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MHD free convective oscillatory Rivlin-Ericksen fluid flow past a porous plate embedded in a porous medium in the presence of heat source with chemical reaction

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ABSTRACT

An analysis is performed to study the unsteady two dimensional MHD free convective oscillatory Rivlin-Ericksen fluid flow of an electrically conducting, incompressible visco-elastic fluid past an infinite vertical porous plate embedded in a porous medium, through which suction occurs with constant velocity and taking into account the effects of both chemical reaction and heat source. A uniform magnetic field is assumed to be applied transversely to the direction of the free stream in the presence of induced magnetic field. The governing equations describing the problem are solved by using the perturbation method. Numerical results for the velocity, temperature and concentration distributions as well as skin friction are obtained and reported in tabular form as well as graphically for several values of pertinent parameters which are of physical and engineering interest. The numerical results of the skin friction of visco-elastic fluid are compared with the corresponding flow problems for an ordinary fluid.

Key words: MHD, Rivlin-Ericksen fluid, porous medium, heat source, chemical reaction.

INTRODUCTION

The influence of magnetic field on viscous incompressible flow of an electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications are in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

In most of the investigations of elastico-viscous fluids, the flow has been considered slow and parameters characterizing the elastic properties of the fluid have been assumed small. In fact the increase emergence of non-Newtonian fluids such as the molten plastics, pulps, emulsions, aqueous solutions of polyacrylamid and polyisobutylene etc., as important raw materials and chemical products in a large variety of industrial processes has stimulated a considerable attention in recent years to the study of non-Newtonian fluids and their related transport processes. General stress-strain relations are expressed by highly complicated non-linear differential equations; to work out solutions for such a class of fluids even for slow flows is not an easy task. The stress-strain velocity relations of classical hydrodynamics and the Rheological behavior of the non-Newtonian liquids have been studied by Rivlin [1] and Rivlin-Ericksen [2].

Jonah Philliph et al. [17] studied the effects of thermal radiation and MHD on the unsteady free convection and mass transform flow past an exponentially accelerated vertical plate with variable temperature. Gireesh Kumar et al. [13] discussed the effects of chemical reaction on transient MHD convection flow past a vertical surface embedded in a porous medium with oscillating temperature. Hemanth Poonia and Chaudhary [15] analyzed the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation. Girish Kumar et al. [14] examined the mass transfer effects on MHD flows exponentially accelerated vertical plate in the presence of chemical reaction through porous media.

The study of convective fluid flow with mass transfer along a vertical porous plate in the presence of magnetic field and internal heat generation receiving considerable attention due to its useful applications in different branches of science and technology such as cosmical and geophysical science, fire engineering, combustion modeling etc. Vajravelu [25] studied natural convection flow along a heated semi-infinite vertical plate with internal heat generation.

Chamkha [7] discussed unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat generation. Ahmed [4] looked the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Sharma and Singh [22] discussed the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Sharma et al. [21] analyzed the heat and mass transfer effects on unsteady MHD free convective flow along a vertical porous plate with internal heat generation and variable suction. Soundalgekar [23] investigated the unsteady free convection flow along vertical porous plate with different boundary conditions and viscous dissipation effect.

Convection in porous medium has important applications in many areas including thermal energy storage, flow through filtering devices, utilization of geothermal energy, oil extraction, high performance insulation for buildings, paper industry etc. Hence combined study may give some vital information which will surely be helpful in developing other relevant areas. Kishore et al. [18] analyzed the effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions. Israel – Cookey et al. [16] studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction.

Chaudhary and Arpita Jain [9] discussed the MHD heat and mass diffusion flow by natural convection past a surface embedded in a porous medium. Seethamahalakshmi et al. [20] examined the effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow fast a semi-infinite vertical moving in a porous medium with heat source and suction. Abdel-Nasser Osman et al. [3] investigated the analytical solution of thermal radiation and chemical reaction effects on unsteady MHD convection through porous medium with heat source/sink. Chamkha and Khaled [7] looked the effects of hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in fluid saturated porous medium.

Mass diffusion rates can changed tremendously with chemical reactions. In majority cases, a chemical reaction depends on the concentration the concentration of the species itself. A reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself [Cussler [10]]. A few representative areas of interest in which heat and mass transfer combined along with chemical reaction play an important role in chemical industries like in food processing and polymer production.

Chambre and Young [6] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al. [11, 12] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with uniform heat flux and mass transfer. Again, they are also discussed the mass transfer effects on moving isothermal infinite vertical plate in the presence of chemical reaction. The dimensionless governing equations were solved by the usual Laplace Transform technique. Sudheer Babu et al. [24] have examined the radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation. Muthucumaraswamy and Kulaivel [19] looked the chemical reaction effects on moving infinite vertical plate with uniform heat flux and variable mass diffusion. Sreekanth et. al. [26] has studied the hydromagnetic unsteady Hele-Shaw flow of a visco-elastic Rivlin-Ericksen fluid through porous media. Heat and mass transfer in MHD visco-elastic fluid flow via a porous medium over stretching sheet with chemical reaction were analyzed by Alharbi et. al.[27]. Recently, Sreelatha et. al. [28] studied the chemical reaction effects on MHD free convective oscillatory flow past a porous plate in a porous medium with heat source.

The main objective of the present analysis is to study the unsteady two-dimensional MHD free convective oscillatory Rivlin-Ericksen fluid flow of an electrically conducting incompressible visco-elastic fluid past an infinite vertical porous plate embedded in a porous medium, in which suction occurs with constant velocity and chemical reaction in the presence of a heat source. The equations of continuity, momentum, energy and diffusion which govern the flow field are solved to the best possible solution. The effects of various governing parameters on the velocity, temperature and concentration are presented graphically. The values of skin friction coefficient for both viscous as well as visco-elastic fluid are tabulated. The present investigations can be utilized as a basis for studying more complex systems that arise in engineering and industrial application.

MATERIALS AND METHODS

We consider the unsteady two-dimensional MHD free convective oscillatory flow of an electrically conducting incompressible visco-elastic fluid past an infinite vertical porous plate embedded in a porous medium, through which suction occurs with constant velocity and chemical reaction in the presence of a heat source. The x' - axis is along the plate in the upward direction and the y' - axis is normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). There is a chemical reaction between the diffusing species and the fluid. The foreign mass present in the flow is assumed to be a low level and hence Soret and Dafour effects are negligible. Under these assumptions, the governing equations of the flow field are:

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum equation

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial p'}{\partial x'} - \rho g_{x'} + \nu\rho\left(1 + \lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \rho\frac{\nu'}{k'}u' \tag{2}$$

Energy equation

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q'}{\rho C_p}(T' - T'_\infty) \tag{3}$$

Diffusion equation

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'(C' - C'_\infty) \tag{4}$$

where u' and v' are the components of the velocity parallel and perpendicular to the plate, $'$ - the permeability of the porous medium and λ_1 is the visco-elastic coefficient.

The boundary conditions are

$$\left.\begin{aligned} u' = 0, v' = -v_0,\ \frac{\partial T'}{\partial y'} = -\frac{q'}{\kappa},\ C' = C'_w \quad &\text{at} \quad y' = 0 \\ u' \to U' = U_0\left(1 + \varepsilon e^{i\omega' t'}\right),\ T' \to T'_\infty,\ C' \to C'_\infty \quad &\text{as} \quad y' \to \infty \end{aligned}\right\} \tag{5}$$

where v_0 is the constant suction velocity and negative sign indicates that it is towards the plate. Here ε (ε<<1) - a constant quantity.

