

# Approximating the Probability of Traffic Sharing by Numerical Analysis Techniques between Two Operators in a Computer Network

Diwakar Shukla<sup>1</sup>, Kapil Verma<sup>\*2</sup> and Sharad Gangele<sup>3</sup>

<sup>1</sup>*Department of Mathematics and Statistics, H.S Gour University,  
Sagar M. P., India*

<sup>2</sup>*Department of Computer Science, M. P. Bhoj (Open) University,  
Bhopal, M. P., India*

*Department of Computer Science, R.K.D.F University,  
Bhopal, M. P., India*

**\*Corresponding Email:** [Kapil\\_mca100@rediffmail.com](mailto:Kapil_mca100@rediffmail.com)

## ABSTRACT

Many authors have derived the expressions for the probability of traffic sharing between two operators in computer networking environment. The mathematical expressions for these probabilities are generally in quadratic function of blocking probabilities of computer network along with the probability of initial choice of customer. The expressions are not easily integrated and, therefore, effort is difficult to obtain the total bounded area under the probability curve. It is a challenging problem to obtain the probability density computation of the traffic sharing parameter. This paper presents a methodology to obtain the approximate bounded area and probability computation using methods of numerical analysis. The main focus is on utilization of Simpson 1/3 rule and Weddle rule along with the mutual comparisons.

**Keywords:** Probability, Area estimation, Trapezoidal rule, Network blocking, Operator. Markov chain, Simpson 1/3 rule, Weddle's rule.

## INTRODUCTION

The internet management problem is a burning avenue of research the field of computer sciences and network management. The dial-up connection for Internet services are being utilized by many under-developed countries around the world with special reference to operator based network services for customers. The Internet services are provided by operators, and users

are free to adopted either of them. It develops competition into market for capturing more and more customers-share in order to earn the maximum profit.<sup>1</sup> has developed a Markov chain model for the study of relationship between traffic sharing probability and network blocking probability faced by operators. The same was extended by<sup>2-4</sup> using the Markov chain model.

Moreover,<sup>5-7</sup> has developed multiple relations for traffic sharing and blocking probability in the presence of dis-connectivity effect, rest state effect, cyber-crime effect and iso-share effect analysis. All these have extended wider applicability of network analysis with special context to establishing the mathematical relationship among various model parameters.<sup>8-11</sup> examined the elasticity variations along with reattempt connectivity aspect.

The<sup>1</sup> has expressed following basic relation,

$$\bar{P}_1 = (1 - L_1) \frac{p + (1 - p)(1 - p_A)L_2}{1 - L_1L_2(1 - p_A)^2}$$

Where  $\bar{P}_1$  is traffic share by first operator  $O_1$ ,  $p$  is user preference,  $L_1$ ,  $L_2$  are the network blocking probabilities experienced by both operators  $O_1$  and  $O_2$ .

Have<sup>12-14</sup> estimated the bounded area of the curve generated by above expression using the trapezoidal rule available in the literature of numerical analysis.

The trapezoidal rule of numerical integration theory is weak in the sense that it requires strictly the linearity in the relationship. For curvilinear relationship, this method is not suitable enough. This paper presents the estimation of bounded area by the Simpson 1/3 rule and Weddle rule. A comparative approach has been adopted for evaluating of computation of bounded area by two methodologies. Some other similar contribution are by<sup>15-30</sup> presents Probability Density Estimation Function of Browser Share Curve for Users Web Browsing Behaviour and Two-Call Index Based Internet Traffic Sharing Analysis.<sup>31,32</sup>, D. has Developed Area Computation of Internet Traffic Share Problem with Special Reference to Cyber Crime Environment.

### Computational methodologies

In the literature of numerical analysis we have two computational approaches listed below: Consider the following integral

$$I = \int_b^a f(x)dx$$

Where  $a$  to  $b$  is range of integration and  $f(x)$  is a function to be integrated. Let us divide the range  $a$  to  $b$  into 20 equal parts  $y_0, y_1, y_2, y_3, y_4, y_5, \dots, y_{11}, y_{20}$  each of length  $h = (y_i - y_{i-1})$ ,  $i=1, 2, 3, 4, \dots, 20$ .

Simpson 1/3 rule: (for  $n=20$ )

$$I = \int_b^a f(x)dx = \int_b^a ydx = \frac{h}{3} \left[ (y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) \right]$$

### Weddle's rule

$$I = \int_b^a f(x)dx = \frac{3h}{10} [y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6 + 5y_7 + y_8 + \dots] + \frac{3h}{10} [y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12} + \dots]$$

### APPLICATIONS

Consider the formula,

$$\bar{P}_1 = f(L_1) = (1 - L_1) \frac{p + (1 - p)(1 - p_A)L_2}{1 - L_1L_2(1 - p_A)^2}$$

Where  $L_1$  is variable but all other parameters  $p$ ,  $p_A$ ,  $L_2$  are constants. While  $L_1$  is varying, the different values of corresponding  $\bar{P}_1$  are given in tables. The total bounded area is computed by using Simpson 1/3 rule and Weddle's rule and both are compound. Graphs are also shown in different figure containing the bounded area between  $L_1=0.0$  to  $L_1=1.0$  at X-axis.

