_{K_2} = \tilde{G}_{K_2} \cup_P \tilde{F}_{K_1}$

$\tilde{F}_{K_1} \cap_P \tilde{G}_{K_2} = \tilde{G}_{K_2} \cap_P \tilde{F}_{K_1}$

Proof 1:

$\tilde{F}_{K_1} \cup_P \tilde{G}_{K_2} = \tilde{H}_{K_3}$ where $K_3 = K_1 \cup K_2$
$\tilde{H}(e_i) = \tilde{F}(e_i)$, if $e_i \in K_1 - K_2$
$= \tilde{G}(e_i)$, if $e_i \in K_2 - K_1$
$= \tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ if $e_i \in K_1 \cap K_2$

Here,
$\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$
$\tilde{G}_{K_2} \cup_P \tilde{F}_{K_1} = \tilde{R}_{K_3}$,
$\tilde{R}_{K_3} = \tilde{G}(e_i)$, if $e_i \in K_2 - K_1$
$= \tilde{F}(e_i)$, if $e_i \in K_1 - K_2$
$= \tilde{G}(e_i) \vee_P \tilde{F}(e_i)$, if $e_i \in K_1 \cap K_2$.
$\tilde{G}(e_i) \vee_P \tilde{F}(e_i) = \{< u, r\max\{B_{e_i}(u), A_{e_i}(u)\}, r\max\{\lambda_{2(e_i)}(u), \lambda_{1(e_i)}(u)\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$
$= \{< u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$
$= \tilde{F}(e_i) \vee_P \tilde{G}(e_i)$.

Hence the proof.

Proof 2:

$\tilde{S}_{K_3} = \tilde{F}_{K_1} \cap_P \tilde{G}_{K_2}$
$= \{< u, r\min\{A_{e_i}(u), B_{e_i}(u)\}, r\min\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$
$\tilde{G}_{K_2} \cap_P \tilde{F}_{K_1} = \tilde{Q}_{K_3}$
$= \{< u, r\min\{B_{e_i}(u), A_{e_i}(u)\}, r\min\{\lambda_{2(e_i)}(u), \lambda_{1(e_i)}(u)\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$
$= \{< u, r\min\{A_{e_i}(u), B_{e_i}(u)\}, r\min\{\lambda_{1(e_i)}(u, \lambda_{2(e_i)}(u))\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$
$= \tilde{F}_{K_1} \cap_P \tilde{G}_{K_2}$.

Hence the proof.

Definition: R-union and R-intersection

Let $\tilde{F}_{K_1}$ and $\tilde{G}_{K_2}$ be two NCSSs over U. Then R-union is denoted as $\tilde{F}_{K_1} \cup_R \tilde{G}_{K_2} = \tilde{N}_{K_3}$, where $K_3 = K_1 \cup K_2$ and $\tilde{N}_{K_3}$. Then R-union is defined as

$\tilde{N}_{K_3} = \tilde{N}(e_i) = \tilde{F}(e_i)$, if $e_i \in K_1 - K_2$
$= \tilde{G}(e_i)$, if $e_i \in K_2 - K_1$
$= \tilde{F}(e_i) \vee_R \tilde{G}(e_i)$, if $e_i \in K_1 \cap K_2$.

Here $\tilde{F}(e_i) \vee_R \tilde{G}(e_i)$ defined as
$\tilde{F}(e_i) \vee_R \tilde{G}(e_i) = \{< u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\min\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} >, \forall u \in U, \forall e_i \in K_1 \cap K_2\}$

R-intersection is denoted as $\tilde{F}_{K_1} \cap_R \tilde{G}_{K_2} = \tilde{J}_{K_3}$ where $K_3 \in K_1 \cap K_2$.Then R-intersection is defined as:

$\widetilde{J}_{K_3} = \{<u, r\min\{A_{e_i}(u), B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}>, \forall u \in U, \forall e_i \in K_3\}$

Theorem 3

Let U be the initial universe and I, J, L, S any four subsets of E, then for four corresponding neutrosophic cubic soft sets $\widetilde{F}_I, \widetilde{G}_J, \widetilde{H}_L, \widetilde{T}_S$ the following properties hold

i. If $\widetilde{F}_I \subseteq_P \widetilde{G}_J$ and $\widetilde{G}_J \subseteq_P \widetilde{H}_L$, then $\widetilde{F}_I \subseteq_P \widetilde{H}_L$

Proof: $\widetilde{F}_I \subseteq_P \widetilde{G}_J \Rightarrow \widetilde{F}(e_i) \le \widetilde{G}(e_i) \forall e_i \in I, I \subseteq J$

$\widetilde{G}_J \subseteq_P \widetilde{H}_L \Rightarrow J \subseteq L$ and $\widetilde{G}(e_i) \le \widetilde{H}(e_i)\ \forall e_i \in J$

$e_i \in I \Rightarrow e_i \in J$, since $I \subseteq J$

$\widetilde{G}(e_i) \le \widetilde{H}(e_i)\ \forall e_i \in I$

$\widetilde{F}(e_i) \le \widetilde{G}(e_i) \le \widetilde{H}(e_i)$

$\Rightarrow \widetilde{F}(e_i) \le \widetilde{H}(e_i), \forall e_i \in I \subseteq J \subseteq L$

$\Rightarrow \widetilde{F}_I \subseteq_P \widetilde{H}_L$.

Hence the proof.

ii. $\widetilde{F}_I \subseteq_P \widetilde{G}_J$ then $(\widetilde{G}_J)^c \subseteq_P (\widetilde{F}_I)^c$ if I = J.

