

Computer Graphics 2018: On geometry and equilibrium of forces in 3D – Masoud Akbarzadeh- USA

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Geometry is usually referred to as a branch of mathematics concerned with questions of shape, size, the relative position of figures, and therefore the properties of space. Fewer realize the importance of geometry in physics and equilibrium of forces. This presentation will expand on the utilization of geometry in describing the static equilibrium of the complex spatial system of forces, and more importantly, how it might help structural designers and designers to style highly efficient structures. This branch of science is named graphic statics where the equilibrium of forces during a structural form is described geometrically/graphically. The geometric representation of forces provides unprecedented control for designers to not only design the geometry of the system, but also design and optimize its internal forces. Traditional graphic statics is predicated on 2D reciprocal diagrams formulated by Maxwell in 1864 and is sort of limited in handling a 3D system of forces. This presentation will specifically expand the present research of the author and show the new development of graphic statics in 3D supported a 150-year proposition by Rankine in Philosophical Magazine. Additionally, it shows how the well-known computer graphic techniques like aggregation/or subdivision of polyhedral cells are often utilized in 3D graphic statics and to get non-conventional, expressive efficient structural forms. Over the past half-century, the sector of structural analysis has become increasingly a matter of matrix-based algebra computation. However, by generalizing methods developed over a century ago by Maxwell, Rankine, Klein et al. , this paper provides an alternate approach based almost purely on geometry. A theory of graphic statics is presented which is applicable to the static equilibrium of rather general three-dimensional frames. We start with some comparatively abstract mathematics to offer a proper statement of the overall theory, before giving examples to demonstrate how it's practical and applicable to real structures. Although the idea will use algebra, the ultimate constructions are going to be diagrammatic, giving geometric visualization of the varied objects like forces and moments.

It should be mentioned that Cremona [7] used an alternate convention. For two-dimensional trusses, a Cremona force diagram is just a 90° rotation of the Maxwell force diagram; such forces are now parallel to their corresponding bars. For three-dimensional trusses, Cremona also represents forces as lines parallel to their corresponding bars, and thus a Cremona three-dimensional force diagram may be a fundamentally different object than a Rankine three-dimensional force diagram. During this paper, we follow the perpendicular convention of Maxwell's lines and Rankine's polygons and generalize these to the case of three-dimensional frames. In "The Principle of Equilibrium of Polyhedral Frames", Rankine [7] proposed how the equilibrium of a spatial system of forces, applied to one point in space, might be described employing a closed polyhedron. Supported Rankine's proposition, Akbarzadeh et al. [1, 2] visualized and clarified the reciprocal relationship between polyhedral form and force diagrams and demonstrated their potential for the planning of complex spatial structures. for instance, they showed how the aggregation of convex polyhedral force cells might be wont to specifically generate complex spatial systems of forces in compression-only (or tension-only) equilibrium (Akbarzadeh et al. [1 2]). However

because the described procedures didn't leave the generation of specific solutions for given boundary conditions, like specific support locations and/or specific applied loads, their applicability in actual design situations remains limited. Therefore, it's clear that to travel beyond open-ended explorations of three-dimensional force equilibrium, and develop an actual three-dimensional version of Graphic Statics, procedures should be established through which specific structural problems involving spatial systems of forces are often addressed during a rigorous and intuitive manner, almost like the way during which specific two-dimensional problems are often addressed with the precise procedures offered by traditional (2D) Graphic Statics. Consider the spatial configuration of forces depicted in Figure 3. As within the two-dimensional case, we discover the magnitude and direction of the resultant force by constructing a Plimsoll line. Then, we use an identical procedure to the choice procedure described in Section 2.2 in reference to Figure 1b, to work out the situation of the road of action of the resultant. First, we construct a plane perpendicular to the road of action of the resultant force. This is often the equilibrium plane, which is that the three-dimensional equivalent of the equilibrium line introduced in Section 2.1. Then, we discover the intersection points of the lines of action of the hundreds with the equilibrium plane, and decompose the hundreds at these intersection points into an in-plane and a traditional component. Note that the traditional components are parallel to the resultant force. As in Section 2.2, the intersection of the road of action of the resultant force with the equilibrium plane is that the centroid of the intersections of the hundreds with the resultant plane, weighted by the magnitude of their normal components. We have presented a purely geometric method for exploring the worldwide equilibrium of various funicular solutions for given three-dimensional boundary conditions, using polyhedral form and force diagrams. The tactic is that the three-dimensional version of the equivalent, the well-known procedure of traditional (2D) Graphic Statics.