As per table 1 the traffic share is a linearly decreasing function of self network blocking probability. But when  $L_2$  increases, the first operator gains the traffic share. The total bounded area is a function of  $L_2$ , but this area is almost independent of  $L_2$ . It is because very little fluctuation observed.

The figure 1 supports facts as observed in table 1 displaying linearly increasing trend between  $P_1$  and  $L_2$

The bounded area level increases with the increase of initial choice probability  $p$ . Moreover, it seems the traffic share is function of  $L_2$  and  $p$  both. Figure 2 reveals that bounded area below the curve increases with increment to  $p$ .

With respect to  $p_A$  variation, the bounded area total reduces from 2.4% to 1.9% using Simpson 1/3 rule. It supports the fact that total bounded area is a function of  $L_2$ ,  $p$ , and  $p_A$ .

Figure 3 is in favour of the downward trend towards over the variation of  $p_A$  over the computation of bounded area.

With reference to table 4, using Weddle's rule, the bounded area estimation range is 16% to 42% which is higher than pattern observed by Simpson's rule.

The figure 4.0 support the fact that while opponent operator  $O_2$  suffers with high blocking probability, the first operator gains the bounded area in the linear increasing pattern.

With reference to table 4, the bounded area variation is between 19% to 52% which is higher than obtained by the Simpson 1/3 rule.

The figure 5.0 shows almost straight line increase in the pattern with respect to variation over initial choice.

In above table, the area estimate decreases from 29% to 10 %. The area is inversionally proportional to the parameter  $p_A$ .

The figure 6 supports the downward trend of the computation of bounded area using Weddle's rule.

### CONCLUDING REMARKS

The bounded area under the probability curve is a function of many input parameters. Most of cases, it bears the linearly increasing or decreasing trend on the variation of model parameters. The  $L_2$  and  $p$ , if increases, the corresponding area also increases but with the increase of  $p_A$  the reverse pattern exists. Area computation by Weddle's method is constantly higher than the Simpson 1/3 rule. The highest bounded area computation by Simpson procedure is 39% at  $p=0.9$  while the highest by Weddle rule is 52% at  $p=0.9$ . Overall Weddle rule

based computed values are constantly above than the Simpson 1/3 rule.

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**Table 1.** Using Simpson 1/3 rule

| Table 1 -For Fig. (1) [ Where Fixed $p=0.25$ , $p_A = 0.35$ , , $h=0.05$ ] |             |             |             |             |             |             |             |             |             |
|----------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $L_2$                                                                      | 0.1         | 0.2         | 0.3         | 0.4         | 0.5         | 0.6         | 0.7         | 0.8         | 0.9         |
| $L_1$                                                                      | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ |
| 0                                                                          | 0.299       | 0.348       | 0.396       | 0.445       | 0.494       | 0.543       | 0.591       | 0.64        | 0.689       |
| 0.05                                                                       | 0.284       | 0.332       | 0.379       | 0.426       | 0.474       | 0.522       | 0.57        | 0.618       | 0.667       |
| 0.1                                                                        | 0.27        | 0.315       | 0.361       | 0.407       | 0.454       | 0.501       | 0.548       | 0.596       | 0.644       |
| 0.15                                                                       | 0.256       | 0.299       | 0.343       | 0.388       | 0.433       | 0.479       | 0.526       | 0.573       | 0.621       |
| 0.2                                                                        | 0.241       | 0.283       | 0.325       | 0.368       | 0.412       | 0.457       | 0.503       | 0.549       | 0.596       |
| 0.25                                                                       | 0.226       | 0.266       | 0.307       | 0.348       | 0.391       | 0.434       | 0.479       | 0.524       | 0.571       |
| 0.3                                                                        | 0.212       | 0.25        | 0.288       | 0.328       | 0.369       | 0.411       | 0.454       | 0.499       | 0.544       |
| 0.35                                                                       | 0.197       | 0.233       | 0.27        | 0.307       | 0.347       | 0.387       | 0.429       | 0.472       | 0.516       |
| 0.4                                                                        | 0.182       | 0.216       | 0.25        | 0.286       | 0.324       | 0.362       | 0.402       | 0.444       | 0.487       |
| 0.45                                                                       | 0.167       | 0.199       | 0.231       | 0.265       | 0.3         | 0.337       | 0.375       | 0.415       | 0.457       |
| 0.5                                                                        | 0.153       | 0.181       | 0.212       | 0.243       | 0.276       | 0.311       | 0.347       | 0.385       | 0.425       |
| 0.55                                                                       | 0.138       | 0.164       | 0.192       | 0.221       | 0.251       | 0.284       | 0.318       | 0.354       | 0.392       |
| 0.6                                                                        | 0.123       | 0.146       | 0.172       | 0.198       | 0.226       | 0.256       | 0.288       | 0.321       | 0.357       |
| 0.65                                                                       | 0.108       | 0.129       | 0.151       | 0.175       | 0.2         | 0.227       | 0.256       | 0.287       | 0.32        |
| 0.7                                                                        | 0.092       | 0.111       | 0.13        | 0.151       | 0.174       | 0.198       | 0.224       | 0.252       | 0.282       |
| 0.75                                                                       | 0.077       | 0.093       | 0.109       | 0.127       | 0.147       | 0.167       | 0.19        | 0.214       | 0.241       |
| 0.8                                                                        | 0.062       | 0.075       | 0.088       | 0.103       | 0.119       | 0.136       | 0.155       | 0.175       | 0.198       |
| 0.85                                                                       | 0.046       | 0.056       | 0.067       | 0.078       | 0.09        | 0.104       | 0.118       | 0.135       | 0.153       |
| 0.9                                                                        | 0.031       | 0.038       | 0.045       | 0.052       | 0.061       | 0.07        | 0.081       | 0.092       | 0.105       |
| 0.95                                                                       | 0.016       | 0.019       | 0.023       | 0.027       | 0.031       | 0.036       | 0.041       | 0.047       | 0.054       |
| Area(A)                                                                    | 0.021       | 0.022       | 0.022       | 0.023       | 0.024       | 0.025       | 0.026       | 0.027       | 0.027       |



