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Mathematical mode for high purity distillation

C. S. Ajinomoh[1]*, P. W. Igbagara[2] and O. R. Momoh[1]

[1]Department of Chemical Engineering, Ahmadu Bello University, Zaria
[2]Department of Chemical Engineering, University of Uyo, Nigeria

ABSTRACT

Technological diagrams for obtaining high purity distillate have been examined. Base on the analysis, it has been suggested that to improve vapour mixture (distillate, condensate) or volatile mixture are as a result of boiling liquid carry –over in the vapour stream is possible. Expressions for finding add–mixture concentration in various number of distillers (Single to multiple distillers) as in the schemes and the ratio of the withdrawn product (from initial steam) has been determined. Analysis has been carried out for the derived and the optimal distribution of stream in a scheme. Base on the result, it has been fixed that the purity of a distillate reduces when the rate of product withdrawal increases.

Key words: carry-over, distiller, vapour, product

INTRODUCTION

Distilled water is commonly used in chemical, petrochemical, food, pharmaceutical and other branches of industries, beside that it also has preformed application in laboratory piratical. Water is commonly obtained by distillation. The source of distillate normally is form condensate of produced vapour. All dissolved non volatile substances in water are deposited as residue in the cube.

Most of the time attempts are made to obtain pure distillated water and this sometime leads to condensation of bi – distillate products which is common in the art of distillation. In practice, it is still a big challenge to distill product of up – to 100% purity. However progress are made sometime to obtain a distillate of a very close value to that mentioned above Gelorin (2002). This attempt is only made possible when a cascade of distillation system is set up. The resulting product which is a form of condensed vapour together with entranced impurities known as "carry–over" which when compared to the targeted product are in minute ratio. Arrangement for such binary distillation is presented in the following scheme for following bi – distillate with a material stream illustrated in figure 1- initial mass stream of liquid S – water.

An increment in concentration of impurities in the feed S to the first distiller and the residue expelled together with stream K_1 and concentration, a_1. The vapour $W_1 \approx W$ entrained along with some impurities and a composition, a^I. Hence the condensed vapour later act as feed $W_2 \approx P$ to the second distiller chamber 2. From the chamber residue is further expelled as K_2 with concentration a_2 leaving a product W_2 of composition a^{11}. During boiling of liquid, it is assumed that the whole mixture are evenly distributed and the concentration of impurities in the bubbles are equal that of the liquid, in the distiller.

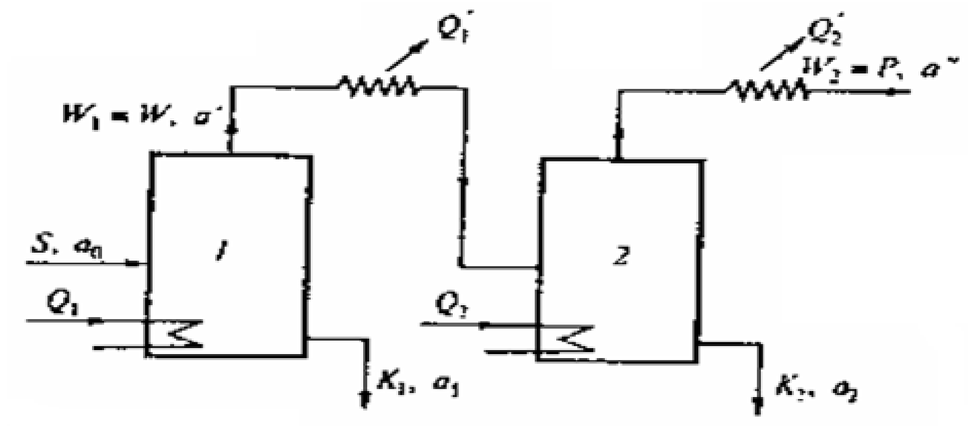


Fig. 1.: Clacualtion shceme for obtaining bi – distillate. 1,2– distillers; Q_1, Q_2 – preheater streams; Q^1_1, Q^1_2- condensed streams.

This case represents the mass ratio of the bubble carry–over by x; and the x represents the number of distiller. In our next analysis the following are to be determine:

(i) the final concentration of impurities in the product at a given material steam;
(ii) the optimum steam and final concentration of impurities in the product or optimal concentration at a given initial and final steam S and P respectively.

From the binary – distillation scheme (fig 1) we take the and material balance around the first distiller, by stream

$$S = W + K_1 \quad (1)$$

$$Sa_o=Wa^1 + K_1a_1 \quad (2)$$

In respect to the applied expression for concentration a^1 and a_1 above for impurities as for vapour and boiling liquid receptively, the following expression is formed.

$$a^1 = xa_1$$

and in a more general form

$$a^{(i)} = xa_i \quad (3)$$

The upper and lower indexes are related receptively to vapour and liquids leaving the distiller.