For the free stream, equation (2) becomes

$$\rho \frac{dU'}{dt'} = -\frac{\partial p'}{\partial x'} - \rho_\infty g_{x\prime} - \sigma B_0^2 U' - \rho \frac{\nu'}{k'} U' \tag{6}$$

with the help of equation (6), equation (2) becomes

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = \rho\frac{dU'}{dt'} + g_{x\prime}(\rho_\infty - \rho) + \nu\rho\left(1 + \lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} - \left(\sigma B_0^2 + \rho\frac{\nu'}{k'}\right)(u' - U') \tag{7}$$

The state equation is

$$g_{x'}(\rho_\infty - \rho) = g_{x'}\rho\beta(T' - T'_\infty) + g_{x'}\rho\beta^*(C' - C'_\infty) \tag{8}$$

From equations (7) and (8), we have

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g_{x'}\beta(T' - T'_\infty) + g_{x'}\beta^*(C' - C'_\infty) + \nu\left(1 + \lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu'}{k'}\right)(u' - U') \tag{9}$$

From equation (1)-

$$v' = -v_0 \quad (v_0 > 0) \tag{10}$$

With the help of equation (10), equations (9), (3) and (4) can be written as-

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g_{x'}\beta(T' - T'_\infty) + g_{x'}\beta^*(C' - C'_\infty) + \nu\left(1 + \lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu'}{k'}\right)(u' - U') \tag{11}$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q'}{\rho C_p}(T' - T'_\infty) \tag{12}$$

$$\frac{\partial C'}{\partial t'} - v_0\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'(C' - C'_\infty) \tag{13}$$

Using the transformations

$$\left.\begin{aligned} &y = \frac{y' v_0}{\nu},\ t = \frac{t' v_0^2}{4\nu},\ u = \frac{u'}{U_0},\ U = \frac{U'}{U_0},\ \omega = \frac{4\nu\omega'}{v_0^2},\ T = \frac{T' - T'_\infty}{(\nu q'/k v_0)},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ &Gr = \frac{g_{x'}\beta\nu^2 q'}{kU_0 v_0^3},\ Gc = \frac{\nu g_{x'}\beta^*(C'_w - C'_\infty)}{U_0 v_0^2},\ Pr = \frac{\rho\nu C_p}{\kappa},\ Ec = \frac{kU_0^2 v_0}{\nu C_p q'},\ M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \\ &Q = \frac{\nu^2 Q'}{k v_0^2},\ Kr = \frac{K'_r \nu}{v_0^2},\ Sc = \frac{\nu}{D},\ k = \frac{\nu^2 k'}{v_0^2},\ \lambda = \frac{\lambda_1 v_0^2}{4\nu} \end{aligned}\right\} \tag{14}$$

Using the non-dimensional quantities (14), equations (11), (12) and (13) reduce to-

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4}\frac{dU}{dt} + GrT + GcC + \left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - N(u - U) \tag{15}$$

$$Pr\left(\frac{1}{4}\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} + PrEc\left(\frac{\partial u}{\partial y}\right)^2 + QT \tag{16}$$

$$Sc\left(\frac{1}{4}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y}\right) = \frac{\partial^2 C}{\partial y^2} - KrScC \tag{17}$$

where $N = M + \frac{1}{k}$ and λ is the visco-elastic parameter.

With boundary conditions

$$\left.\begin{array}{ll} u = 0,\ \frac{\partial T}{\partial y} = -1,\ C = 1 & \text{at} \quad y = 0 \\ u \to U(t) = 1 + \varepsilon e^{i\omega t},\ T \to 0,\ C \to 0 & \text{as} \quad y \to \infty \end{array}\right\} \quad (18)$$

SOLUTION

Equations (15) – (17) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate as:

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \cdots\cdots\cdots\cdots\cdots\cdots \quad (19)$$
$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \cdots\cdots\cdots\cdots\cdots\cdots \quad (20)$$
$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) + \cdots\cdots\cdots\cdots\cdots\cdots \quad (21)$$

Substituting equations (19)-(21) in equations (15)-(17), we obtain the following system of differential equations-

$$\frac{d^2u_0}{dy^2} + \frac{du_0}{dy} - Nu_0 = -[GrT_0 + GcC_0 + N] \quad (22)$$
$$(1 + i\lambda\omega)\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} - \left(\frac{i\omega}{4} + N\right)u_1 = -\left[\frac{i\omega}{4} + GrT_1 + GcC_1 + N\right] \quad (23)$$
$$\frac{d^2T_0}{dy^2} + Pr\frac{dT_0}{dy} + QT_0 = -PrEc\left(\frac{du_0}{dy}\right)^2 \quad (24)$$
$$\frac{d^2T_1}{dy^2} + Pr\frac{dT_1}{dy} + \left(Q - \frac{i\omega}{4}Pr\right)T_1 = -2PrEc\left(\frac{du_0}{dy}\right)\left(\frac{du_1}{dy}\right) \quad (25)$$
$$\frac{d^2C_0}{dy^2} + Sc\frac{dC_0}{dy} - KrScC_0 = 0 \quad (26)$$
$$\frac{d^2C_1}{dy^2} + Sc\frac{dC_1}{dy} - Sc\left(\frac{i\omega}{4} - Kr\right)C_1 = 0 \quad (27)$$

The corresponding boundary conditions are-

$$\left.\begin{array}{ll} u_0 = 0, u_1 = 0,\ \frac{dT_0}{dy} = -1,\ \frac{dT_1}{dy} = 0,\ C_0 = 1,\ C_1 = 0 & \text{at} \quad y = 0 \\ u_0 \to 1, u_1 \to 1, T_0 \to 0, T_1 \to 0,\ C_0 \to 0,\ C_1 \to 0 & \text{as} \quad y \to \infty \end{array}\right\} \quad (28)$$

In order to solve the system of differential equations (22)-(27), we put-

$$\left.\begin{array}{l} u_0(y) = u_{01}(y) + Ecu_{02}(y) \\ T_0(y) = T_{01}(y) + EcT_{02}(y) \end{array}\right\} \quad (29)$$

and

$$\left.\begin{array}{l} u_1(y) = u_{11}(y) + Ecu_{12}(y) \\ T_1(y) = T_{11}(y) + EcT_{12}(y) \end{array}\right\} \quad (30)$$

Using equations (29) and (30) in equations (22)-(25) and equating the coefficients of Ec^0 and Ec^1, we obtain

$$\frac{d^2u_{01}}{dy^2} + \frac{du_{01}}{dy} - Nu_{01} = -[GrT_{01} + GcC_{01} + N] \quad (31)$$
$$\frac{d^2u_{02}}{dy^2} + \frac{du_{02}}{dy} - Nu_{02} = -[GrT_{02} + GcC_{02}] \quad (32)$$
$$\frac{d^2T_{01}}{dy^2} + Pr\frac{dT_{01}}{dy} + QT_{01} = 0 \quad (33)$$
$$\frac{d^2T_{02}}{dy^2} + Pr\frac{dT_{02}}{dy} + QT_{02} = -PrEc\left(\frac{du_{01}}{dy}\right)^2 \quad (34)$$
$$(1 + i\lambda\omega)\frac{d^2u_{11}}{dy^2} + \frac{du_{11}}{dy} - \left(\frac{i\omega}{4} + N\right)u_{11} = -\left[\frac{i\omega}{4} + GrT_{11} + GcC_{11} + N\right] \quad (35)$$
$$(1 + i\lambda\omega)\frac{d^2u_{12}}{dy^2} + \frac{du_{12}}{dy} - \left(\frac{i\omega}{4} + N\right)u_{12} = -[GrT_{12} + GcC_{12}] \quad (36)$$
$$\frac{d^2T_{11}}{dy^2} + Pr\frac{dT_{11}}{dy} + \left(Q - \frac{i\omega}{4}Pr\right)T_{11} = 0 \quad (37)$$
$$\frac{d^2T_{12}}{dy^2} + Pr\frac{dT_{12}}{dy} + \left(Q - \frac{i\omega}{4}Pr\right)T_{12} = -2PrEc\left(\frac{du_{01}}{dy}\right)\left(\frac{du_{11}}{dy}\right) \quad (38)$$

The corresponding boundary conditions (28) becomes-

$$\left.\begin{array}{l}\left.\begin{array}{l} u_{01}=0, u_{02}=0, u_{11}=0, u_{12}=0, \\ \frac{dT_{01}}{dy}=-1,\ \frac{dT_{02}}{dy}=0, \frac{dT_{11}}{dy}=0,\ \frac{dT_{12}}{dy}=0, \\ C_0=1,\ C_1=0 \end{array}\right\} \text{ at } \ y=0 \\ \\ \left.\begin{array}{l} u_{01}\to 1, u_{02}\to 0, u_{11}\to 1, u_{12}\to 0, \\ T_{01}\to 0,\ T_{02}\to 0, T_{11}\to 0,\ T_{12}\to 0, \\ C_0\to 0,\ C_1\to 0 \end{array}\right\} \text{ as } \ y\to\infty \end{array}\right\} \tag{39}$$