**Table 2.** Using Simpson 1/3 rule

| Table 2 -For Fig. (2) [ Where Fixed $p_A=0.25$ , $L_2 = 0.35$ , , $h=0.05$ ] |             |             |             |             |             |             |             |             |             |
|------------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| p                                                                            | 0.1         | 0.2         | 0.3         | 0.4         | 0.5         | 0.6         | 0.7         | 0.8         | 0.9         |
| $L_1$                                                                        | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ |
| 0                                                                            | 0.336       | 0.41        | 0.484       | 0.558       | 0.631       | 0.705       | 0.779       | 0.853       | 0.926       |
| 0.05                                                                         | 0.323       | 0.393       | 0.464       | 0.535       | 0.606       | 0.676       | 0.747       | 0.818       | 0.889       |
| 0.1                                                                          | 0.309       | 0.376       | 0.444       | 0.512       | 0.58        | 0.647       | 0.715       | 0.783       | 0.85        |
| 0.15                                                                         | 0.295       | 0.359       | 0.424       | 0.488       | 0.553       | 0.617       | 0.682       | 0.747       | 0.811       |
| 0.2                                                                          | 0.28        | 0.341       | 0.403       | 0.464       | 0.526       | 0.587       | 0.649       | 0.71        | 0.771       |
| 0.25                                                                         | 0.265       | 0.323       | 0.382       | 0.44        | 0.498       | 0.556       | 0.614       | 0.672       | 0.731       |
| 0.3                                                                          | 0.25        | 0.305       | 0.36        | 0.415       | 0.47        | 0.524       | 0.579       | 0.634       | 0.689       |
| 0.35                                                                         | 0.235       | 0.286       | 0.338       | 0.389       | 0.441       | 0.492       | 0.544       | 0.595       | 0.647       |
| 0.4                                                                          | 0.219       | 0.267       | 0.315       | 0.363       | 0.411       | 0.459       | 0.507       | 0.555       | 0.603       |
| 0.45                                                                         | 0.203       | 0.247       | 0.292       | 0.336       | 0.381       | 0.425       | 0.47        | 0.514       | 0.559       |
| 0.5                                                                          | 0.186       | 0.227       | 0.268       | 0.309       | 0.35        | 0.391       | 0.432       | 0.473       | 0.514       |
| 0.55                                                                         | 0.17        | 0.207       | 0.244       | 0.281       | 0.319       | 0.356       | 0.393       | 0.43        | 0.467       |
| 0.6                                                                          | 0.153       | 0.186       | 0.219       | 0.253       | 0.286       | 0.32        | 0.353       | 0.387       | 0.42        |
| 0.65                                                                         | 0.135       | 0.165       | 0.194       | 0.224       | 0.253       | 0.283       | 0.313       | 0.342       | 0.372       |
| 0.7                                                                          | 0.117       | 0.143       | 0.168       | 0.194       | 0.22        | 0.245       | 0.271       | 0.297       | 0.322       |
| 0.75                                                                         | 0.099       | 0.12        | 0.142       | 0.164       | 0.185       | 0.207       | 0.228       | 0.25        | 0.272       |
| 0.8                                                                          | 0.08        | 0.097       | 0.115       | 0.132       | 0.15        | 0.167       | 0.185       | 0.202       | 0.22        |
| 0.85                                                                         | 0.061       | 0.074       | 0.087       | 0.1         | 0.114       | 0.127       | 0.14        | 0.154       | 0.167       |
| 0.9                                                                          | 0.041       | 0.05        | 0.059       | 0.068       | 0.077       | 0.086       | 0.095       | 0.104       | 0.113       |
| 0.95                                                                         | 0.021       | 0.025       | 0.03        | 0.034       | 0.039       | 0.043       | 0.048       | 0.052       | 0.057       |
| Area(A)                                                                      | 0.021       | 0.023       | 0.024       | 0.025       | 0.026       | 0.028       | 0.029       | 0.03        | 0.031       |