Proof: If I = J $\Rightarrow$ $I \subseteq J$ and $J \subseteq I$

$\widetilde{F}_I \subseteq_P \widetilde{G}_J \Rightarrow I \subseteq J$ and $\widetilde{F}(e_i) \le \widetilde{G}(e_i)$ (by definition)

$\widetilde{F}(e_i) \le \widetilde{G}(e_i) \Rightarrow A(e_i) \subseteq B(e_i)$ and $\lambda_1(e_i) \subseteq \lambda_2(e_i)$ for all $u \in U$,

$T^-_{A(e_i)}(u) \le T^-_{B(e_i)}(u), T^+_{A(e_i)}(u) \le T^+_{B(e_i)}(u), I^-_{A(e_i)}(u) \ge I^-_{B(e_i)}(u), I^+_{A(e_i)}(u) \ge I^+_{B(e_i)}(u),$

$F^-_{A(e_i)}(u) \ge F^-_{B(e_i)}(u), F^+_{A(e_i)}(u) \ge F^+_{B(e_i)}(u)$ and

$T_{\lambda_1(e_i)}(u) \le T_{\lambda_2(e_i)}(u)$, $I_{\lambda_1(e_i)}(u) \ge I_{\lambda_2(e_i)}(u)$ $F_{\lambda_1(e_i)}(u) \ge F_{\lambda_2(e_i)}(u)$

$(\widetilde{F}_I)^c = \{<u, A^c_{e_i}(u), \lambda^c_{1(e_i)}(u)>: \forall u \in U, e_i \in I\}$ where

$A^c_{e_i}(u) = \{<u, [1-T^-_{A(e_i)}(u), 1-T^+_{A(e_i)}(u)], [1-I^-_{A(e_i)}(u), 1-I^+_{A(e_i)}(u)], [1-F^-_{A(e_i)}(u), 1-F^+_{A(e_i)}(u)]>, \forall u \in U\},$

$\lambda^c_{1(e_i)}(u) = \{<u, 1-T_{\lambda_1(e_i)}(u), 1-I_{\lambda_1(e_i)}(u), 1-F_{\lambda_1(e_i)}(u)>, \forall u \in U\}$

And

$(\widetilde{G}_J)^c = \{<u, B^c_{e_i}(u), \lambda^c_{2(e_i)}(u)>: \forall u \in U, e_i \in I\}$, where

$B^c_{e_i}(u) = \{<u, [1-T^-_{B(e_i)}(u), 1-T^+_{B(e_i)}(u)], [1-I^-_{B(e_i)}(u), 1-I^+_{B(e_i)}(u)], [1-F^-_{B(e_i)}(u), 1-F^+_{B(e_i)}(u)]>, \forall u \in U\},$

$\lambda^c_{2(e_i)}(u) = \{<u, 1-T_{2(e_i)}(u), 1-I_{2(e_i)}(u), 1-F_{\lambda_2(e_i)}(u)>, \forall u \in U\}$

Now, $T^-_{A(e_i)}(u) \le T^-_{B(e_i)}(u) \Rightarrow -T^-_{A(e_i)}(u) \ge -T^-_{B(e_i)}(u)$

Adding 1 both sides we get, $1-T^-_{A(e_i)}(u) \ge 1-T^-_{B(e_i)}(u)$,

Similarly, $1-T^+_{A(e_i)}(u) \ge 1-T^+_{B(e_i)}(u)$, $1-I^+_{A(e_i)}(u) \le 1-I^-_{B(e_i)}(u)$,

$1-I^+_{A(e_i)}(u) \le 1-I^+_{B(e_i)}(u)$, $1-F^-_{A(e_i)}(u) \le 1-F^-_{B(e_i)}(u)$,

$1-F^+_{A(e_i)}(u) \le 1-F^+_{B(e_i)}(u)$ (1)

And, $1-T_{\lambda_1(e_i)}(u) \ge 1-T_{\lambda_2(e_i)}(u)$, $1-I_{\lambda_1(e_i)}(u) \le 1-I_{\lambda_2(e_i)}(u)$,

$1-F_{\lambda_1(e_i)}(u) \le 1-F_{\lambda_2(e_i)}(u)$ (2)

& (2) $\Rightarrow$ $(\widetilde{G}_J)^c \subseteq_P (\widetilde{F}_I)^c$ since $J \subseteq I$.

Hence the proof.

Theorem 4

Let $\widetilde{F}_I$ be a NCSS over U,

If $\widetilde{F}_I$ is an INCSS then $\widetilde{F}^c_I$ is also an INCSS.

If $\widetilde{F}_I$ is an ENCSS then $\widetilde{F}^c_I$ is also an ENCSS.

Proof

i. $<u, A_{e_i}(u), \lambda_{1(e_i)}(u)>: \forall u \in U$, is an INCSS,

$T^-_{A(e_i)}(u) \le T_{\lambda_1(e_i)}(u) \le T^+_{A(e_i)}(u)$,

$I^-_{A(e_i)}(u) \le I_{\lambda_1(e_i)}(u) \le I^+_{A(e_i)}(u)$,

$F^-_{A(e_i)}(u) \le F_{\lambda_1(e_i)}(u) \le F^+_{A(e_i)}(u)$, $\forall u \in U, \forall e_i \in I$

Now, $1-T^-_{A(e_i)}(u) \ge 1-T_{\lambda_1(e_i)}(u) \ge 1-T^+_{A(e_i)}(u)$,

$1-I^-_{A(e_i)}(u) \le 1-I_{\lambda_1(e_i)}(u) \le 1-I^+_{A(e_i)}(u)$,

$1-F^-_{A(e_i)}(u) \le 1-F_{\lambda_1(e_i)}(u) \le -F^+_{A(e_i)}(u)$ $\forall u \in U, \forall e_i \in I$.