Solving the differential equations (26), (27), (31)-(38), using boundary conditions (39), we get

$$u_{01}=1+a_3e^{b_2y}+a_4e^{c_2y}+a_5e^{a_2y} \tag{40}$$

$$u_{02}=a_6e^{b_2y}+a_7e^{2a_2y}+a_8e^{2b_2y}+a_9e^{2c_2y}+a_{10}e^{(a_2+b_2)y}+a_{11}e^{(b_2+c_2)y}+a_{12}e^{(c_2+a_2)y}+a_{13}e^{a_2y} \tag{41}$$

$$u_{11}=1-e^{a_{15}y} \tag{42}$$

$$u_{12}=a_{16}e^{b_{11}y}+a_{17}e^{(a_2+a_{15})y}+a_{18}e^{(b_2+a_{15})y}+a_{19}e^{(c_2+a_{15})y}+a_{20}e^{a_{15}y} \tag{43}$$

$$T_{01}=-\frac{1}{b_2}e^{b_2y} \tag{44}$$

$$T_{02}=b_3e^{2a_2y}+b_4e^{2b_2y}+b_5e^{2c_2y}+b_6e^{(a_2+b_2)y}+b_7e^{(b_2+c_2)y}+b_8e^{(c_2+a_2)y}+b_9e^{b_2y} \tag{45}$$

$$T_{11}=0 \tag{46}$$

$$T_{12}=b_{12}e^{(a_2+a_{15})y}+b_{13}e^{(b_2+a_{15})y}+b_{14}e^{(c_2+a_{15})y}+b_{15}e^{b_{11}y} \tag{47}$$

$$C_0=e^{c_2y} \tag{48}$$

$$C_1=0 \tag{49}$$

With the help of (40)-(47), the equations (29) and (30) becomes

$$u_0=[1+a_3e^{b_2y}+a_4e^{c_2y}+a_5e^{a_2y}] \\ +Ec\left[a_6e^{b_2y}+a_7e^{2a_2y}+a_8e^{2b_2y}+a_9e^{2c_2y}+a_{10}e^{(a_2+b_2)y}+a_{11}e^{(b_2+c_2)y}+a_{12}e^{(c_2+a_2)y}+a_{13}e^{a_2y}\right] \tag{50}$$

$$T_0=\left[-\frac{1}{b_2}e^{b_2y}\right]+Ec\left[b_3e^{2a_2y}+b_4e^{2b_2y}+b_5e^{2c_2y}+b_6e^{(a_2+b_2)y}+b_7e^{(b_2+c_2)y}+b_8e^{(c_2+a_2)y}+b_9e^{b_2y}\right] \tag{51}$$

$$u_1=[1-e^{a_{15}y}]+Ec\left[a_{16}e^{b_{11}y}+a_{17}e^{(a_2+a_{15})y}+a_{18}e^{(b_2+a_{15})y}+a_{19}e^{(c_2+a_{15})y}+a_{20}e^{a_{15}y}\right] \tag{52}$$

$$T_1=Ec\left[b_{12}e^{(a_2+a_{15})y}+b_{13}e^{(b_2+a_{15})y}+b_{14}e^{(c_2+a_{15})y}+b_{15}e^{b_{11}y}\right] \tag{53}$$

Finally, using (48)-(53) in equations (19)-(21), the velocity, temperature and concentration fields are as follows-

$$u(y)=[\{1+a_3e^{b_2y}+a_4e^{c_2y}+a_5e^{a_2y}\} \\ +Ec\left\{a_6e^{b_2y}+a_7e^{2a_2y}+a_8e^{2b_2y}+a_9e^{2c_2y}+a_{10}e^{(a_2+b_2)y}+a_{11}e^{(b_2+c_2)y}+a_{12}e^{(c_2+a_2)y}+a_{13}e^{a_2y}\right\}] \\ +\varepsilon(\cos\omega t+i\sin\omega t)[1-e^{a_{15}y}+Ec\left\{a_{16}e^{b_{11}y}+a_{17}e^{(a_2+a_{15})y}+a_{18}e^{(b_2+a_{15})y}+a_{19}e^{(c_2+a_{15})y}+\right. \\ \left.a_{20}e^{a_{15}y}\right\}] \tag{54}$$

$$T(y)=\left[-\frac{1}{b_2}e^{b_2y}+Ec\left\{b_3e^{2a_2y}+b_4e^{2b_2y}+b_5e^{2c_2y}+b_6e^{(a_2+b_2)y}+b_7e^{(b_2+c_2)y}+b_8e^{(c_2+a_2)y}+b_9e^{b_2y}\right\}\right] \\ +\varepsilon(\cos\omega t+i\sin\omega t)\left[Ec\left\{b_{12}e^{(a_2+a_{15})y}+b_{13}e^{(b_2+a_{15})y}+b_{14}e^{(c_2+a_{15})y}+b_{15}e^{b_{11}y}\right\}\right] \tag{55}$$

$$C(y)=e^{c_2y} \tag{56}$$

The skin friction at the wall of the plate is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=[\{a_3b_2+a_4c_2+a_5a_2\} \\ +Ec\{a_6b_2+2a_7a_2+2a_8b_2+2a_9c_2+a_{10}(a_2+b_2)+a_{11}(b_2+c_2)+a_{12}(c_2+a_2)+a_{13}a_2\}] \\ +\varepsilon(\cos\omega t+i\sin\omega t)[-a_{15}+Ec\{a_{16}b_{11}+a_{17}(a_2+a_{15})+a_{18}(b_2+a_{15})+a_{19}(c_2+a_{15})+a_{20}a_{15}\}]$$

RESULTS AND DISCUSSION

In the preceding sections, the problem of an unsteady MHD free convective oscillatory Rivlin-Ericksen fluid flow of a visco-elastic, incompressible, electrically conducting and chemically reacting fluid past an infinite vertical porous plate in a porous medium with heat source was formulated and solved by perturbation method. The approximate solutions were obtained for velocity, temperature and concentration field in addition to skin friction coefficient. To illustrate the behavior of these physical quantities, numeric values were computed with respect to the variations in the governing parameters viz., chemical reaction parameter (*K*r), visco-elastic parameter (λ), heat source parameter (Q), Prandtl number (*P*r), Eckert number (*E*c), Schmidt number (*S*c), Grashof number (*G*r and *G*c), magnetic parameter (M) and permeability parameter (k).

