Some Results of Intuitionistic Fuzzy Soft Matrix Theory

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ABSTRACT

The concept of soft set is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parametrization tool of soft set theory enhance the flexibility of its applications. In this paper, we define intuitionistic fuzzy soft matrices and their operations which are more functional to make theoretical studies in the intuitionistic fuzzy soft set theory. We also define five types of products and some results are established.

Keywords: Soft sets, Intuitionistic fuzzy soft sets, Soft matrices, Intuitionistic fuzzy soft matrices and Products of intuitionistic fuzzy soft matrices.

INTRODUCTION

Most of our real life problems in medical sciences, engineering, management, environment and social sciences often involve data which are not always all crisp, precise and determinist in character because of various uncertainties typical for these problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy sets, intuitionistic fuzzy sets, interval mathematics and rough sets etc. However, Molodtsov[8] has shown that each of the above topics suffers from some inherent difficulties due to inadequacy of their parametrization tools and introduced a concept called ‘Soft Set Theory’ having parametrization tools for successfully dealing with various types of uncertainties. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Research on soft sets has been very wide spread and many important results have been achieved in the theoretical aspect. Maji et al. introduced several algebraic operations in soft set theory and published a detailed theoretical study on soft sets[7]. The same authors also extended crisp soft sets to fuzzy soft sets[4] and intuitionistic fuzzy soft sets[6]. At the same time, there has been some progress concerning practical applications of soft set theory, especially the use of soft sets in decision making. Recently, Çağman et al.[1] introduced soft matrix and applied it in decision making problems. In one of our earlier work[2], we proposed the idea of ‘Fuzzy Soft Matrix Theory’ in sequel to [1] defining some operations. The present paper aims to define intuitionistic fuzzy soft matrix and establish some results on them. This style of representation is useful for storing an intuitionistic fuzzy soft set in computer memory and which are very useful and applicable.

2. Preliminaries

Definition 2.1[8]

Let \( U \) be an initial universe, \( P(U) \) be the power set of \( U \), \( E \) be the set of all parameters and \( A \subseteq E \). A soft set on the universe \( U \) is defined by the set of ordered pairs \( (f_A, E) = \{(e, f_A(e)) | e \in E, \text{ and } f_A(e) \in P(U)\} \), where \( f_A : E \rightarrow P(U) \) such that \( f_A(e) = \emptyset \text{ if } e \notin A \).

Here, \( f_A \) is called an approximate function of the soft set \( (f_A, E) \). The set \( f_A(e) \) is called \( e \)-approximate value set or \( e \)-approximate set which consists of related objects of the parameter \( e \in E \).
Example 2.1
Let \( U = \{c_1, c_2, c_3\} \) be the set of three cars and \( E = \{\text{costly}(e_1), \text{metallic colour}(e_2), \text{cheap}(e_3)\} \) be the set of parameters, where \( A = \{e_1, e_2, e_3\} \subseteq E \). Then \( f_2(e_1) = \{c_1, c_2, c_3\} \), \( f_2(e_2) = \{c_1, e_3\} \), and \( f_2(e_3) = \{e_1, e_3\} \). Let \( f_0, E = \{(e_1, \{c_1, c_2, c_3\})(e_2, \{c_1, e_3\})\} \) over \( U \) which describes the “attractiveness of the cars” which Mr. S (say) is going to buy.

Definition 2.2 \([3,5]\)
Let \( U \) be a universal set, \( E \) a set of parameters and \( A \subseteq E \). Let \( F(U) \) denotes the set of all fuzzy subsets of \( U \). A fuzzy soft set \( (f_0, E) \) on the universe \( U \) is defined as the set of ordered pairs \( (f_0, E) = \{(e, f_0(e)) : e \in E, f_0(e) \in F(U)\} \), where \( f_0 : E \rightarrow F(U) \).

Here, \( f_0 \) is called an approximate function of the fuzzy soft set \( (f_0, E) \). The set \( f_0(e) \) is called e-approximate value set or e-approximate set which consists of related objects of the parameter \( e \in E \).