$\Rightarrow$ $\widetilde{F}^c_I$ is an INCSS.

ii. For $\widetilde{F}_K$, an ENCSS, then

$T_{\lambda_1(e_i)}(u) \notin (T^-_{A(e_i)}(u), T^+_{A(e_i)}(u)$,

$I_{\lambda_1(e_i)}(u) \notin (I^-_{A(e_i)}(u), I^+_{A(e_i)}(u)$,

$F_{\lambda_1(e_i)}(u) \notin (F^-_{A(e_i)}(u), F^+_{A(e_i)}(u)$, $\forall u \in U, \forall e_i \in I$,

and $0 \le T^-_{A(e_i)}(u) \le T^+_{A(e_i)}(u) \le 1$, $0 \le I^-_{A(e_i)}(u) \le I^+_{A(e_i)}(u) \le 1$, $0 \le F^-_{A(e_i)}(u) \le F^+_{A(e_i)}(u) \le 1$.

$\Rightarrow T_{\lambda_1(e_i)}(u) \le T^-_{A(e_i)}(u)$ or $T_{\lambda_1(e_i)}(u) \ge T^+_{A(e_i)}(u)$,

$\Rightarrow I_{\lambda_1(e_i)}(u) \le I^-_{A(e_i)}(u)$ or $I_{\lambda_1(e_i)}(u) \ge I^+_{A(e_i)}(u)$,

$\Rightarrow F_{\lambda_1(e_i)}(u) \le F^-_{A(e_i)}(u)$ or $F_{\lambda_1(e_i)}(u) \ge F^+_{A(e_i)}(u)$ $\forall u \in U, \forall e_i \in I$

$\Rightarrow 1-T_{\lambda_1(e_i)}(u) \ge 1-T^-_{A(e_i)}(u)$ or $1-T_{\lambda_1(e_i)}(u) \le 1-T^+_{A(e_i)}(u)$,

$\Rightarrow 1-I_{\lambda_1(e_i)}(u) \ge 1-I^-_{A(e_i)}(u)$ or $1-I_{\lambda_1(e_i)}(u) \le 1-I^+_{A(e_i)}(u)$

$\Rightarrow 1-F_{\lambda_1(e_i)}(u) \ge 1-F^-_{A(e_i)}(u)$ or $1-F_{\lambda_1(e_i)}(u) \le 1-F^+_{A(e_i)}(u)$

Thus,

$(1-T_{\lambda_1(e_i)}(u) \notin (1-T^-_{A(e_i)}(u), (1-T^+_{A(e_i)}(u)$, $(1-I_{\lambda_1(e_i)}(u) \notin (1-I^-_{A(e_i)}(u), (1-I^+_{A(e_i)}(u)$

and $(1-F_{\lambda_1(e_i)}(u)) \notin (1-F^-_{A(e_i)}(u), (1-F^+_{A(e_i)}(u))\ \forall u \in U, \forall e_i \in I. \forall u \in U, \forall e_i \in I.$

Hence $\widetilde{F}^c_I$ is an ENCSS.

Theorem 5

Let $\widetilde{F}_I$ and $\widetilde{G}_J$ be any two INCSSs then

$\widetilde{F}_I \cup_P \widetilde{G}_J$ is an INCSS.

$\widetilde{F}_I \cap_P \widetilde{G}_J$ is an INCSS.

Proof

Since $\widetilde{F}_I$ and $\widetilde{G}_J$ are INCSSs, so for $\widetilde{F}_I$ we have

$T^-_{A(e_i)}(u) \le T_{\lambda_1(e_i)}(u) \le T^+_{A(e_i)}(u)$,

$I^-_{A(e_i)}(u) \le I_{\lambda_1(e_i)}(u) \le I^+_{A(e_i)}(u)$

$F^-_{A(e_i)}(u) \le F_{\lambda_1(e_i)}(u) \le F^+_{A(e_i)}(u)$, for all $u \in U$, $e_i \in I$.

Also for $\widetilde{G}_J$ we have

$T^-_{B(e_i)}(u) \le T_{\lambda_2(e_i)}(u) \le T^+_{B(e_i)}(u)$,

$I^-_{B(e_i)}(u) \le I_{\lambda_2(e_i)}(u) \le I^+_{B(e_i)}(u)$,

$F^-_{B(e_i)}(u) \le F_{\lambda_2(e_i)}(u) \le F^+_{B(e_i)}(u)$, $\forall u \in U$ and $\forall e_i \in J$.

Now, $\max\{T^-_{A(e_i)}(u), T^-_{B(e_i)}(u)\} \le \max\{T_{\lambda_1(e_i)}(u), T_{\lambda_2(e_i)}(u)\} \le \max\{T^+_{A(e_i)}(u), T^+_{B(e_i)}(u)\}$,

$\max\{I^-_{A(e_i)}(u), I^-_{B(e_i)}(u)\} \le \max\{I_{\lambda_1(e_i)}(u), I_{\lambda_2(e_i)}(u)\} \le \max\{I^+_{A(e_i)}(u), I^+_{B(e_i)}(u)\}$,

$\max\{F^-_{A(e_i)}(u), F^-_{B(e_i)}(u)\} \le \max\{F_{\lambda_1(e_i)}(u), F_{\lambda_2(e_i)}(u)\} \le \max\{F^+_{A(e_i)}(u), F^+_{B(e_i)}(u)\}$ $\forall u \in U, \forall e_i \in I \cup J$

Now by the definition of P-union $\tilde{F}_I \cup_P \tilde{G}_J$ is an INCSS.