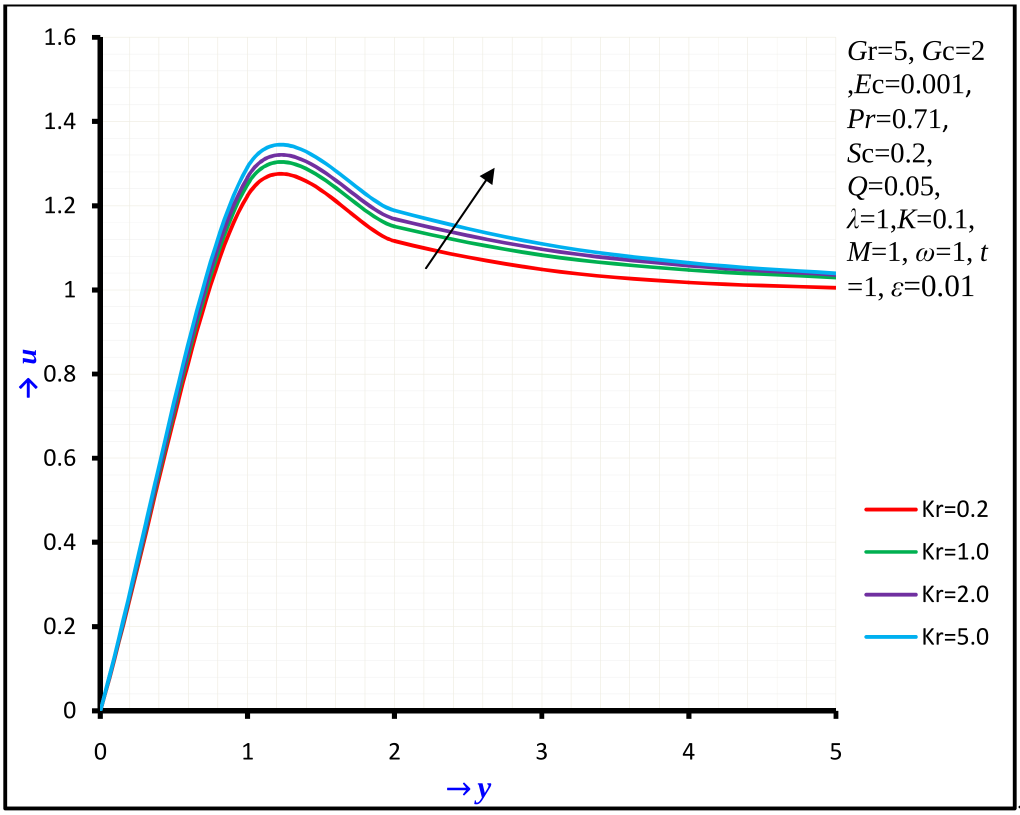

Fig. 1(a): Velocity profiles for different values of Kr

Figs. 1(a) and 1(b) illustrate the effect of chemical reaction parameter *K*r on velocity and concentration distributions. From Fig. 1(a), it is clear that the velocity at the start of the boundary layer increases slowly till it attains the maximum value, after that velocity decreases as y increases and this trend is seen for all the values of *K*r. Also as *K*r increases, the velocity increases. Fig. 1(b) shows that an increasing *K*r results in decreasing concentration of species in the boundary layer. This is due to the fact that destructive chemical reduces the solutal boundary thickness and increases the mass transfer.

The effect of heat source parameter Q on the velocity and temperature is shown in Figs. 2(a) and 2(b). Fig. 2(a) indicates that when heat is generated, the buoyancy force increase which induces the flow rate to increase giving rise to the increase in the velocity profiles. Also from Fig. 2(b), it is seen that when Q increases, the temperature T also increases.

The influence of magnetic parameter M on the velocity is displayed in Fig. 3. It is seen that the existence of the magnetic field reduces the velocity which shows that the velocity decreases in the presence of magnetic field, as compared to its absence. The application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive type of force called Lorentz force. This force has the tendency to slow down the fluid. This trend is apparent in Fig. 3.