Example 2.2
Let \( U = \{c_1, c_2, c_3\} \) be the set of three cars and \( E = \{\text{costly}(e_1), \text{metallic colour}(e_2), \text{getup}(e_3)\} \) be the set of parameters, where \( A = \{e_1, e_2, e_3\} \subseteq E \). Then \( (G, A) = (G(e_1) = \{e_1, c_2, c_3, e_3\}, G(e_2) = \{c_1, 5, c_7, c_8\}) \) is the fuzzy soft set over \( U \) and describes the “attractiveness of the cars” which Mr. S (say) is going to buy.

Definition 2.3 \([1]\)
Let \( (f_0, E) \) be a soft set over \( U \). Then a subset of \( U \times E \) is uniquely defined by \( R_A = \{(u, e) : e \in A, u \in f_0(e)\} \) which is called a relation form of \( (f_0, E) \). The characteristic function of \( R_A \) is written by \( \chi_{R_A} : U \times E \rightarrow \{0, 1\} \), \( \chi_{R_A}(u, e) = 1 \), \( (u, e) \in R_A \).

If \( U = \{u_1, u_2, \ldots, u_m\} \), \( E = \{e_1, e_2, \ldots, e_n\} \) and \( A \subseteq E \), then the \( R_A \) can be presented by a table as given below:

\[
\begin{array}{ccc}
\begin{array}{cccc}
R_A & e_1 & e_2 & e_n \\
\hline
u_1 & \chi_{R_A}(u_1, e_1) & \chi_{R_A}(u_1, e_2) & \cdots & \chi_{R_A}(u_1, e_n) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
u_m & \chi_{R_A}(u_m, e_1) & \chi_{R_A}(u_m, e_2) & \cdots & \chi_{R_A}(u_m, e_n) \\
\end{array}
\end{array}
\]

If \( a_{ij} = \chi_{R_A}(u_i, e_j) \), we can define a matrix

\[
[a_{ij}] = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

which is called an \( m \times n \) soft matrix of the soft set \((f_0, E)\) over \( U \).

According to this definition, a soft set \((f_0, E)\) is uniquely characterized by the matrix \([a_{ij}]_{m \times n}\). It means that a soft set \((f_0, E)\) is formally equal to its soft matrix \([a_{ij}]_{m \times n}\).

3. Fuzzy soft matrices:

Definition 3.1 \([2]\)
Let \( (f_0, E) \) be a fuzzy soft set over \( U \). Then a subset of \( U \times E \) is uniquely defined by \( R_A = \{(u, e) : e \in A, u \in f_0(e)\} \) which is called a relation form of \((f_0, E)\). The characteristic function of \( R_A \) is written by \( \mu_{R_A} : U \times E \rightarrow [0, 1] \), \( \mu_{R_A}(u, e) \in [0, 1] \) is the membership value of \( u \in U \) for each \( e \in E \).
If \( \mu_{ij} = \mu_{R_{i}}(u_{i}, e_{j}) \), we can define a matrix

\[
[\mu_{ij}]_{m \times n} = 
\begin{bmatrix}
\mu_{11} & \mu_{12} & \ldots & \mu_{1n} \\
\mu_{21} & \mu_{22} & \ldots & \mu_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{m1} & \mu_{m2} & \ldots & \mu_{mn}
\end{bmatrix}
\]

which is called an \( m \times n \) fuzzy soft matrix of the fuzzy soft set \((f_{A}, E)\) over \( U \).

Therefore, we can say that a fuzzy soft set \((f_{A}, E)\) is uniquely characterized by the matrix \([\mu_{ij}]_{m \times n}\) and both concept are interchangeable.

The set of all \( m \times n \) fuzzy soft matrices over \( U \) will be denoted by \( FSM_{m \times n} \).

**Example 3.1**

Assume that \( U = \{u_{1}, u_{2}, u_{3}, u_{4}, u_{5}\} \) is a universal set and \( E = \{e_{1}, e_{2}, e_{3}, e_{4}\} \) is a set all parameters. If \( A \subseteq E = \{e_{2}, e_{3}, e_{4}\} \) and \( f_{A}(e_{2}) = \{u_{1}/.4, u_{2}/.5, u_{3}/1, u_{4}/.3, u_{5}/.6\} \), \( f_{A}(e_{3}) = \{u_{1}/.3, u_{2}/.4, u_{3}/.6, u_{4}/.5, u_{5}/1\} \), \( f_{A}(e_{4}) = \{u_{1}/.5, u_{2}/.6, u_{3}/.4, u_{4}/.3, u_{5}/.9\} \).