$\tilde{F}_I \cap_P \tilde{G}_J = \tilde{H}_K$, where $K \in I \cap J$.

ii. Now, $\tilde{H}_K = \tilde{F}(e_i) \wedge_P \tilde{G}(e_i)$ and by definition,

$\tilde{F}(e_i) \wedge_P \tilde{G}(e_i) = \{<u, r\min\{A_{(e_i)}(u)\ B_{(e_i)}(u)\}, r\min\{\lambda_{1(e_i)}(u)\ \lambda_{2(e_i)}(u)\} >: u \in U, e_i \in K\}$,

Since $\tilde{G}_J$ and $\tilde{G}_J$ are INCSSs then we have for $\tilde{F}_I$,

$T^-_{A(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^+_{A(e_i)}(u)$, $I^-_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{A(e_i)}(u)$

$F^-_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^+_{A(e_i)}(u)$, $\forall$ u ∈ U, $\forall$ $e_i \in I$

And for $\tilde{G}_J$,

$T^-_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^+_{B(e_i)}(u)$,

$I^-_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^+_{B(e_i)}(u)$,

$F^-_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^+_{B(e_i)}(u)$, $\forall$ u ∪ U, $\forall$ $e_i \in J$

$\Rightarrow \min\{T^-_{A(e_i)}(u), T^-_{B(e_i)}(u)\} \le \min\{T_{\lambda_1(e_i)}(u), T_{\lambda 2(e_i)}(u)\} \le \min\{T^+_{A(e_i)}(u), T^+_{B(e_i)}(u)\}$,

$\min\{I^-_{A(e_i)}(u), I^-_{B(e_i)}(u)\} \le \min\{I_{\lambda 1(e_i)}(u), I_{\lambda_2(e_i)}(u)\} \le \min\{I^+_{A(e_i)}(u), I^+_{B(e_i)}(u)\}$

$\min\{F^-_{A(e_i)}(u), F^-_{B(e_i)}(u)\} \le \min\{F_{\lambda_1(e_i)}(u), F_{\lambda_2(e_i)}(u)\} \le \min\{F^+_{A(e_i)}(u), F^+_{B(e_i)}(u)\}$ $\forall$ $e_i \in K$.

Hence $\tilde{H}_K$ is an INCSS.

Theorem 6

Let $\tilde{F}_I$ and $\tilde{G}_J$ be any two INCSSs over U having the conditions:

$\max\{T^-_{A(e_i)}(u), T^-_{B(e_i)}(u)\} \le \min\{T_{\lambda_1(e_i)}(u), T_{\lambda_2(e_i)}(u)\}$,

$\max\{I^-_{A(e_i)}(u), I^-_{B(e_i)}(u)\} \le \min\{I_{\lambda 1(e_i)}(u), I_{\lambda_2(e_i)}(u)\}$ and

$\max\{F^-_{A(e_i)}(u), F^-_{B(e_i)}(u)\} \le \min\{F_{\lambda_1(e_i)}(u), F_{\lambda_2(e_i)}(u)\}$ $\forall$ $e_i \in I \cap J$, $\forall$ u ∈ U.

Then R-union of $\tilde{F}_I$ and $\tilde{G}_J$ is also INCSS.

Proof

Since $\tilde{F}_I$ and $\tilde{G}_J$ are INCSSs in U.

So for $\tilde{F}_I$, we have

$I^-_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{A(e_i)}(u)$, $I^-_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{A(e_i)}(u)$,

$F^-_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^+_{A(e_i)}(u)$, $\forall$ u ∈ U, $\forall$ $e_i \in I$.

Also for $\tilde{G}_J$ we have

$T^-_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^+_{B(e_i)}(u)$,

$I^-_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^+_{B(e_i)}(u)$,

$F^-_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^+_{B(e_i)}(u)$, $\forall$ u ∈ U, $\forall$ $e_i \in J$

From $\tilde{F}_I$ and $\tilde{G}_J$ we get

$\min\{T_{\lambda_1(e_i)}(u), T_{\lambda 2(e_i)}(u)\} \le \max\{T^+_{A(e_i)}(u), T^+_{B(e_i)}(u)\}$,

$\min\{I_{\lambda 1(e_i)}(u), I_{\lambda_2(e_i)}(u)\} \le \max\{I^+_{A(e_i)}(u), I^+_{B(e_i)}(u)\}$ and

$\min\{F_{\lambda_1(e_i)}(u), F_{\lambda_2(e_i)}(u)\} \le \max\{F^+_{A(e_i)}(u), F^+_{B(e_i)}(u)\}$ $\forall$ $e_i \in K$.

Also given that $\max\{T^-_{A(e_i)}(u), T^-_{B(e_i)}(u)\} \le \min\{T_{\lambda_1(e_i)}(u), T_{\lambda_2(e_i)}(u)\}$,

$\max\{I^-_{A(e_i)}(u), I^-_{B(e_i)}(u)\} \le \min\{I_{\lambda 1(e_i)}(u), I_{\lambda_2(e_i)}(u)\}$ and

$\max\{F^-_{A(e_i)}(u), F^-_{B(e_i)}(u)\} \le \min\{F_{\lambda_1(e_i)}(u), F_{\lambda_2(e_i)}(u)\}$ $\forall$ $e_i \in I \cap J$, $\forall$ u ∈ U.

Now, $\tilde{F}_I \cup_R \tilde{G}_J = \tilde{H}_K$ where $K = I \cup J$ and

$\tilde{H}_K = \tilde{F}(e_i)$, if $e_i \in$ I-J.

$= \tilde{G}(e_i)$, if $e_i \in$ I-J.

$= \tilde{F}(e_i) \vee_R \tilde{G}(e_i)$, if $e_i \in I \cap J$.

Her $\tilde{F}(e_i) \vee_R \tilde{G}(e_i)$ defined as

$\tilde{F}(e_i) \vee_R \tilde{G}(e_i) = \{<u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}, \forall u \in U, \forall e_i \in I \cap J\}$,

Since $\tilde{F}_I$ and $\tilde{G}_J$ are INCSSs so from the given condition and definition of INCSS we can write,

$\max\{A^-_{e_i}(u), B^-_{e_i}(u)\} \le \min\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} \le \max\{A^+_{e_i}(u), B^+_{e_i}(u)\}$, $\forall$ $e_i \in I \cap J$, $\forall$ u∈U.

If $e_i \in$ I - J or $e_i \in$ J - I then, the result is trivial.

Thus $\tilde{F}_I \cup_R \tilde{G}_J$ is an INCSS.