**Table 3.** Using Simpson 1/3 rule

| Table 3 -For Fig. (3) [ Where Fixed $p=0.15$ , $L_2 = 0.45$ , $h=0.05$ ] |             |             |             |             |             |             |             |             |             |
|--------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $p_A$                                                                    | 0.1         | 0.2         | 0.3         | 0.4         | 0.5         | 0.6         | 0.7         | 0.8         | 0.9         |
| $L_1$                                                                    | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ |
| 0                                                                        | 0.494       | 0.456       | 0.418       | 0.38        | 0.341       | 0.303       | 0.265       | 0.227       | 0.188       |
| 0.05                                                                     | 0.478       | 0.44        | 0.401       | 0.363       | 0.326       | 0.289       | 0.252       | 0.215       | 0.179       |
| 0.1                                                                      | 0.462       | 0.423       | 0.384       | 0.347       | 0.311       | 0.275       | 0.239       | 0.204       | 0.17        |
| 0.15                                                                     | 0.444       | 0.405       | 0.367       | 0.331       | 0.295       | 0.26        | 0.226       | 0.193       | 0.16        |
| 0.2                                                                      | 0.426       | 0.387       | 0.35        | 0.314       | 0.279       | 0.246       | 0.214       | 0.182       | 0.151       |
| 0.25                                                                     | 0.408       | 0.369       | 0.332       | 0.297       | 0.263       | 0.231       | 0.201       | 0.171       | 0.141       |
| 0.3                                                                      | 0.388       | 0.349       | 0.313       | 0.279       | 0.247       | 0.217       | 0.188       | 0.159       | 0.132       |
| 0.35                                                                     | 0.368       | 0.33        | 0.294       | 0.262       | 0.231       | 0.202       | 0.175       | 0.148       | 0.123       |
| 0.4                                                                      | 0.347       | 0.309       | 0.275       | 0.243       | 0.214       | 0.187       | 0.161       | 0.137       | 0.113       |
| 0.45                                                                     | 0.325       | 0.288       | 0.255       | 0.225       | 0.198       | 0.172       | 0.148       | 0.126       | 0.104       |
| 0.5                                                                      | 0.302       | 0.266       | 0.235       | 0.206       | 0.181       | 0.157       | 0.135       | 0.114       | 0.094       |
| 0.55                                                                     | 0.278       | 0.244       | 0.214       | 0.187       | 0.164       | 0.142       | 0.122       | 0.103       | 0.085       |
| 0.6                                                                      | 0.253       | 0.221       | 0.193       | 0.168       | 0.146       | 0.127       | 0.109       | 0.092       | 0.076       |
| 0.65                                                                     | 0.227       | 0.196       | 0.171       | 0.148       | 0.129       | 0.111       | 0.095       | 0.08        | 0.066       |
| 0.7                                                                      | 0.199       | 0.171       | 0.148       | 0.128       | 0.111       | 0.096       | 0.082       | 0.069       | 0.057       |
| 0.75                                                                     | 0.17        | 0.145       | 0.125       | 0.108       | 0.093       | 0.08        | 0.068       | 0.057       | 0.047       |
| 0.8                                                                      | 0.14        | 0.119       | 0.101       | 0.087       | 0.075       | 0.064       | 0.055       | 0.046       | 0.038       |
| 0.85                                                                     | 0.107       | 0.091       | 0.077       | 0.066       | 0.057       | 0.048       | 0.041       | 0.035       | 0.028       |
| 0.9                                                                      | 0.074       | 0.062       | 0.052       | 0.044       | 0.038       | 0.032       | 0.027       | 0.023       | 0.019       |
| 0.95                                                                     | 0.038       | 0.031       | 0.026       | 0.022       | 0.019       | 0.016       | 0.014       | 0.012       | 0.009       |
| Area(A)                                                                  | 0.024       | 0.023       | 0.023       | 0.022       | 0.022       | 0.021       | 0.02        | 0.02        | 0.019       |