Then the fuzzy soft set \((f_{A}, E)\) is a parametrized family \( \{f_{A}(e_{2}), f_{A}(e_{3}), f_{A}(e_{4})\} \) of all fuzzy sets over \( U \). Then the relation form of \((f_{A}, E)\) is written by

\[
R_{A} = 
\begin{array}{cccc}
e_{1} & e_{2} & e_{3} & e_{4} \\
0.4 & 0.3 & 0.5 \\
0.5 & 0.4 & 0.5 \\
1 & 0.6 & 0.4 \\
0.3 & 0.5 & 0.3 \\
0.6 & 1 & 0.9
\end{array}
\]

Hence the fuzzy soft matrix \([\mu_{ij}]\) is written as

\[
[\mu_{ij}] = 
\begin{bmatrix}
0 & 0.4 & 0.3 & 0.4 \\
0 & 0.5 & 0.4 & 0.5 \\
0 & 1 & 0.6 & 0.4 \\
0 & 0.3 & 0.5 & 0.3 \\
0 & 0.6 & 1 & 0.4
\end{bmatrix}
\]

**Definition 3.2[2]**

Let \([\mu_{ij}] \in FSM_{m \times n}\). Then \([\mu_{ij}]\) is called

(a) a zero fuzzy soft matrix, denoted by \([0]\), if \( \mu_{ij} = 0 \) for all \( i \) and \( j \).

(b) a universal fuzzy soft matrix, denoted by \([1]\), if \( \mu_{ij} = 1 \) for all \( i \) and \( j \).

(c) \([\mu_{ij}]\) is a fuzzy soft submatrix of \([\lambda_{ij}]\), denoted by \([\mu_{ij}] \subseteq \lambda_{ij}\), if \( \mu_{ij} \leq \lambda_{ij} \) for all \( i \) and \( j \).
(d) $[\mu_{ij}]$ and $[\lambda_{ij}]$ are fuzzy soft equal matrices, denoted by $[\mu_{ij}] = [\lambda_{ij}]$, if $\mu_{ij} = \lambda_{ij}$
for all $i$ and $j$.

**Definition 3.3**

Let $[\mu_{ij}]$, $[\lambda_{ij}] \in \text{FSM}_{m \times n}$. Then the fuzzy soft matrix $[v_{ij}]$ is called
(a) union of $[\mu_{ij}]$ and $[\lambda_{ij}]$, denoted by $[\mu_{ij}] \cup [\lambda_{ij}]$ if $v_{ij} = \max\{\mu_{ij}, \lambda_{ij}\}$ for all $i$ and $j$.
(b) intersection of $[\mu_{ij}]$ and $[\lambda_{ij}]$, denoted by $[\mu_{ij}] \cap [\lambda_{ij}]$ if $v_{ij} = \min\{\mu_{ij}, \lambda_{ij}\}$
for all $i$ and $j$.
(c) complement of $[\mu_{ij}]$, denoted by $[\mu_{ij}]^{\circ}$, if $v_{ij} = 1 - \mu_{ij}$ for all $i$ and $j$.

**Example 3.2**

Let $[\mu_{ij}] = \begin{bmatrix} 0.2 & 0.4 & 0.5 & 0.6 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0.6 & 0.3 \end{bmatrix}$ and $[\lambda_{ij}] = \begin{bmatrix} 0.3 & 0.4 & 0.4 & 0.6 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0.6 & 0.3 \end{bmatrix}$. Then

$[\mu_{ij}] \cup [\lambda_{ij}] = \begin{bmatrix} 0.4 & 0.4 & 0.5 & 0.6 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0.6 & 0.3 \end{bmatrix}$ and $[\mu_{ij}]^{\circ} = [\lambda_{ij}]^{\circ}$.