Theorem 7

Let $\tilde{F}_I$ and $\tilde{G}_J$ be any two INCSSs over U satisfying the condition: $r\min\{A^+_{e_i}(u), B^+_{e_i}(u)\} \ge r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}$, $\forall$ $e_i \in I \cap J$, $\forall$ u ∈ U.

Then $\tilde{F}_I \cap_R \tilde{G}_J$ is an INCSS.

Proof

Since $\tilde{F}_I$ and $\tilde{G}_J$ are INCSSs in U,

we have, $I^-_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{A(e_i)}(u)$, $I^-_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{A(e_i)}(u)$

and $F^-_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^+_{A(e_i)}(u)$, $\forall$ u ∈ U, $\forall$ $e_i \in I$.

Also for $\tilde{G}_J$ we have, $T^-_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^+_{B(e_i)}(u)$,

$I^-_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^+_{B(e_i)}(u)$ and

$F^-_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^+_{B(e_i)}(u)$, $\forall$ u ∈ U, $\forall$ $e_i \in J$.

$\Rightarrow \min\{A^-_{e_i}(u), B^-_{e_i}(u)\} \le \max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}$, also $\tilde{F}_I \cap_R \tilde{G}_J = \tilde{H}_K$, where K = I ∩ J and $\tilde{F}_I \cap_R \tilde{G}_J = \tilde{F}(e_i) \wedge_R \tilde{G}(e_i)$, if $e_i \in I \cap J$, then $\tilde{F}(e_i) \wedge_R \tilde{G}(e_i)$ defined as

$\tilde{F}(e_i) \wedge_R \tilde{G}(e_i) = \{< u, r\min\{A_{e_i}(u)\ B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u)\ \lambda_{2(e_i)}(u)\}, \forall u \in U, \forall e_i \in K\}$.

Given condition that $r\min\{A^+_{e_i}(u), B^+_{e_i}(u)\} \ge r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}$,

$\forall$ $e_i \in I \cap J$, $\forall$ u ∈ U. Thus from given condition and definition of INCSSs

$r\min\{A^-_{e_i}(u), B^-_{e_i}(u)\} \le r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} \le r\min\{A^+_{e_i}(u), B^+_{e_i}(u)\}$.

Hence $\tilde{F}_I \cap_R \tilde{G}_J$ is an INCSS.

Theorem 8

Let $\tilde{F}_I$ and $\tilde{G}_J$ be any two ENCSSs then

$\tilde{F}_I \cup_P \tilde{G}_J$ is an ENCSS.

$\tilde{F}_I \cap_P \tilde{G}_J$ is an ENCSS.

Proof

Since $\tilde{F}_I$ and are ENCSSs, we have

$T_{\lambda_1(e_i)}(u) \notin (T^-_{A(e_i)}(u), T^+_{A(e_i)}(u))$, $I_{\lambda_1(e_i)}(u) \notin (I^-_{A(e_i)}(u), I^+_{A(e_i)}(u))$,

$F_{\lambda 1(e_i)}(u) \notin (F^-_{A(e_i)}(u)\ F^+_{A(e_i)}(u))$, $\forall$ u ∈ U, $\forall$ $e_i \in I$

and $0 \le T^-_{A(e_i)}(u) \le T^+_{A(e_i)}(u) \le 1$, $0 \le I^-_{A(e_i)}(u) \le I^+_{A(e_i)}(u) \le 1$, $0 \le F^-_{A(e_i)}(u) \le F^+_{A(e_i)}(u) \le 1$.

$\Rightarrow T_{\lambda_1(e_i)}(u) \le T^-_{A(e_i)}(u)$ or $T_{\lambda_1(e_i)}(u) \ge T^+_{A(e_i)}(u)$,

$\Rightarrow I_{\lambda_1(e_i)}(u) \le I^-_{A(e_i)}(u)$ or $I_{\lambda_1(e_i)}(u) \ge I^+_{A(e_i)}(u)$,

$\Rightarrow F_{\lambda_1(e_i)}(u) \le F^-_{A(e_i)}(u)$ or $F_{\lambda_1(e_i)}(u) \ge F^+_{A(e_i)}(u)$, $\forall$ u ∈ U, $\forall$ $e_i \in I$.

$\Rightarrow \max\{\lambda_{1(e_i)}(u),\lambda_{2(e_i)}(u)\} \notin \{\max\{A^-_{e_i}(u),B^-_{e_i}(u)\},\max\{A^+_{e_i}(u),B^+_{e_i}(u)\}$, $\forall$ $e_i \in I \cap J$, $\forall$ $u \in U$.

Now by definition of P-union, $\tilde{F}_I \cup_P \tilde{G}_J = \tilde{H}_K$, where $K = I \cup J$.

$\tilde{H}_K = \tilde{F}(e_i)$, if $\forall$ $e_i \in I - J$.

$= \tilde{G}(e_i)$, if $\forall$ $e_i \in J - I$.

$= \tilde{F}(e_i) \vee_P \tilde{G}(e_i)$, if $\forall$ $e_i \in I \cap J$.

Here $\tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ defined as

$\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}$, for all $u \in U$ and $e_i \in I \cap J\}$.

Thus $\tilde{F}_I \cup_P \tilde{G}_J$ is a ENCSSs if $\forall$ $e_i \in I \cap J$.

For $e_i \in I - J$ or $e_i \in J - I$ these results are trivial.

Hence the proof.

ii. Since $\tilde{F}_I$ and $\tilde{G}_J$ are ENCSSs, then

$T_{\lambda(e_i)}(u) \notin (T^-_{A(e_i)}(u), T^+_{A(e_i)}(u))$, $I_{\lambda(e_i)}(u) \notin (I^-_{A(e_i)}(u), I^+_{A(e_i)}(u))$,

$F_{\lambda(e_i)}(u) \notin (F^-_{A(e_i)}(u), F^+_{A(e_i)}(u))$, $\forall$ $u \in U$, $\forall$ $e_i \in J$

and $0 \le T^-_{A(e_i)}(u) \le T^+_{A(e_i)}(u) \le 1$, $0 \le I^-_{A(e_i)}(u) \le I^+_{A(e_i)}(u) \le 1$ and $0 \le F^-_{A(e_i)}(u) \le F^+_{A(e_i)}(u) \le 1$.