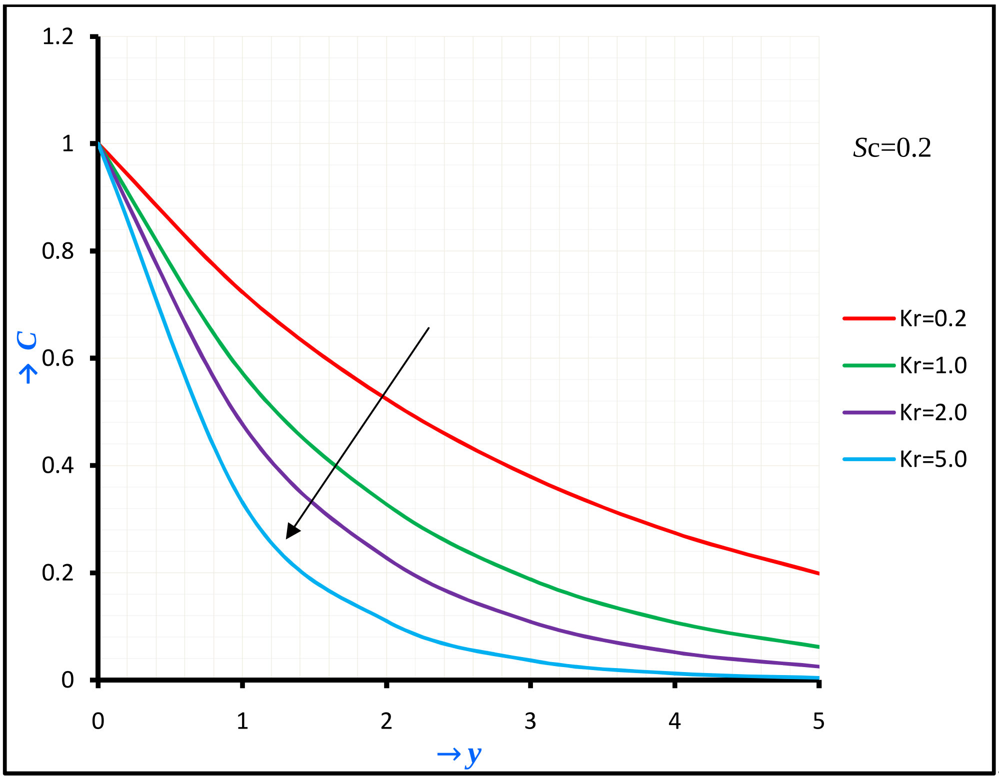

Fig. 1(b): Concentration profiles for different values of Kr

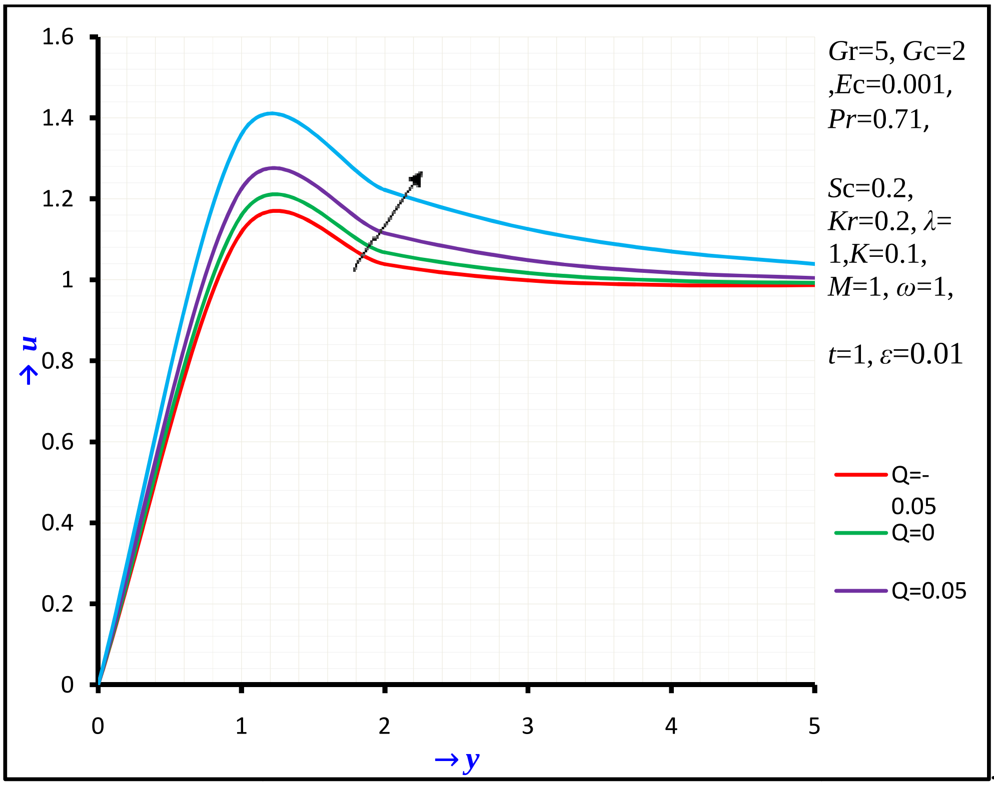

Fig. 2(a): Velocity profiles for different values of Q

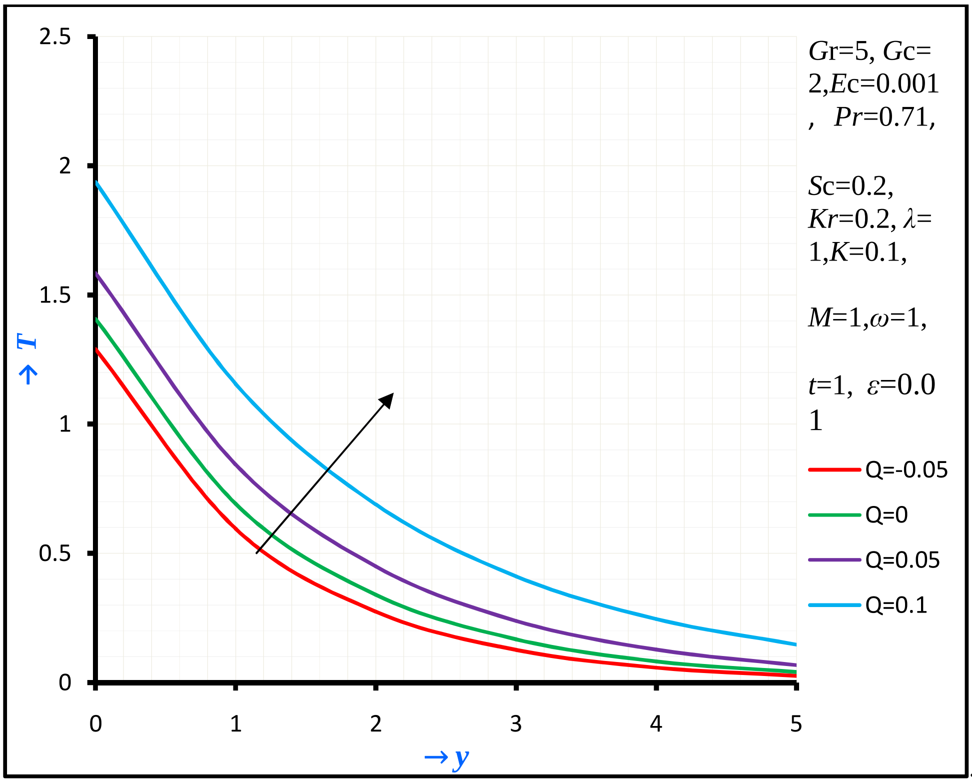

Fig. 2(b): Temperature profiles for different values of Q

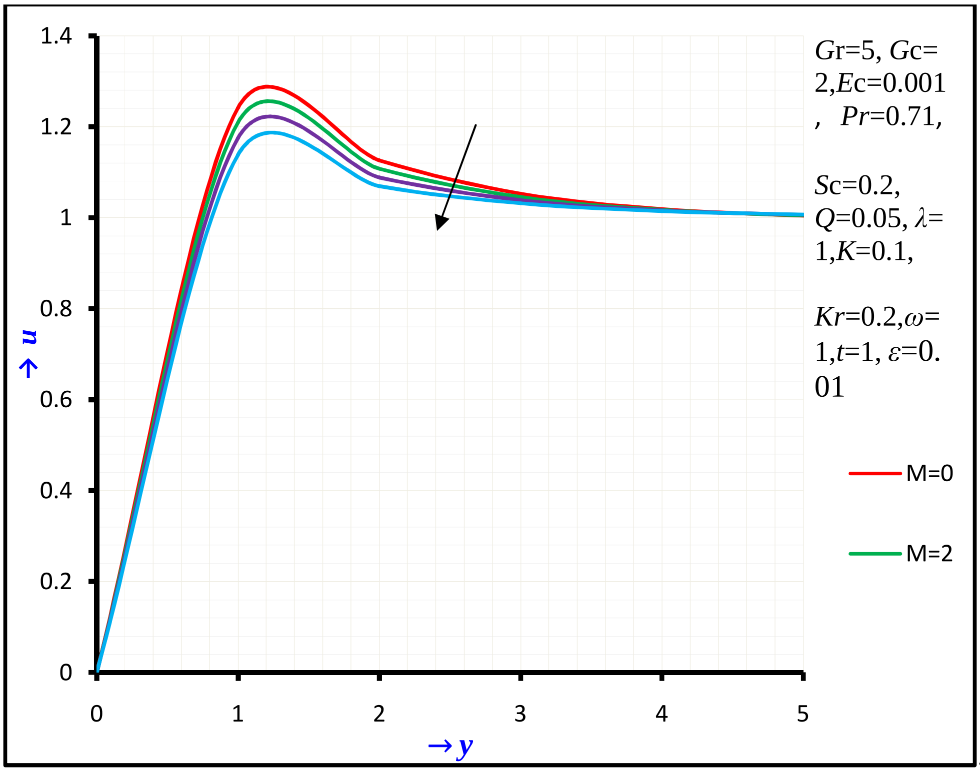

Fig.3: Velocity profiles for different values of M

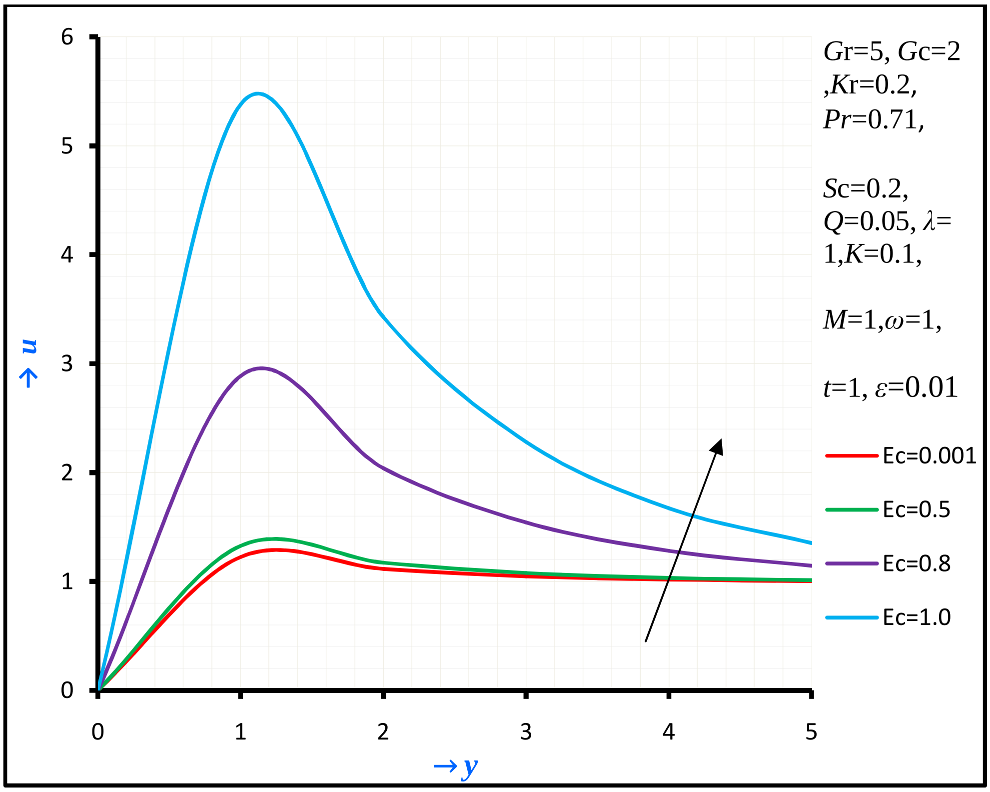

Fig.4: Velocity profiles for different values of Ec

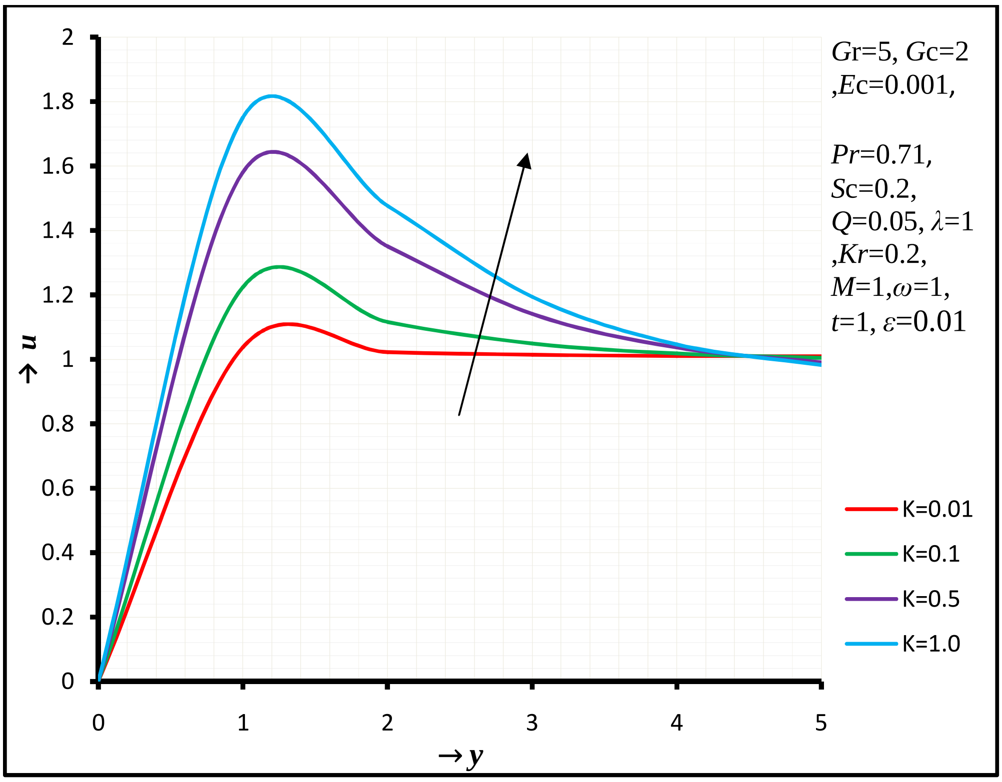

Fig.5: Velocity profiles for different values of K

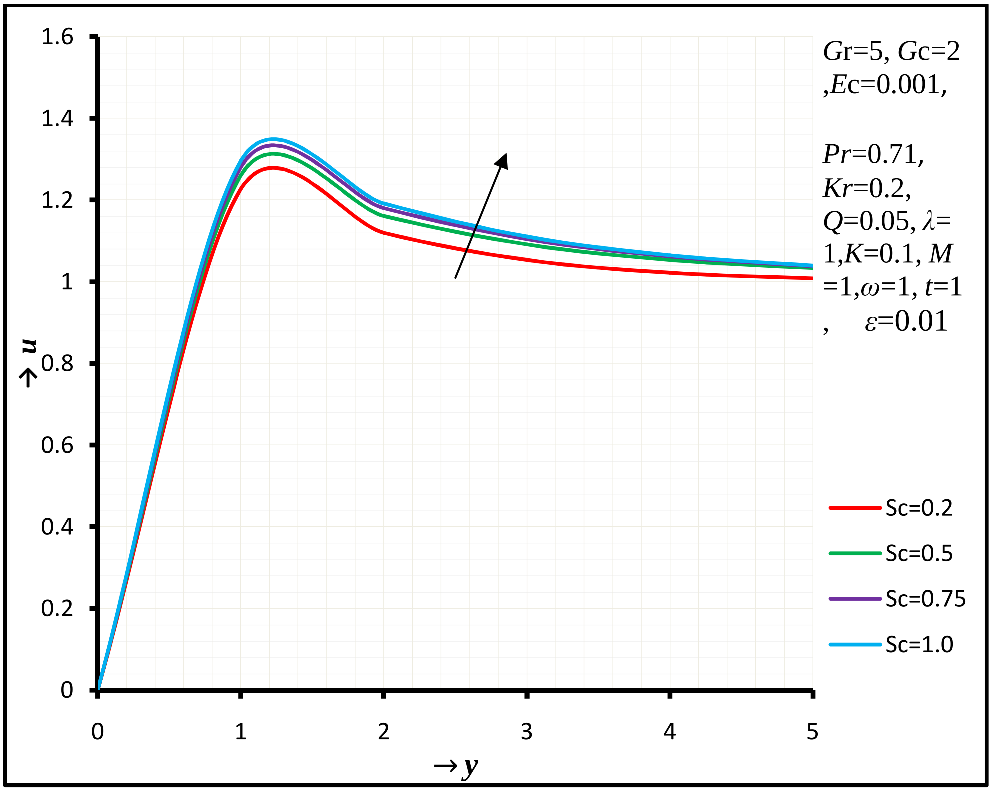

Fig. 6(a): Velocity profiles for different values of Sc

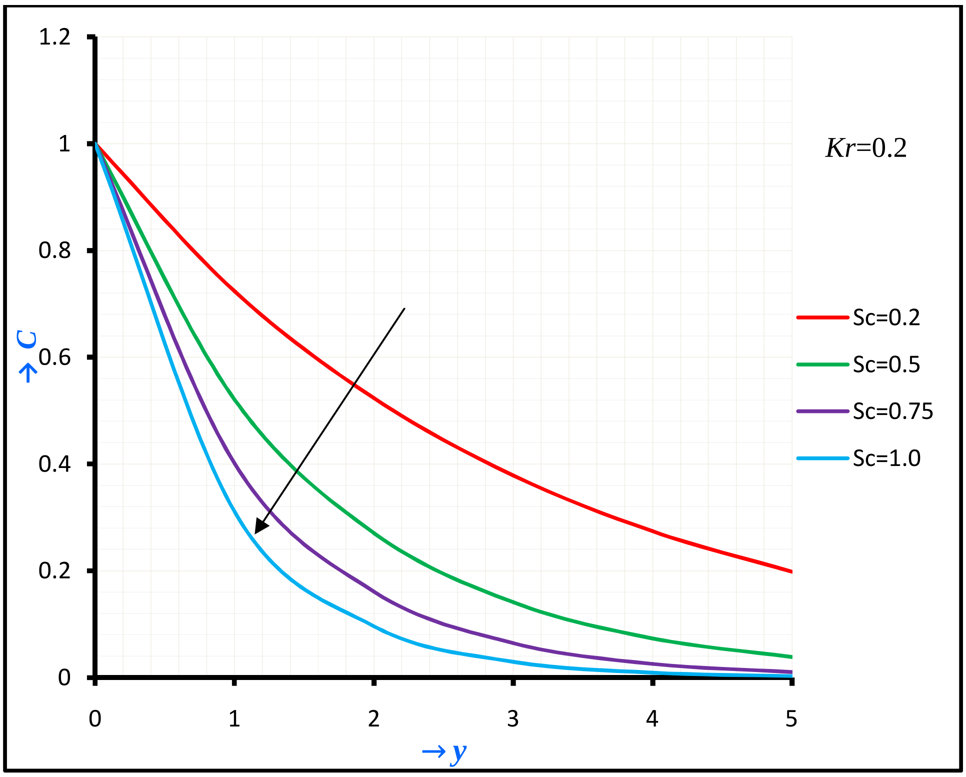

Fig. 6(b): Concentration profiles for different values of Sc

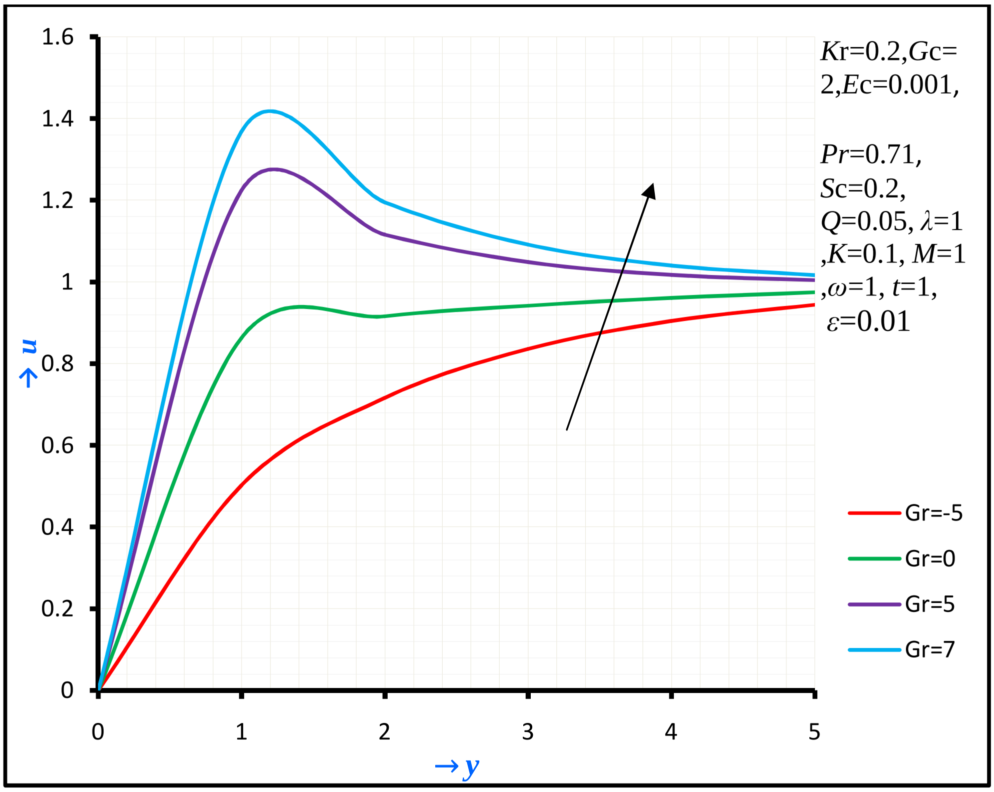

Fig. 7(a): Velocity profiles for different values of Gr

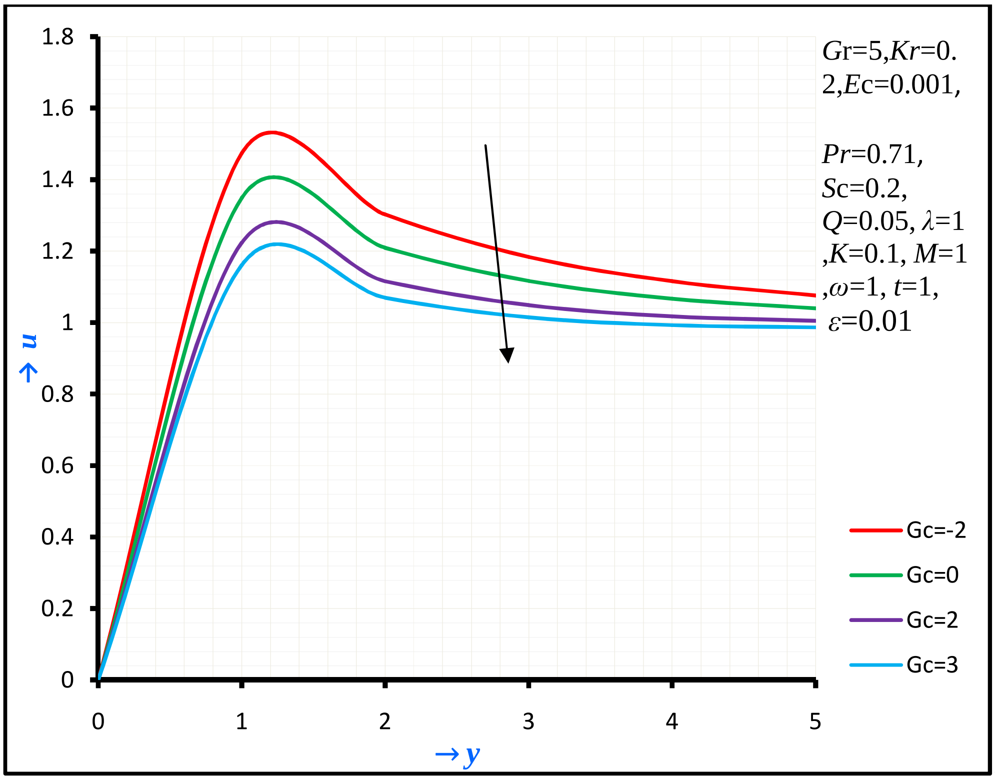

Fig. 7(b): Velocity profiles for different values of Gc

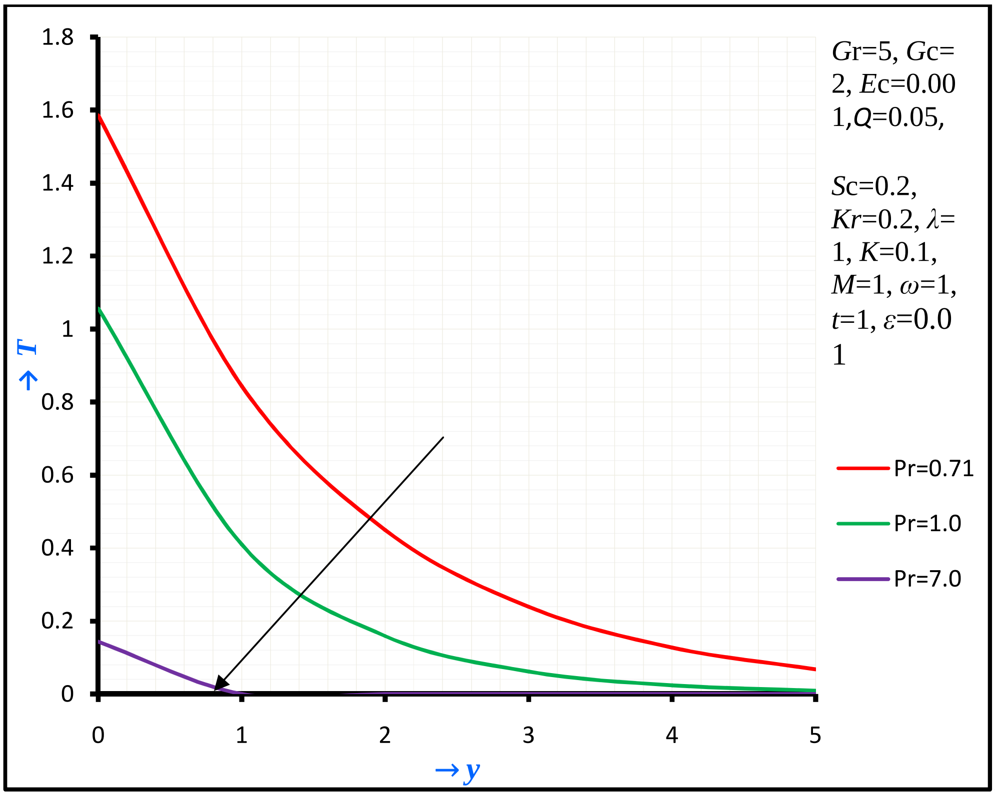

Fig.8: Temperature profiles for different values of Pr

The effect of Eckert number *E*c on the velocity is shown in Fig. 4. It is obvious that the velocity increases as the Eckert number increases.

Fig. 5 demonstrates the velocity profiles for various values of permeability parameter *K*. It is clear that as *K* increases, the peak value of velocity across the boundary layer tends to increase rapidly near wall of the porous plate.