**Proposition 3.1**

Let $[\mu_{ij}]$, $[\lambda_{ij}] \in \text{FSM}_{m \times n}$. Then
(i) $([\mu_{ij}] \cup [\lambda_{ij}])^{\circ} = [\mu_{ij}]^{\circ} \cap [\lambda_{ij}]^{\circ}$
(ii) $([\mu_{ij}] \cap [\lambda_{ij}])^{\circ} = [\mu_{ij}]^{\circ} \cup [\lambda_{ij}]^{\circ}$

**Proof:** (i) For all $i$ and $j$,

$(([\mu_{ij}] \cup [\lambda_{ij}])^{\circ} = \max\{\mu_{ij}, \lambda_{ij}\}^{\circ}$

$= [1 - \max\{\mu_{ij}, \lambda_{ij}\}]$

$= [\min\{1 - \mu_{ij}, 1 - \lambda_{ij}\}]$

$= [\mu_{ij}]^{\circ} \cap [\lambda_{ij}]^{\circ}$

(ii) Similar to (i).

**Proposition 3.2**

Let $[\mu_{ij}]$, $[v_{ij}]$, $[\lambda_{ij}] \in \text{FSM}_{m \times n}$, then
(i) $[\mu_{ij}] \cup ([(v_{ij}] \cap [\lambda_{ij}]) = ([\mu_{ij}] \cup [v_{ij}]) \cap ([\mu_{ij}] \cup [\lambda_{ij}])$
(ii) $[\mu_{ij}] \cap ([(v_{ij}] \cup [\lambda_{ij}]) = ([\mu_{ij}] \cap [v_{ij}]) \cup ([\mu_{ij}] \cap [\lambda_{ij}])$

4. Product of fuzzy soft matrices

In this section, four types of products of fuzzy soft matrices are defined in continuation to four special products of soft matrices introduced by Çagman et al.[1].

**Definition 4.1**

Let $[\mu_{ij}]$, $[v_{ij}] \in \text{FSM}_{m \times n}$. Then And-product of $[\mu_{ij}]$ and $[v_{ij}]$ is defined by
$\&: \text{FSM}_{m \times n} \times \text{FSM}_{m \times n} \rightarrow \text{FSM}_{m \times n}$, $[\mu_{ij}] \& [v_{ij}] = [\lambda_{ij}]$ where $\lambda_{ij} = \min\{\mu_{ij}, v_{ij}\}$ such that $p = n(j-1)+k$.

**Definition 4.2**

Let $[\mu_{ij}]$, $[v_{ij}] \in \text{FSM}_{m \times n}$. Then Or-product of $[\mu_{ij}]$ and $[v_{ij}]$ is defined by
$\lor: \text{FSM}_{m \times n} \times \text{FSM}_{m \times n} \rightarrow \text{FSM}_{m \times n}$, $[\mu_{ij}] \lor [v_{ij}] = [\lambda_{ij}]$ where $\lambda_{ij} = \max\{\mu_{ij}, v_{ij}\}$ such that $p = n(j-1)+k$. 

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Definition 4.3\[2\]
Let \([\mu_{ij}], [\nu_{ik}] \in \text{FSM}_{m \times n}\). Then And-Not-product of \([\mu_{ij}]\) and \([\nu_{ij}]\) is defined by
\[
\wedge: \text{FSM}_{m \times n} \times \text{FSM}_{m \times n} \rightarrow \text{FSM}_{m \times n}, 
[\mu_{ij}] \wedge [\nu_{ik}] = [\lambda_{ip}],
\]
where \(\lambda_{ip} = \min\{\mu_{ij}, 1-\nu_{ik}\}\) such that \(p = n(j-1)+k\).

Definition 4.4\[2\]
Let \([\mu_{ij}], [\nu_{ik}] \in \text{FSM}_{m \times n}\). Then Or-Not-product of \([\mu_{ij}]\) and \([\nu_{ik}]\) is defined by
\[
\vee: \text{FSM}_{m \times n} \times \text{FSM}_{m \times n} \rightarrow \text{FSM}_{m \times n}, 
[\mu_{ij}] \vee [\nu_{ik}] = [\lambda_{ip}],
\]
where \(\lambda_{ip} = \max\{\mu_{ij}, 1-\nu_{ik}\}\) such that \(p = n(j-1)+k\).