$\Rightarrow T_{\lambda_2(e_i)}(u) \le T^-_{B(e_i)}(u)$ or $T_{\lambda_2(e_i)}(u) \ge T^+_{B(e_i)}(u)$,

$\Rightarrow I_{2(e_i)}(u) \le I^-_{B(e_i)}(u)$ or $I_{2(e_i)}(u) \ge I^+_{B(e_i)}(u)$,

$\Rightarrow F_{\lambda_2(e_i)}(u) \le F^-_{B(e_i)}(u)$ or $F_{\lambda_2(e_i)}(u) \ge F^+_{B(e_i)}(u)$, $\forall$ $u \in U$, $\forall$ $e_i \in I$

For $\tilde{G}_J$, we have

$T_{\lambda 2(e_i)}(u) \notin (T^-_{B(e_i)}(u), T^+_{B(e_i)}(u))$,

$I_{\lambda 2(e_i)}(u) \notin (I^-_{B(e_i)}(u), I^+_{B(e_i)}(u))$,

$F_{\lambda 2(e_i)}(u) \notin (F^-_{B(e_i)}(u), F^+_{B(e_i)}(u))$, $\forall$ $u \in U$, $\forall$ $e_i \in I$

and $0 \le T^-_{B(e_i)}(u) \le T^+_{B(e_i)}(u) \le 1$, $0 \le I^-_{B(e_i)}(u) \le I^+_{B(e_i)}(u) \le 1$, $0 \le F^-_{B(e_i)}(u) \le F^+_{B(e_i)}(u) \le 1$

$\Rightarrow T_{\lambda_2(e_i)}(u) \le T^-_{B(e_i)}(u)$ or $T_{\lambda_2(e_i)}(u) \ge T^+_{B(e_i)}(u)$,

$\Rightarrow I_{2(e_i)}(u) \le I^-_{B(e_i)}(u)$ or $I_{2(e_i)}(u) \ge I^+_{B(e_i)}(u)$,

$\Rightarrow F_{\lambda_2(e_i)}(u) \le F^-_{B(e_i)}(u)$ or $F_{\lambda_2(e_i)}(u) \ge F^+_{B(e_i)}(u)$, $\forall$ $u \in U$, $\forall$ $e_i \in I$

$\Rightarrow \min\{\lambda_{1(e_i)}(u),\lambda_{2(e_i)}(u)\} \notin \{\min\{A^-_{e_i}(u),B^-_{e_i}(u)\},\min\{A^+_{e_i}(u),B^+_{e_i}(u)\}$, $\forall$ $e_i \in I \cap J$ $\forall$ $u \in U$

Now, by definition of P- intersection $\tilde{F}_I \cap_P \tilde{G}_J = \tilde{H}_K$, where $K = I \cap J$, we have

$\tilde{F}_I \cap_P \tilde{G}_J = \tilde{F}(e_i) \wedge_P \tilde{G}(e_i)$, if $e_i \in I \cap J$

Here $\tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ defined as

$\tilde{F}(e_i) \wedge_P \tilde{G}(e_i) = \{< u, r\min\{A_{e_i}(u), B_{e_i}(u)\}, r\min\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}, \forall u \in U, \forall e_i \in I \cap J\}$.

Thus $\tilde{F}_I \cap_P \tilde{G}_J$ is an ENCSS.

Theorem 9

Let $\tilde{F}_I$ and $\tilde{G}_J$ be any two INCSSs in U such that

$\min\{\lambda_{1(e_i)}(u),\lambda_{2(e_i)}(u)\} \le \max\{A^-_{e_i}(u),B^-_{e_i}(u), \forall e_i \in I, \forall e_i \in J$ and $\forall u \in U$, then $\tilde{F}_I \cup_R \tilde{G}_J$ is an ENCSS.

Proof:

Since $\tilde{F}_I$ and $\tilde{G}_J$ are INCSSs in U.

we have, $T^-_{A(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^+_{A(e_i)}(u)$,

$I^-_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{A(e_i)}(u)$,

$F^-_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^+_{A(e_i)}(u)$, $\forall$ $u \in U$, $\forall$ $e_i \in I$

and $T^-_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^+_{B(e_i)}(u)$,

$I^-_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^+_{B(e_i)}(u)$,

$F^-_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^+_{B(e_i)}(u)$, $\forall$ $u \in U$, $\forall$ $e_i \in J$.

Since, $\tilde{F}_I \cup_R \tilde{G}_J = \tilde{H}_K$, where $K = I \cup J$

$\tilde{H}_K = \tilde{F}(e_i)$, if $\forall$ $e_i \in I - J$.

$= \tilde{G}(e_i)$, if $\forall$ $e_i \in J - I$.

$= \tilde{F}(e_i) \vee_R \tilde{G}(e_i)$, if $\forall e_i \in I \cap J$.

Here $\tilde{F}(e_i) \vee_R \tilde{G}(e_i)$ defined as

$\tilde{F}(e_i) \vee_R \tilde{G}(e_i) = \{< u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\min\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\} >, \forall u \in U, \forall e_i \in I \cap J\}$

Given condition is $\min\{\lambda_{1(e_i)}(u),\lambda_{2(e_i)}(u)\} \le \max\{A^-_{e_i}(u),B^-_{e_i}(u), \forall e_i \in I, \forall e_i \in J$ and $\forall u \in U$.

$\Rightarrow \min\{\lambda_{1(e_i)}(u),\lambda_{2(e_i)}(u)\} \notin (\max\{A^-_{e_i}(u),B^-_{e_i}(u)\},\max\{A^+_{e_i}(u),B^+_{e_i}(u)\})$

Hence $\tilde{F}_I \cup_R \tilde{G}_J$ is an ENCSS.