Figs. 6(a) and 6(b) are plotted to illustrate the effect of Schmidt number *S*c on the velocity and concentration. It is found that the velocity of the fluid increases as *S*c increases while reverse trend is seen in the concentration profiles. The velocity profiles for different values of thermal Grashof number *G*r and mass Grashof number *G*c are described in Figs. 7(a) and 7(b). From Fig. 7(a), it is seen that an increase in *G*r leads to a rise in the values of velocity. Here the positive values of *G*r correspond to cooling of the plate. Furthermore, it is observed that the velocity increases rapidly near wall of the plate as *G*r increases. From Fig. 7(b), it is observed that an increase in *G*c leads to a fall in the values of the velocity.

Fig. 8 reveals the influence of Prandtl number *P*r on the temperature distribution. It is observed that an increase in *P*r results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of *P*r are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of *P*r.

Finally, we have prepared a table to display the numerical values of skin friction for visco-elastic as well as viscous fluid for several values of flow and material parameters. General trend shows that the skin friction is slightly more for ordinary fluid as compared with the visco-elastic fluid. The results also reveal that the skin friction is more in the presence of magnetic field as compared to its absence. Furthermore, skin friction increases when *G*r, *S*c, *K*r and *Q* increase and decreases when *G*c, *P*r and *K* increases.