**Table 4.** Using Weddle's rule

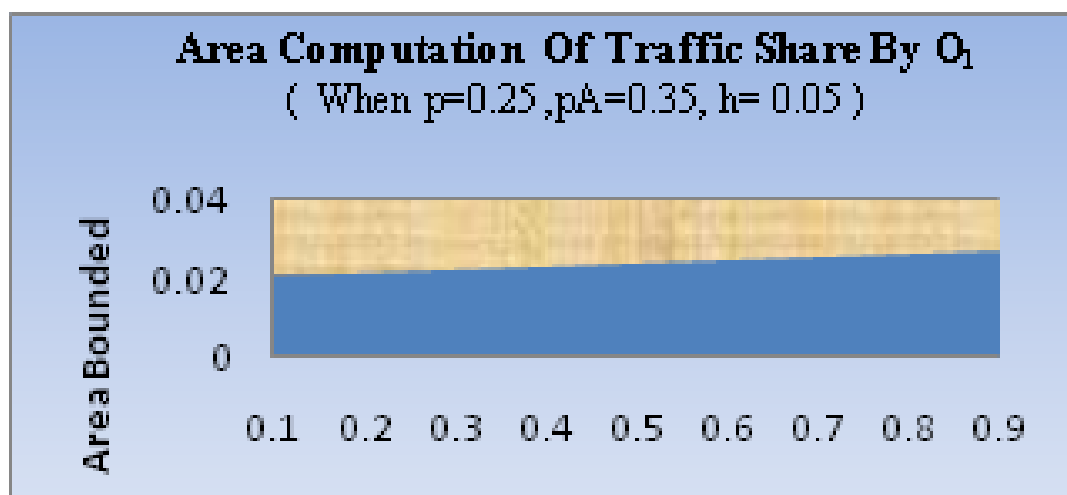
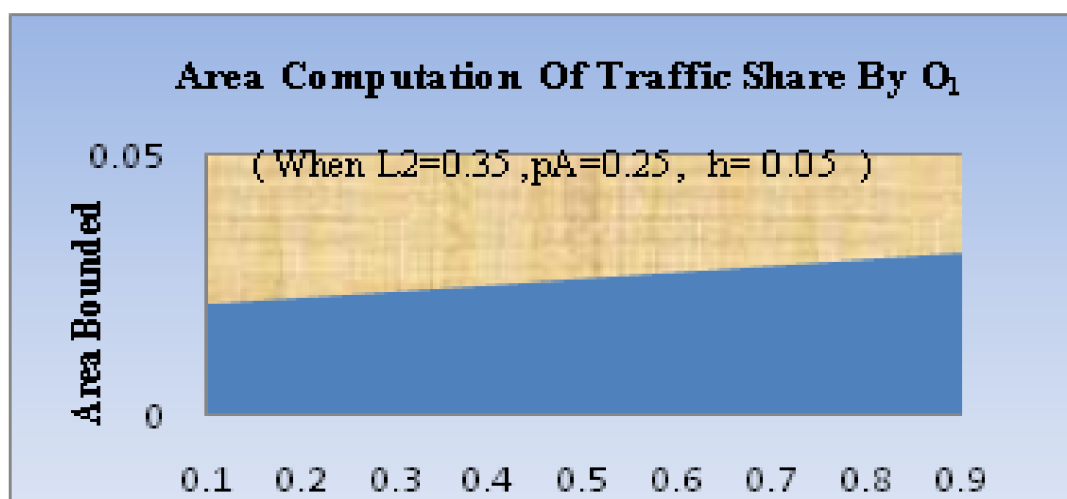
| Table 4 -For Fig. (4) [ Where Fixed $p_A=0.35$ , $p = 0.25$ , $h=0.05$ ] |             |             |             |             |             |             |             |             |             |
|--------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $L_2$                                                                    | 0.1         | 0.2         | 0.3         | 0.4         | 0.5         | 0.6         | 0.7         | 0.8         | 0.9         |
| $L_1$                                                                    | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ |
| 0                                                                        | 0.299       | 0.348       | 0.396       | 0.445       | 0.494       | 0.543       | 0.591       | 0.64        | 0.689       |
| 0.05                                                                     | 0.284       | 0.332       | 0.379       | 0.426       | 0.474       | 0.522       | 0.57        | 0.618       | 0.667       |
| 0.1                                                                      | 0.27        | 0.315       | 0.361       | 0.407       | 0.454       | 0.501       | 0.548       | 0.596       | 0.644       |
| 0.15                                                                     | 0.256       | 0.299       | 0.343       | 0.388       | 0.433       | 0.479       | 0.526       | 0.573       | 0.621       |
| 0.2                                                                      | 0.241       | 0.283       | 0.325       | 0.368       | 0.412       | 0.457       | 0.503       | 0.549       | 0.596       |
| 0.25                                                                     | 0.226       | 0.266       | 0.307       | 0.348       | 0.391       | 0.434       | 0.479       | 0.524       | 0.571       |
| 0.3                                                                      | 0.212       | 0.25        | 0.288       | 0.328       | 0.369       | 0.411       | 0.454       | 0.499       | 0.544       |
| 0.35                                                                     | 0.197       | 0.233       | 0.27        | 0.307       | 0.347       | 0.387       | 0.429       | 0.472       | 0.516       |
| 0.4                                                                      | 0.182       | 0.216       | 0.25        | 0.286       | 0.324       | 0.362       | 0.402       | 0.444       | 0.487       |
| 0.45                                                                     | 0.167       | 0.199       | 0.231       | 0.265       | 0.3         | 0.337       | 0.375       | 0.415       | 0.457       |
| 0.5                                                                      | 0.153       | 0.181       | 0.212       | 0.243       | 0.276       | 0.311       | 0.347       | 0.385       | 0.425       |
| 0.55                                                                     | 0.138       | 0.164       | 0.192       | 0.221       | 0.251       | 0.284       | 0.318       | 0.354       | 0.392       |
| 0.6                                                                      | 0.123       | 0.146       | 0.172       | 0.198       | 0.226       | 0.256       | 0.288       | 0.321       | 0.357       |
| 0.65                                                                     | 0.108       | 0.129       | 0.151       | 0.175       | 0.200       | 0.227       | 0.256       | 0.287       | 0.32        |
| 0.7                                                                      | 0.092       | 0.111       | 0.13        | 0.151       | 0.174       | 0.198       | 0.224       | 0.252       | 0.282       |
| 0.75                                                                     | 0.077       | 0.093       | 0.109       | 0.127       | 0.147       | 0.167       | 0.19        | 0.214       | 0.241       |
| 0.8                                                                      | 0.062       | 0.075       | 0.088       | 0.103       | 0.119       | 0.136       | 0.155       | 0.175       | 0.198       |
| 0.85                                                                     | 0.046       | 0.056       | 0.067       | 0.078       | 0.09        | 0.104       | 0.118       | 0.135       | 0.153       |
| 0.9                                                                      | 0.031       | 0.038       | 0.045       | 0.052       | 0.061       | 0.07        | 0.081       | 0.092       | 0.105       |
| Area(A)                                                                  | 0.161       | 0.19        | 0.219       | 0.25        | 0.282       | 0.314       | 0.348       | 0.384       | 0.42        |