Definition

Let $\tilde{F}_I = \tilde{F}(e_i) = \{<u, A_{(e_i)}(u), \lambda_{1(e_i)}(u)>: \forall u \in U, \forall e_i \in I\}$ and $\tilde{G}_J = \tilde{G}(e_i) = \{<u, B_{(e_i)}(u), \lambda_{2(e_i)}(u)>: \forall u \in U, \forall e_i \in J\}$ are two NCSSs in U. We defined new NCSSs by interchanging the neutrosophic part of the two NCSSs. We denoted its by $\ddot{F}_I$, $\ddot{G}_J$ and defined by $\ddot{F}_I = \{<u, A(e_i), \lambda_2(e_i)>: \forall u \in U, e_i \in I\}$, $\ddot{G}_J = \{<u, B(e_i), \lambda_1(e_i)>: \forall u \in U, e_i \in J\}$ respectively.

Theorem 10

$\tilde{F}_I$ and $\tilde{G}_J$ are ENCSSs and $\ddot{F}_I$ and $\ddot{G}_J$ are INCSSs in U. Then $\tilde{F}_I \cup_P \tilde{G}_J$ is an INCSS in U.

Proof

Since $\tilde{F}_I = \{<u, A_{(e_i)}(u), \lambda_{1(e_i)}(u)>: \forall u \in U, \forall e_i \in I\}$ and $\tilde{G}_J = \{<u, B_{(e_i)}(u), \lambda_{2(e_i)}(u)>: \forall u \in U, \forall e_i \in J\}$ are ENCSSs, we have

For $\tilde{F}_I$

$T_{\lambda 1(e_i)}(u) \notin (T^-_{A(e_i)}(u), T^+_{A(e_i)}(u))$,

$I_{\lambda 1(e_i)}(u) \notin (I^-_{A(e_i)}(u), I^+_{A(e_i)}(u))$,

$F_{\lambda 1(e_i)}(u) \notin (F^-_{A(e_i)}(u), F^+_{A(e_i)}(u))$, $\forall$ $u \in U$, $\forall$ $e_i \in I$

For $\tilde{G}_J$

$T_{\lambda 2(e_i)}(u) \notin (T^-_{B(e_i)}(u), T^+_{B(e_i)}(u))$,

$F_{\lambda 2(e_i)}(u) \notin (F^-_{B(e_i)}(u), F^+_{B(e_i)}(u))$,

$F_{\lambda 2(e_i)}(u) \notin (F^-_{B(e_i)}(u), F^+_{B(e_i)}(u))$, $\forall$ $u \in U$, $\forall$ $e_i \in I$.

and $\ddot{F}_I = \{<u, A_{(e_i)}(u), \lambda_{2(e_i)}(u)>: \forall u \in U, \forall e_i \in I\}$ and $\ddot{G}_J = \{<u, B_{(e_i)}(u), \lambda_{1(e_i)}(u)>: \forall u \in U, \forall e_i \in J\}$

are INCSSs, then

$T^-_{A(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^+_{A(e_i)}(u)$,

$I^-_{A(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^+_{A(e_i)}(u)$,

$F^-_{A(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^+_{A(e_i)}(u)$, $\forall$ $u \in U$, $e_i \in I$.

and $T^-_{B(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^+_{B(e_i)}(u)$,

$I^-_{B(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^+_{B(e_i)}(u)$,

$F^-_{B(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^+_{B(e_i)}(u)$, $\forall$ $u \in U$, $\forall$ $e_i \in J$. By the definition of ENCSSs and INCSSs all the possibility are as under:

i(a). $T_{\lambda_1(e_i)}(u) \le T^{-}_{A(e_i)}(u) \le T_{\lambda_2(e_i)}(u) \le T^{+}_{A(e_i)}(u)$,

$I_{\lambda 1(e_i)}(u) \le I^{-}_{A(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^{+}_{A(e_i)}(u)$,

$F_{\lambda 1(e_i)}(u) \le F^{-}_{A(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^{+}_{A(e_i)}(u)$, $\forall$ u $\in$ U, $\forall$ $e_i \in$ I.

i(b). $T_{\lambda 2(e_i)}(u) \le T^{-}_{B(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^{+}_{B(e_i)}(u)$,

$I_{\lambda 2(e_i)}(u) \le I^{-}_{B(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^{+}_{B(e_i)}(u)$

$F_{\lambda 2(e_i)}(u) \le F^{-}_{B(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^{+}_{B(e_i)}(u)$, $\forall$ u $\in$ U, $\forall$ $e_i \in$ I

ii(a). $T^{-}_{A(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^{+}_{A(e_i)}(u) \le T_{\lambda 1(e_i)}(u)$

$I^{-}_{A(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^{+}_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u)$

$F^{-}_{A(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^{+}_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u), \forall e_i \in I, \forall u \in U.$

ii(b). $T^{-}_{B(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^{+}_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u)$

$I^{-}_{B(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^{+}_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u)$,

$F^{-}_{B(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^{+}_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u)$, $\forall$ $e_i \in$ I, $\forall$ u $\in$ U

iii(a). $T_{\lambda 1(e_i)}(u) \le T^{-}_{A(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^{+}_{A(e_i)}(u)$,

$I_{\lambda 1(e_i)}(u) \le I^{-}_{A(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^{+}_{A(e_i)}(u)$,

$F_{\lambda 1(e_i)}(u) \le F^{-}_{A(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^{+}_{A(e_i)}(u)$, $\forall$ $e_i \in$ I and $\forall$ u $\in$ U

iii(b). $T^{-}_{B(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^{+}_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u)$

$I^{-}_{B(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^{+}_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u)$