By eliminating K1 stream form equation (2) with the aid of equation (1) and substituting it in the obtained expression (3), the value $a_1=a^I/x_1$, we obtain an expression $Sao = \frac{a^1}{x_1}(S - W + X_1W)$

$$a^1 = Sa_ox_1/ [S – W(1 – X_1)] \; a^1 = Sa_ox_1 / [S-W (1-x_1)] \quad (4)$$

If a similar material balance is written for space contour capturing the second distiller (see fig 1), hence after similar transformation, taken into consideration that in agreement to expression (3), $a_2 = a^{11}/x_2$,

we obtain

$$a^{11} = Wa^1x_2 / [W – p (1 – x_2)] \quad (5)$$

By introducing into expression (5), the value a^1 from expression (4),

$$\mathbf{a^{11} = a_o\, x_1\, x_2}\ \frac{SW}{[S - W(1 - X_1][W - P_1 - X_2]} \qquad \textbf{(6)}$$

The optimal process is considered to be in such a fixed stream W at which the final concentration of impurities a^{11} in the product P will be minimal. Optimization condition for W can be written as $\partial a^{11} / \partial W$ = O by differentiating the expression (6) gives.

$$\frac{\partial a^{11}}{\partial W} = SaoX_1X_2\frac{(S - Wb_1)(W - Pb_2)}{(S - Wb_1)^2(W - Pb_2)^{2-1}} - \frac{W(S - Wb1) + Wb_1(W - Pb_2)}{(S - Wb1)^2(w - Pb_2)^2} = O \qquad \textbf{(7)}$$

Where $b_1 \equiv (1 - x_2)$

The denominator in the expression (7) whose value is positive, and by adopting the numerator of the expression to be equal to zero, and after opening the bracket and simplifying we obtained

$$W = W_{opt}\sqrt{PSb_2/b_1} \qquad \textbf{(8)}$$

At $b_1 = b_2$ (or $x_1 = x_2$) and the optimal stream W_{opt} equal to mean geometric quality from initial S and final P streams:

$$W_{opt} = \sqrt{PS} \qquad \textbf{(9)}$$

It is observed that due to infinesimal x value, b tend to unity, therefore the equality (9) is satisfied at $x_1 \neq x_2$, at optimal stream $W = W_{opt}$, concentration of impurities in the binary distillate consists,

$$a^{11} = a_o x_1 x_2 \quad \frac{\sqrt{PS(b_2/b_1)}}{S - \sqrt{PS(b_2/b_1)}(1 - x_1)}\, x\, \frac{1}{\sqrt{Ps(b_2/b_1)} - P(1 - x_2)}$$

Figure 2 illustrates the scheme for multiple distillations with arbitrary number of distillate. If similar analysis is carried out for scheme consists of three distillers, then concentration of impurities in the distillate – final product P consist of:

$$\mathbf{a^{111} = a_o x_1 x_2 x_3}\ \frac{SW_1W_2}{[(S - W_1)(1 - x1][W_1 - W_2(1 - x_2)][W_2 - P(1 - x_3)]} \qquad \textbf{(10)}$$

For such scheme, in optimizing, (at a given number of n by initial and final streams S and P repetitively) lies two intermediate streams – W_1 and W_2. For determining the streams, two system of equations has to be solved:

$$\partial a^{11}/\partial W_1 = O$$
$$\partial a^{11}/\partial W_2 = O$$

Their differentiation leads to algebraic systems of equations:

$W_1^2b_1 = W_2Sb_2$

$W_2^2b_2 = PW_1b_3$

Solution of the system givens optimal value of streams W1 and W2

$$W_{1opt} = \sqrt[3]{S^2 P\left({b_2 b_3}\Big/{b_1} \right)}$$
$$W_{2opt} = \sqrt[3]{S^2 P\left({b_2 b_3}\Big/{b_2} \right)}$$

In the case of scheme consisting four distiller, similar analysis givens the expression a^{1111}, in respect of the expression type (6) and (10). In this case the optimal values for intermediate streams are:-

$$W_{1opt} = \sqrt[4]{S^3 p\left(b_2 b_3 b_4 / b_1^3\right)}$$
$$W_{2opt} = \sqrt[4]{S^2 P^2 \left({b_3 b_4^2}\Big/{\left(b_1^2 b_2^2\right)} \right)}$$
$$W_{3opt} = \sqrt[4]{S P^3 \left({b_4^3}\Big/{\left(b_1 b_2 b_3\right)} \right)}$$

In the case of several numbers of arbitrary distillers n, in the scheme, the general expression for concentration of impurities in the final distillate P takes the following form:

$$\mathbf{a^{(n)}} = \mathbf{a_o} \prod_{i=1}^{n} xi \left\{ \prod_{i=o}^{n-1} W_i \Big/ \prod_{i=1}^{n} \left[W_i - 1 - W_i (1 - xi) \right] \right\} \quad \textbf{(11)}$$

This implies that, at i=0, $W_i \equiv$ Wo S; also when i = n, $W_i \equiv W_n \equiv$P.