**Table 5.** Using Weddle's rule

| Table 5 -For Fig. (5) [ Where Fixed $p_A=0.25$ , $L_2 = 0.35$ , $h=0.05$ ] |             |             |             |             |             |             |             |             |             |
|----------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| p                                                                          | 0.1         | 0.2         | 0.3         | 0.4         | 0.5         | 0.6         | 0.7         | 0.8         | 0.9         |
| L1                                                                         | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ |
| 0                                                                          | 0.336       | 0.41        | 0.484       | 0.558       | 0.631       | 0.705       | 0.779       | 0.853       | 0.926       |
| 0.05                                                                       | 0.323       | 0.393       | 0.464       | 0.535       | 0.606       | 0.676       | 0.747       | 0.818       | 0.889       |
| 0.1                                                                        | 0.309       | 0.376       | 0.444       | 0.512       | 0.58        | 0.647       | 0.715       | 0.783       | 0.85        |
| 0.15                                                                       | 0.295       | 0.359       | 0.424       | 0.488       | 0.553       | 0.617       | 0.682       | 0.747       | 0.811       |
| 0.2                                                                        | 0.28        | 0.341       | 0.403       | 0.464       | 0.526       | 0.587       | 0.649       | 0.71        | 0.771       |
| 0.25                                                                       | 0.265       | 0.323       | 0.382       | 0.44        | 0.498       | 0.556       | 0.614       | 0.672       | 0.731       |
| 0.3                                                                        | 0.25        | 0.305       | 0.36        | 0.415       | 0.47        | 0.524       | 0.579       | 0.634       | 0.689       |
| 0.35                                                                       | 0.235       | 0.286       | 0.338       | 0.389       | 0.441       | 0.492       | 0.544       | 0.595       | 0.647       |
| 0.4                                                                        | 0.219       | 0.267       | 0.315       | 0.363       | 0.411       | 0.459       | 0.507       | 0.555       | 0.603       |
| 0.45                                                                       | 0.203       | 0.247       | 0.292       | 0.336       | 0.381       | 0.425       | 0.47        | 0.514       | 0.559       |
| 0.5                                                                        | 0.186       | 0.227       | 0.268       | 0.309       | 0.35        | 0.391       | 0.432       | 0.473       | 0.514       |
| 0.55                                                                       | 0.17        | 0.207       | 0.244       | 0.281       | 0.319       | 0.356       | 0.393       | 0.43        | 0.467       |
| 0.6                                                                        | 0.153       | 0.186       | 0.219       | 0.253       | 0.286       | 0.32        | 0.353       | 0.387       | 0.42        |
| 0.65                                                                       | 0.135       | 0.165       | 0.194       | 0.224       | 0.253       | 0.283       | 0.313       | 0.342       | 0.372       |
| 0.7                                                                        | 0.117       | 0.143       | 0.168       | 0.194       | 0.22        | 0.245       | 0.271       | 0.297       | 0.322       |
| 0.75                                                                       | 0.099       | 0.12        | 0.142       | 0.164       | 0.185       | 0.207       | 0.228       | 0.25        | 0.272       |
| 0.8                                                                        | 0.08        | 0.097       | 0.115       | 0.132       | 0.15        | 0.167       | 0.185       | 0.202       | 0.22        |
| 0.85                                                                       | 0.061       | 0.074       | 0.087       | 0.1         | 0.114       | 0.127       | 0.14        | 0.154       | 0.167       |
| 0.9                                                                        | 0.041       | 0.05        | 0.059       | 0.068       | 0.077       | 0.086       | 0.095       | 0.104       | 0.113       |
| Area(A)                                                                    | 0.191       | 0.233       | 0.275       | 0.316       | 0.358       | 0.4         | 0.442       | 0.484       | 0.526       |