Example 4.1

Let \([\mu_{ij}], [\nu_{ik}] \in \text{FSM}_{5 \times 4}\) where
\[
\begin{bmatrix}
0.2 & 0.4 & 0.5 & 0.6 \\
0.3 & 0.5 & 0.1 & 1 \\
1 & 0.9 & 0.7 & 0.5 \\
0.6 & 0.5 & 0.6 & 0.3
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
0.3 & 0.4 & 0.4 & 0.5 \\
0.2 & 0.4 & 0.5 & 0.4 \\
1 & 0.9 & 0.7 & 0.5 \\
0.6 & 0.5 & 0.6 & 0.3
\end{bmatrix}
\]
then
\[
[\mu_{ij}] \wedge [\nu_{ik}] =
\begin{bmatrix}
0.2 & 0.2 & 0.2 & 0.3 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 & 0.6 \\
0.3 & 0.3 & 0.1 & 0.3 & 0.3 & 0.5 & 0.1 & 0.1 & 0.1 & 0.1 & 0.3 & 0.5 & 1 \\
0.5 & 0.1 & 0.7 & 0.5 & 0.1 & 0.7 & 0.5 & 0.1 & 0.7 & 0.5 & 0.1 & 0.7 & 0.5 \\
0.5 & 0.6 & 0.4 & 0.6 & 0.5 & 0.4 & 0.5 & 0.6 & 0.4 & 0.6 & 0.3 & 0.3 & 0.3
\end{bmatrix}
\]
Similarly, the other products can also be obtained.

Remark: Commutatively is not valid for the products of fuzzy soft matrices.

Proposition 4.1\[2\]
Let \([\mu_{ij}], [\lambda_{ik}] \in \text{FSM}_{m \times n}\), then
\[(i) \quad ([\mu_{ij}] \vee [\lambda_{ik}])^o = [\mu_{ij}]^o \wedge [\lambda_{ik}]^o \]
\[(ii) \quad ([\mu_{ij}] \wedge [\lambda_{ik}])^o = [\mu_{ij}]^o \vee [\lambda_{ik}]^o \]
\[(iii) \quad ([\mu_{ij}] \wedge [\lambda_{ik}])^o = [\mu_{ij}]^o \wedge [\lambda_{ik}]^o \]
\[(iv) \quad ([\mu_{ij}] \vee [\lambda_{ik}])^o = [\mu_{ij}]^o \vee [\lambda_{ik}]^o \]

5. Intuitionistic Fuzzy soft matrices (IFSMs).
In this section, we define intuitionistic fuzzy soft matrices.

Definition 5.1
Let \(U\) be an initial universe, \(E\) be the set of parameters and \(A \subseteq E\). Let \((f_A, E)\) be an intuitionistic fuzzy soft set (IFSS) over \(U\). Then a subset of \(U \times E\) is uniquely defined by \(R_A = \{(u, e); e \in A, u \in f_A (e)\}\) which is called a relation form of \((f_A, E)\). The membership function and non membership function are written by \(\mu_{R_A} : U \times E \rightarrow [0,1]\) and \(\nu_{R_A} : U \times E \rightarrow [0,1]\) where \(\mu_{R_A} : (u, e) \in [0,1]\) and \(\nu_{R_A} : (u, e) \in [0,1]\) are the membership value and non membership value respectively of \(u \in U\) for each \(e \in E\).

If \((\mu_j, \nu_j) = (\mu_{R_j} (u_j, e_j), \nu_{R_j} (u_j, e_j))\), we can define a matrix
which is called an $m \times n$ IFSM of the IFSS $(f_A, E)$ over $U$.
Therefore, we can say that a IFSS $(f_A, E)$ is uniquely characterized by the matrix $[(\mu_{ij}, \nu_{ij})]_{mn}$ and both concepts are interchangeable. The set of all $m \times n$ IFS matrices over $U$ will denoted by IFSM$_{m \times n}$.

**Example 5.1**

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters. If $A \subseteq E = \{e_2, e_3, e_4\}$ and

\[
\begin{align*}
&f_A(e_2) = \{u_1 / (0, .4, 5), u_2 / (0, .5, 3), u_3 / (0, 1, 0), u_4 / (0, .5, 6), u_5 / (0, 6, 2)\} \\
&f_A(e_3) = \{u_1 / (0, .3, 5), u_2 / (0, .4, 6), u_3 / (0, .5, 6), u_4 / (0, 1, 0)\} \\
&f_A(e_4) = \{u_1 / (0, .5, 2), u_2 / (0, .5, 6), u_3 / (0, .4, 6), u_4 / (0, .5, 6), u_5 / (0, 9, 1)\}
\end{align*}
\]