$F^{-}_{B(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^{+}_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u)$, $\forall$ $e_i \in$ I $\forall$ u $\in$ U,

iv(a). $T^{-}_{A(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^{+}_{A(e_i)}(u) \le T_{\lambda 1(e_i)}(u)$,

$I^{-}_{A(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^{+}_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u)$,

$F^{-}_{A(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^{+}_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u), \forall e_i \in I, \forall u \in U.$

iv(b). $T_{\lambda 2(e_i)}(u) \le T^{-}_{B(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^{+}_{B(e_i)}(u)$,

$I_{\lambda 2(e_i)}(u) \le I^{-}_{B(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^{+}_{B(e_i)}(u)$,

$F_{\lambda 2(e_i)}(u) \le F^{-}_{B(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^{+}_{B(e_i)}(u)$, $\forall$ u $\in$ U, $\forall$ $e_i \in$ I

Since p-union of $\tilde{F}_I \cup_P \tilde{G}_J = \tilde{H}_C$, where $C = I \cup J$.

$\tilde{F}_I \cup_P \tilde{G}_J = \tilde{F}(e_i)$ if $e_i \in$ I - J

$= \tilde{G}(e_i)$, if $e_i \in$ J – I.

$= \tilde{F}(e_i) \vee_P \tilde{G}(e_i)$, if $e_i \in I \cap J$.

Here $\tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ defined as

$\tilde{F}(e_i) \vee_P \tilde{G}(e_i) = \{< u, r\max\{A_{e_i}(u), B_{e_i}(u)\}, r\max\{\lambda_{1(e_i)}(u), \lambda_{2(e_i)}(u)\}, \forall u \in U \text{ and } \forall e_i \in I \cap J\}$.

Case 1

If $\tilde{H}_C = \tilde{F}(e_i)$, if $e_i \in$ I – J, then from i(a)., and ii(a). We have

$T_{\lambda 1(e_i)}(u) = T^{-}_{A(e_i)}(u)$

$I_{\lambda 1(e_i)}(u) = I^{-}_{A(e_i)}(u)$, $F_{\lambda 1(e_i)}(u) = F^{-}_{A(e_i)}(u)$ and $T_{\lambda 1(e_i)}(u) = T^{+}_{A(e_i)}(u)$,

$I_{\lambda 1(e_i)}(u) = I^{+}_{A(e_i)}(u)$, $F_{\lambda 1(e_i)}(u) = F^{+}_{A(e_i)}(u)$, $\forall$ $e_i \in$ I and $\forall$ u $\in$ U.

Thus

$T^{-}_{A(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^{+}_{A(e_i)}(u)$, $I^{-}_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^{+}_{A(e_i)}(u)$,

$F^{-}_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^{+}_{A(e_i)}(u)$

$\forall$ $e_i \in$ I $\forall$ u $\in$ U.

Case 2

$\tilde{H}_C = e_i \in J - I$, if $e_i \in J - I$ then from i(b) and ii(b)., we have

$T_{\lambda 2(e_i)}(u) = T^{-}_{B(e_i)}(u)$,

$I_{\lambda 2(e_i)}(u) = I^{-}_{B(e_i)}(u)$, $F_{\lambda 2(e_i)}(u) = F^{-}_{B(e_i)}(u)$ and

$T_{\lambda 2(e_i)}(u) = T^{+}_{B(e_i)}(u)$, $I_{\lambda 2(e_i)}(u) = I^{+}_{B(e_i)}(u)$,

$F_{\lambda 2(e_i)}(u) = F^{+}_{B(e_i)}(u)$, $\forall$ $e_i \in$ J $\forall$ u $\in$ U. Thus

$T^{-}_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^{+}_{B(e_i)}(u)$, $I^{-}_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^{+}_{B(e_i)}(u)$,

$F^{-}_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^{+}_{B(e_i)}(u)$,

$\forall$ $e_i \in$ J - I $\forall$ u $\in$ U.

Case 3

$\tilde{H}_C = \tilde{F}(e_i) \vee_P \tilde{G}(e_i)$ if $e_i \in I \cap J$, then from i(a) and i(b)., we have

$T^{-}_{A(e_i)}(u) \le T_{\lambda 1(e_i)}(u) \le T^{+}_{A(e_i)}(u)$, $I^{-}_{A(e_i)}(u) \le I_{\lambda 1(e_i)}(u) \le I^{+}_{A(e_i)}(u)$,

$F^{-}_{A(e_i)}(u) \le F_{\lambda 1(e_i)}(u) \le F^{+}_{A(e_i)}(u)$ $\forall$ $e_i \in$ J, $\forall$ u $\in$ U and

$T^{-}_{B(e_i)}(u) \le T_{\lambda 2(e_i)}(u) \le T^{+}_{B(e_i)}(u)$, $I^{-}_{B(e_i)}(u) \le I_{\lambda 2(e_i)}(u) \le I^{+}_{B(e_i)}(u)$,

$F^{-}_{B(e_i)}(u) \le F_{\lambda 2(e_i)}(u) \le F^{+}_{B(e_i)}(u)$

$\forall$ $e_i \in$ J, $\forall$ u $\in$ U

Hence, if $e_i \in$ I ∩ J, then

$\max\{A^{-}_{e_i}(u), B^{-}_{e_i}(u)\} \le (\lambda_{1(e_i)}(u) \vee \lambda_{2(e_i)}(u)) \le \max\{A^{+}_{e_i}(u), B^{+}_{e_i}(u)\}$

in all the three cases.

$\tilde{F}_I \cup_P \tilde{G}_J$ is an INCSS in U.

Conclusion

In this paper we have defined some operations such as P-union, P-intersection, R-union, R-intersection for neutrosophic cubic soft sets. We have also defined some operation of INCSSs and ENCSSs. We have proved some theorems on INCSSs and ENCSSs. We have discussed various approaches INCSSs and ENCSSs. We hope that proposed theorems and operations will be helpful to multi attribute group decision making problems in neutrosophic cubic soft set environment.

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