Table 1. values of Skin friction

For fixed ω=1, t=1, ϵ=0.01, Ec=0.001									
Gr	***Gc***	***Pr***	***Sc***	***M***	***Kr***	***K***	***Q***	τ at λ=1	τ at λ=0
5	2	0.71	0.2	1	0.2	0.1	0.05	5.527884658	5.537904353
7	2	0.71	0.2	1	0.2	0.1	0.05	6.437823467	6.447843181
5	3	0.71	0.2	1	0.2	0.1	0.05	5.213192957	5.223212649
5	2	7.0	0.2	1	0.2	0.1	0.05	3.325663878	3.335683562
5	2	0.71	0.5	1	0.2	0.1	0.05	5.58701494	5.597034634
5	2	0.71	0.2	0	0.2	0.1	0.05	5.446559632	5.456244969
5	2	0.71	0.2	1	1.0	0.1	0.05	5.571163186	5.58118288
5	2	0.71	0.2	1	0.2	0.5	0.05	5.18936476	5.196022419
5	2	0.71	0.2	1	0.2	0.1	0.1	6.126255441	6.136275142

CONCLUSION

In this paper, an analysis has been carried out to study the MHD free convective oscillatory Rivlin-Ericksen fluid flow past a porous plate embedded in a porous medium in the presence of heat source with chemical reaction. Analytical and numerical solutions are obtained for velocity, temperature, concentration and skin friction. The effect of several parameters controlling the velocity, temperature and concentration profiles are shown graphically and discussed briefly. Some of the important findings of our analysis obtained by the graphical representation are listed below.