**Table 6.** Using Weddle's rule

| Table 6 -For Fig. (6) [ Where Fixed $p=0.15$ , $L_2 = 0.45$ , $h=0.05$ ] |             |             |             |             |             |             |             |             |             |
|--------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $P_A$                                                                    | 0.1         | 0.2         | 0.3         | 0.4         | 0.5         | 0.6         | 0.7         | 0.8         | 0.9         |
| $L_1$                                                                    | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ | $\bar{P}_1$ |
| 0                                                                        | 0.494       | 0.456       | 0.418       | 0.38        | 0.341       | 0.303       | 0.265       | 0.227       | 0.188       |
| 0.05                                                                     | 0.478       | 0.44        | 0.401       | 0.363       | 0.326       | 0.289       | 0.252       | 0.215       | 0.179       |
| 0.1                                                                      | 0.462       | 0.423       | 0.384       | 0.347       | 0.311       | 0.275       | 0.239       | 0.204       | 0.17        |
| 0.15                                                                     | 0.444       | 0.405       | 0.367       | 0.331       | 0.295       | 0.26        | 0.226       | 0.193       | 0.16        |
| 0.2                                                                      | 0.426       | 0.387       | 0.35        | 0.314       | 0.279       | 0.246       | 0.214       | 0.182       | 0.151       |
| 0.25                                                                     | 0.408       | 0.369       | 0.332       | 0.297       | 0.263       | 0.231       | 0.201       | 0.171       | 0.141       |
| 0.3                                                                      | 0.388       | 0.349       | 0.313       | 0.279       | 0.247       | 0.217       | 0.188       | 0.159       | 0.132       |
| 0.35                                                                     | 0.368       | 0.33        | 0.294       | 0.262       | 0.231       | 0.202       | 0.175       | 0.148       | 0.123       |
| 0.4                                                                      | 0.347       | 0.309       | 0.275       | 0.243       | 0.214       | 0.187       | 0.161       | 0.137       | 0.113       |
| 0.45                                                                     | 0.325       | 0.288       | 0.255       | 0.225       | 0.198       | 0.172       | 0.148       | 0.126       | 0.104       |
| 0.5                                                                      | 0.302       | 0.266       | 0.235       | 0.206       | 0.181       | 0.157       | 0.135       | 0.114       | 0.094       |
| 0.55                                                                     | 0.278       | 0.244       | 0.214       | 0.187       | 0.164       | 0.142       | 0.122       | 0.103       | 0.085       |
| 0.6                                                                      | 0.253       | 0.221       | 0.193       | 0.168       | 0.146       | 0.127       | 0.109       | 0.092       | 0.076       |
| 0.65                                                                     | 0.227       | 0.196       | 0.171       | 0.148       | 0.129       | 0.111       | 0.095       | 0.08        | 0.066       |
| 0.7                                                                      | 0.199       | 0.171       | 0.148       | 0.128       | 0.111       | 0.096       | 0.082       | 0.069       | 0.057       |
| 0.75                                                                     | 0.17        | 0.145       | 0.125       | 0.108       | 0.093       | 0.08        | 0.068       | 0.057       | 0.047       |
| 0.8                                                                      | 0.14        | 0.119       | 0.101       | 0.087       | 0.075       | 0.064       | 0.055       | 0.046       | 0.038       |
| 0.85                                                                     | 0.107       | 0.091       | 0.077       | 0.066       | 0.057       | 0.048       | 0.041       | 0.035       | 0.028       |
| 0.9                                                                      | 0.074       | 0.062       | 0.052       | 0.044       | 0.038       | 0.032       | 0.027       | 0.023       | 0.019       |
| Area(A)                                                                  | 0.299       | 0.268       | 0.239       | 0.213       | 0.188       | 0.165       | 0.142       | 0.121       | 0.1         |