Therefore optimal intermediate stream W_i for n – distiller in this case

$$W_{iopt} = \sqrt[n]{S^{n-i} P^i \left(\prod_{j=i+1}^{n} b_j^i \Big/ \prod_{j=i}^{i} b_j^{n-i} \right)} \quad \textbf{(12)}$$

The later expression givens solution of system of (n – 1) differential equation then algebraic equations, as shown in the scheme with three distillers. To avoid complex using the given method of analysis, a preliminary logarithm of the expression (11) can be taken, then differentiate.

Finally method of dynamic programming can be adopted. In getting the expression 912) method of mathematic induction can be adopted. in equation 11 on.

In order to determine the concentration of impurities in the distillate at an optimal distribution of stream W_i. Along the distillers, in the expression in place of W_i we replace W_{iopt} (20), and form expression (21). In some cases, from the assumption that $b_j = 1$ gives:

$$\mathbf{a^{(n)}} = \mathbf{a_o} \prod_{j=1}^{n} xi \left[1 \Big/ \left(1 - \sqrt[n]{P/S}\right)^n \right] \quad \textbf{(13)}$$

Here it is noted that the error of approximation by taking out $b_j = 1$, which in practical calculations is insignificant; in association with previsions which ratio is very small in determining bj; the precision to some degree is relaxed since the quantities bi appears in the numerator and in the denominator of the expression (12), then also under the root of nth power.

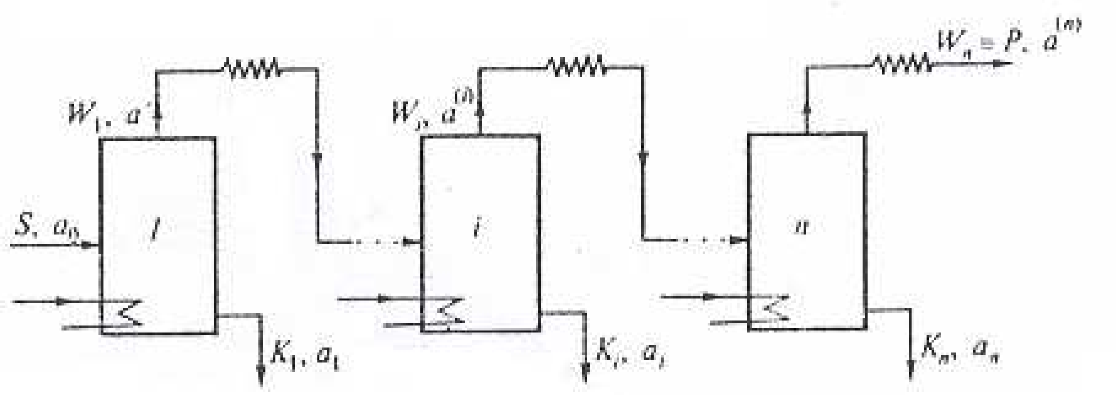
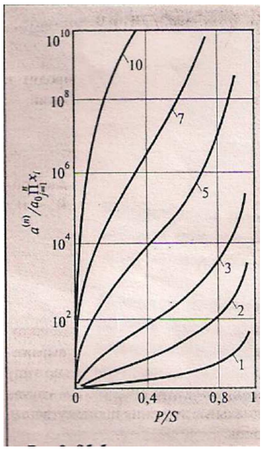


fig. 2. Calculation scheme for determining the distillate from multiple distiller



Fi3: General relationship of Concentration of Impurity on the ratio of distillate product on stream at various stream (see the figure on the curves) number of distillers.

With an aim for extended the generalization of the relationship (13), it can also be represented in the form:

$$\frac{a^{(n)}}{\left(a_o \prod_{i=1}^{n} x_i\right)} = f\left({}^{p}\!/_{s,n}\right)$$

This relationship stands in the case of optimal redistribution of streams $Wi_{,opt}$, as illustrated in fig.3 as half logarithm coordinates. As seen here, that increases in P/S (at constant n), the effectiveness of the purification process decreases: then arise from material balance (1) and (2) that large stream of product P represents large concentration of residues carried along in the vapor phase. By increasing the ordinate when moving to large number of (n) distillers in the technological diagram, however should be considered as reduction in quality of purified water. What this is all about is that, during the derivation of the ratio a^n/a_o, the actual characteristic of water purification, at high values (>1 or >>1) , then arias the need to multiply the ordinate with a very small value Πx_i (or when equal to x_i the with x_i), Gelperin (2002), Holland (1975).

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