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Velocity and acceleration in parabolic cylindrical coordinates

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ABSTRACTS

Instantaneous velocity and acceleration are often studied and expressed in Cartesian, circular cylindrical and spherical coordinates system for applications in mechanics but it is a well-known fact that some bodies cannot not be perfectly described in these coordinates systems, so they require some other curvilinear coordinates systems such as oblate spheroidal, parabolic cylindrical etc. In this paper we derive new expression for position vector, instantaneous velocity and acceleration of bodies and test particle in parabolic cylindrical coordinates system for applications in Newtonian Mechanics, Einstein's Special Relativistic law of motion and Schrödinger's law of Quantum Mechanics.

Keyword: *velocity, acceleration, Newtonian's mechanics, parabolic cylindrical coordinates.*

INTRODUCTION

Velocity and acceleration in Spheroidals Coordinates and Parabolic Coordinates had been established [1, 2].The parabolic cylindrical coordinates system , (ξ, η, z) are defined in terms of the Cartesian coordinates (x, y, z) by [3, 4]

$$x = \xi\eta \tag{1}$$

$$y = \frac{1}{2}(\eta^2 - \xi^2) \tag{2}$$

$$z = z \tag{3}$$

where,

$$-\infty < \xi < \infty; \quad 0 \leq \eta < \infty; \quad -\infty < z < \infty \tag{4}$$

The scale factors $(h_\xi,\ h_\eta, h_z)$ of parabolic cylindrical coordinates are given as

$$h_\xi = (\eta^2 + \xi^2)^{\frac{1}{2}} \tag{5}$$

$$h_\eta = (\eta^2 + \xi^2)^{\frac{1}{2}} \tag{6}$$

$$h_z = 1 \tag{7}$$

These scale factors obtained define the unit vectors, line elements, volume element, as well as gradient, divergence, curl and Laplacian operators in parabolic cylindrical coordinates, according to theory of orthogonal curvilinear coordinates. These quantities are necessary and sufficient for the derivation of the fields of all parabolic distribution of mass and charge and current. Now for the derivation of the equations of motion for test particles in these fields, we shall derive the expression for instantaneous velocity and acceleration in parabolic cylindrical coordinates.

2.0 MATHEMATICAL FORMULATIONS

The Cartesian unit vectors are related to the parabolic cylindrical coordinate unit vector as:

$$\hat{\xi} = \frac{\eta}{(\eta^2 + \xi^2)^{\frac{1}{2}}}\hat{\imath} - \frac{\xi}{(\eta^2 + \xi^2)^{\frac{1}{2}}}\hat{\jmath} \tag{8}$$

and

$$\hat{\eta} = \frac{\xi}{(\eta^2 + \xi^2)^{\frac{1}{2}}}\hat{\imath} + \frac{\eta}{(\eta^2 + \xi^2)^{\frac{1}{2}}}\hat{\jmath} \tag{9}$$

and

$$\hat{k} = \hat{k} \tag{10}$$

The time differential of equation (8), (9) and (10) is obtained by denoting one time differentiation by a dot and some mathematical manipulation as:

$$\dot{\hat{\xi}} = \frac{1}{\xi^2 + \eta^2}[\xi\dot{\eta} - \eta\dot{\xi}]\hat{\eta} \tag{11}$$

$$\dot{\hat{\eta}} = \frac{1}{\xi^2 + \eta^2}[\eta\dot{\xi} - \xi\dot{\eta}]\hat{\xi} \tag{12}$$

and

$$\dot{\hat{k}} = \dot{\hat{k}} \tag{13}$$

Now it follows from definition of instantaneous position vector, $\underline{r}$, as:

$$\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{14}$$

This can now be express in parabolic cylindrical coordinate (ξ, η, z) as:

$$\underline{r} = \eta\xi\hat{\imath} + \frac{1}{2}(\eta^2 - \xi^2)\hat{\jmath} + z\hat{k} \tag{15}$$

Substituting (11), (12), and (13) into (15);

The position vector $\underline{r}$ in parabolic cylindrical coordinates now becomes:

$$\underline{r} = \frac{\xi}{2}(\eta^2 + \xi^2)^{\frac{1}{2}}\hat{\xi} + \frac{\eta}{2}(\eta^2 + \xi^2)^{\frac{1}{2}}\hat{\eta} + k\hat{k} \tag{16}$$

It now follows from definition of instantaneous velocity vector $\underline{u}$ as:

$$\underline{u} = \dot{\underline{r}} \tag{17}$$

and equation (16) and (11)-(14) that the instantaneous velocity may be given as:

$$\vec{u} = u_\xi\hat{\xi} + u_\eta\hat{\eta} + u_k\hat{k} \tag{18}$$

where

$$u_\xi = \left\{\frac{\eta^2 + \xi^2}{(\eta^2 + \xi^2)^{\frac{1}{2}}}\right\}\dot{\xi} \tag{19}$$

$$u_\eta = \left\{\frac{\eta^2 + \xi^2}{(\eta^2 + \xi^2)^{\frac{1}{2}}}\right\}\dot{\eta} \tag{20}$$

and

$$u_k = \dot{k} \tag{21}$$

Therefore equation (18) describes the velocity in a parabolic cylindrical coordinates system

Similarly, it also follows from definition of instantaneous acceleration $\underline{a}$ as

$$\underline{a} = \dot{\underline{u}} \tag{22}$$

and equations (18) – (21) and (11) – (13) that the instantaneous acceleration in parabolic cylindrical coordinates can be expressed as:

$$\vec{a} = a_\xi \hat{\boldsymbol{\xi}} + a_\eta \hat{\boldsymbol{\eta}} + a_k \hat{\boldsymbol{k}} \tag{23}$$

where

$$a_\xi = (\eta^2 + \xi^2)^{\frac{1}{2}} \left\{ \ddot{\xi} + \frac{\dot{\xi}^2 \xi}{\xi^2 + \eta^2} - \frac{\dot{\eta}^2 \xi}{\xi^2 + \eta^2} + \frac{2\eta\dot{\eta}\dot{\xi}}{\xi^2 + \eta^2} \right\} \tag{24}$$

$$a_\eta = (\eta^2 + \xi^2)^{\frac{1}{2}} \left\{ \ddot{\eta} - \frac{\dot{\xi}^2 \eta}{\xi^2 + \eta^2} + \frac{\dot{\eta}^2 \eta}{\xi^2 + \eta^2} + \frac{2\xi\dot{\eta}\dot{\xi}}{\xi^2 + \eta^2} \right\} \tag{25}$$

and

$$a_k = \ddot{k} \tag{26}$$

Therefore equation (23) - (26) describes the acceleration of a body in parabolic cylindrical coordinates system.

RESULTS AND DISCUSSION

In this paper we derived the component of velocity as (18)-(20) and acceleration as (23)-(26) in parabolic cylindrical coordinates system. These results obtained in this paper are necessary and sufficient for expressing all mechanical quantities (linear momentum, kinetic energy, Lagragian and Hamiltonian) in terms of parabolic cylindrical coordinates system. Also results in this paper has pave way for expressing all dynamical law of motion (Newton's laws, Lagrange's law, Hamiltonian's law, Einstein's Special Relativistic Law of Motion and Schrödinger Law of Quantum Mechanics) entirely in parabolic cylindrical coordinates system.

CONCLUSION

Finally, the work in this paper paves for expressing all dynamical laws of motion (Newton's law, Lagrange's law, Hamilton's law, Einstein's Special Relativistic law of motion, and Schrodinger's law of quantum mechanics) entirely in terms of Parabolic Cylindrical Coordinates.

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