Figure 1. Variation over  $L_2$ Figure 2. Variation over  $P$

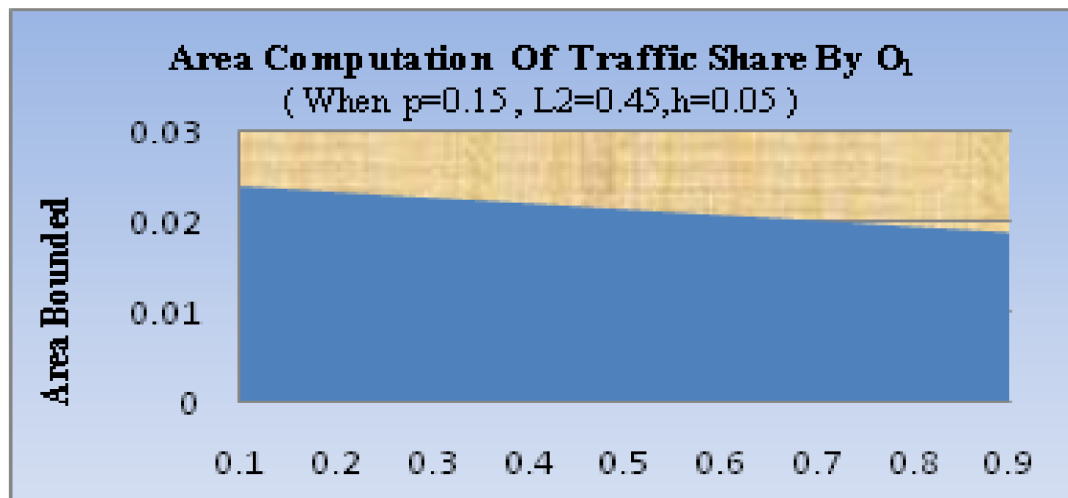


Figure 3. Variation over  $pA$

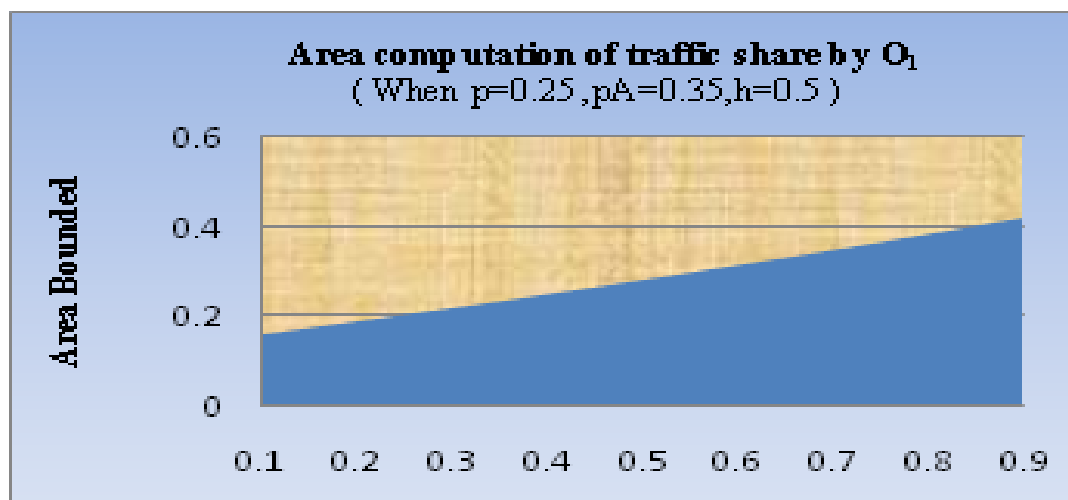


Figure 4. Variation over  $L2$

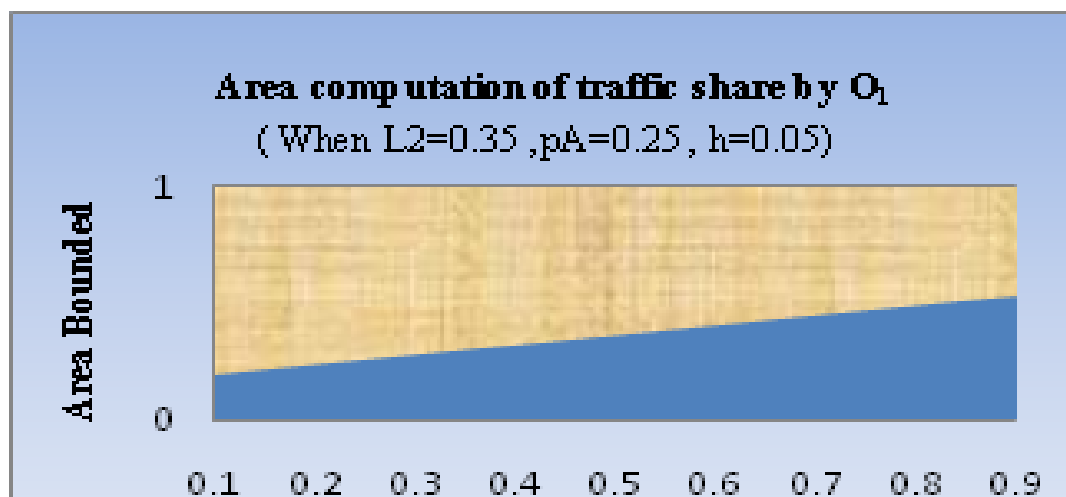


Figure 5. Variation over p

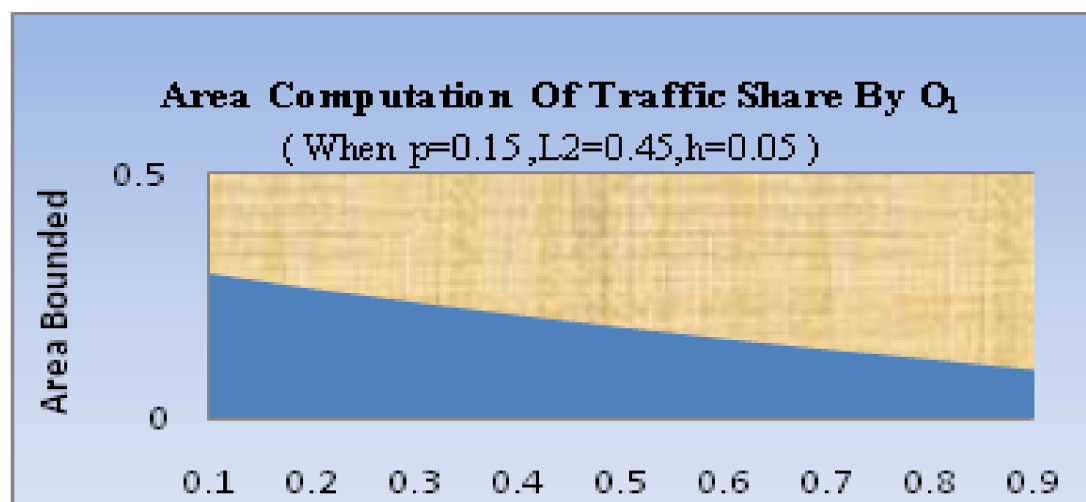


Figure 6. Variation over pA