1. The effect of increasing chemical reaction parameter or heat source parameter or Eckert number or Schmidt number or Grashof number is to accelerate the velocity of the flow field.
2. A growing magnetic parameter or mass Grashof number retards the velocity of the flow field.
3. A growing Prandtl number decreases the temperature of the flow field.
4. Concentration of the flow field decreases if chemical reaction parameter or Schmidt number increases.
5. The skin friction is slightly more for viscous fluid as compared with the visco-elastic fluid.

NOMENCLATURE

C	-	dimensionless concentration, [-]
C'	-	concentration, [molm^{-3}]
C'_w	-	Species concentration at the plate, [molm^{-3}]
C'_∞	-	Species concentration far away from the plate, [molm^{-3}]
c_p	-	specific heat at constant pressure, [$Jkg^{-1}K^{-1}$]
D	-	Chemical diffusivity, [m^2s^{-1}]
Ec	-	Eckert number, [-]
g'_x	-	Acceleration due to gravity, [ms^{-2}]
Gc	-	modified Grashof number, [-]
Gr	-	Grashof number, [-]
B_0	-	applied magnetic field, [-]
k	-	permeability parameter, [-]
κ	-	thermal conductivity, [$Wm^{-1}K^{-1}$]
Pr	-	Prandtl number, [-]
p'	-	pressure, [$kgm^{-1}s^{-2}$]
q'	-	heat flux at the plate, [Wm^{-2}]
Q'	-	coefficient of heat source, [-]
Q	-	heat source parameter, [-]
Sc	-	Schmidt number, [-]
Kr'	-	chemical reaction, [-]
T	-	Dimensionless fluid temperature, [-]
T'	-	fluid temperature, [K]
T'_∞	-	Temperature of the fluid far away from the plate, [K]
t	-	Dimensionless time, [-]
t'	-	the time, [s]
U_0	-	mean free stream velocity, [ms^{-1}]
u	-	dimensionless velocity of the fluid in x'- direction, [-]
u'	-	velocity of the fluid in x'- direction, [ms^{-1}]
v'	-	velocity of the fluid in y'- direction, [-]
v_0	-	suction velocity, [ms^{-1}]
x'	-	co-ordinate axis along the plate, [-]
y'	-	co-ordinate axis normal to the plate, [-]

GREEK SYMBOLS

α - absorption coefficient, $[m^{-1}]$
β - coefficient of thermal expansion, $[K^{-1}]$
β^* - Coefficient of concentration expansion, $[(molm^{-3})^{-1}]$
ν - Kinematic viscosity, $[m^2s^{-1}]$
ρ - fluid density, $[kgm^{-3}]$
σ^* - Stefan – Boltzmann constant, $[Wm^{-2}K^{-4}]$
ω - dimensionless frequency of vibration of the fluid, [-]
ω' - frequency of vibration of the fluid, $[rads^{-1}]$

APPENDIX

$N = M + \frac{1}{k}$, $a_1 = \frac{1}{2}\left[-1 + \sqrt{1+4N}\right]$, $a_2 = \frac{1}{2}\left[-1 - \sqrt{1+4N}\right]$, $a_3 = \frac{Gr}{b_2[b_2^2+b_2-N]}$, $a_4 = \frac{Gc}{c_2^2+c_2-N}$,

$a_5 = -[1 + a_3 + a_4]$, $a_6 = \frac{-Grb_9}{b_2^2+b_2-N}$, $a_7 = \frac{-Grb_3}{(2a_2)^2+2a_2-N}$, $a_8 = \frac{-Grb_4}{(2b_2)^2+2b_2-N}$, $a_9 = \frac{-Grb_5}{(2c_2)^2+2c_2-N}$,

$a_{10} = \frac{-Grb_6}{(a_2+b_2)^2+(a_2+b_2)-N}$, $a_{11} = \frac{-Grb_7}{(b_2+c_2)^2+(b_2+c_2)-N}$, $a_{12} = \frac{-Grb_8}{(c_2+a_2)^2+(c_2+a_2)-N}$,

$a_{13} = -[a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12}]$, $a_{14} = \frac{1}{2(1+i\lambda\omega)}\left[-1 + \sqrt{1 + 4(1+i\lambda\omega)\left(\frac{i\omega}{4} + N\right)}\right]$,

$a_{15} = \frac{-1}{2(1+i\lambda\omega)}\left[1 + \sqrt{1 + 4(1+i\lambda\omega)\left(\frac{i\omega}{4} + N\right)}\right]$, $a_{16} = \frac{-Grb_{15}}{(1+i\lambda\omega)b_{11}^2+b_{11}-\left(\frac{i\omega}{4}+N\right)}$,

$a_{17} = \frac{-Grb_{12}}{(1+i\lambda\omega)(a_2+a_{15})^2+(a_2+a_{15})-\left(\frac{i\omega}{4}+N\right)}$, $a_{18} = \frac{-Grb_{13}}{(1+i\lambda\omega)(b_2+a_{15})^2+(b_2+a_{15})-\left(\frac{i\omega}{4}+N\right)}$,

$a_{19} = \frac{-Grb_{14}}{(1+i\lambda\omega)(c_2+a_{15})^2+(c_2+a_{15})-\left(\frac{i\omega}{4}+N\right)}$, $a_{20} = -(a_{16} + a_{17} + a_{18} + a_{19})$,

$b_1 = \frac{1}{2}\left[-Pr + \sqrt{Pr^2 - 4Q}\right]$, $b_2 = \frac{1}{2}\left[-Pr - \sqrt{Pr^2 - 4Q}\right]$, $b_3 = \frac{-2PrEc(a_2a_5)^2}{(2a_2)^2+Pr(2a_2)+Q}$, $b_4 = \frac{-2PrEc(b_2a_3)^2}{(2b_2)^2+Pr(2b_2)+Q}$

$b_5 = \frac{-2PrEc(c_2a_4)^2}{(2c_2)^2+Pr(2c_2)+Q}$, $b_6 = \frac{-4PrEca_2a_3a_5b_2}{(a_2+b_2)^2+Pr(a_2+b_2)+Q}$, $b_7 = \frac{-4PrEca_3a_4b_2c_2}{(b_2+c_2)^2+Pr(b_2+c_2)+Q}$, $b_8 = \frac{-4PrEca_2a_4a_5c_2}{(c_2+a_2)^2+Pr(c_2+a_2)+Q}$,

$b_9 = \frac{-1}{b_2}[2a_2b_3 + 2b_2b_5 + 2c_2b_5 + b_6(a_2 + b_2) + b_7(b_2 + c_2) + b_8(c_2 + a_2)]$,

$b_{10} = \frac{1}{2}\left[-Pr + \sqrt{Pr^2 + 4\left(\frac{i\omega}{4}Pr - Q\right)}\right]$, $b_{11} = \frac{1}{2}\left[-Pr - \sqrt{Pr^2 + 4\left(\frac{i\omega}{4}Pr - Q\right)}\right]$,

$b_{12} = \frac{2PrEca_2a_5a_{15}}{(a_2+a_{15})^2+Pr(a_2+a_{15})-\left(\frac{i\omega}{4}Pr-Q\right)}$, $b_{13} = \frac{2PrEca_3a_{15}b_2}{(b_2+a_{15})^2+Pr(b_2+a_{15})-\left(\frac{i\omega}{4}Pr-Q\right)}$, $b_{14} = \frac{2PrEca_4a_{15}c_2}{(c_2+a_{15})^2+Pr(c_2+a_{15})-\left(\frac{i\omega}{4}Pr-Q\right)}$,

$b_{15} = \frac{-1}{b_{11}}[b_{12}(a_2 + a_{15}) + b_{13}(b_2 + a_{15}) + b_{14}(c_2 + a_{15})]$,

$c_1 = \frac{1}{2}\left[-Sc + \sqrt{Sc^2 + 4KrSc}\right]$, $c_2 = \frac{1}{2}\left[-Sc - \sqrt{Sc^2 + 4KrSc}\right]$

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