Then the IFS set $(f_A, E)$ is a parameterized family \{$(f_A(e_2), f_A(e_3), f_A(e_4))$\} of all IFS sets over $U$. Then the relation form of $(f_A, E)$ is written as

\[
R_A = \begin{array}{ccccc}
R_A & e_1 & e_2 & e_3 & e_4 \\
\hline
u_1 & 0 & (.4, 5) & (.3, 5) & (.5, 2) \\
u_2 & 0 & (.5, 3) & (.4, 6) & (.5, 5) \\
u_3 & 0 & (1, 0) & (.6, 2) & (.4, 6) \\
u_4 & 0 & (.3, 6) & (.5, 5) & (.3, 6) \\
u_5 & 0 & (.6, 2) & (1, 0) & (.9, 1) \\
\end{array}
\]

Hence the IFSM $[(\mu_{ij}, \nu_{ij})]$ is written by

\[
[(\mu_{54}, \nu_{54})] = \begin{bmatrix}
0 & (.4, 5) & (.3, 5) & (.5, 2) \\
0 & (.5, 3) & (.4, 6) & (.5, 5) \\
0 & (1, 0) & (.6, 2) & (.4, 6) \\
0 & (.3, 6) & (.5, 5) & (.3, 6) \\
0 & (.6, 2) & (1, 0) & (.9, 1)
\end{bmatrix}
\]

**Definition 5.2**

Let $\tilde{A} = [(\mu_{ij}, \nu_{ij})] \in $ IFSM$_{m \times n}$. Then $\tilde{A}$ is called

(a) a zero IFSM, denoted by $\tilde{0} = [(0, 0)]$, if $\mu_{ij} = 0$ and $\nu_{ij} = 0$ for all $i$ and $j$.
(b) a $\mu$ - universal IFSM, denoted by $\tilde{1} = [(1, 0)]$, if $\mu_{ij} = 1$ and $\nu_{ij} = 0$ for all $i$ and $j$.
(c) a $\nu$ - universal IFSM, denoted by $\tilde{I} = [(0, 1)]$, if $\mu_{ij} = 0$ and $\nu_{ij} = 1$ for all $i$ and $j$.

(d) $\tilde{A} = [(\mu_{ij}, \nu_{ij})]$ is a intuitionistic fuzzy soft sub matrix of $\tilde{B} = [(\mu_{ij}', \nu_{ij}')$, denoted by $\tilde{A} \subseteq \tilde{B}$ if $\mu_{ij}' \leq \mu_{ij}$ and $\nu_{ij}' \geq \nu_{ij}$ for all $i$ and $j$. 

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(e) \( \tilde{A} = [(\mu'_i, \nu'_i)] \) and \( \tilde{B} = [(\mu''_i, \nu''_i)] \) are intuitionistic fuzzy soft equal matrices, denoted by \( \tilde{A} = \tilde{B} \) if \( \mu'_i = \mu''_i \) and \( \nu'_i = \nu''_i \) for all \( i \) and \( j \).

**Definition 5.3**

Let \( \tilde{A} = [(\mu'_i, \nu'_i)], \tilde{B} = [(\mu''_i, \nu''_i)] \in \text{IFSM}_{m \times n} \). Then the IFSM \( \tilde{C} = [(\mu''_i, \nu''_i)] \) is called

(a) union of \( \tilde{A} \) and \( \tilde{B} \), denoted by \( \tilde{A} \cup \tilde{B} \) if \( \mu_{ij} = \max\{\mu'_i, \mu''_i\} \) and \( \nu_{ij} = \min\{\nu'_i, \nu''_i\} \) for all \( i \) and \( j \).

(b) intersection of \( \tilde{A} \) and \( \tilde{B} \), denoted by \( \tilde{A} \cap \tilde{B} \) if \( \mu_{ij} = \min\{\mu'_i, \mu''_i\} \) and

\[ \nu_{ij} = \max\{\nu'_i, \nu''_i\} \]

for all \( i \) and \( j \).

(c) complement of \( \tilde{A} = [(\mu_i, \nu_i)] \), denoted by \( \tilde{A}^* = [(\nu_i, \mu_i)] \) for all \( i \) and \( j \).

**Example 5.2**

Let

\[ \tilde{A} = \begin{bmatrix} (.1,2) & (.5,4) & (.3,6) \\ (.4,4) & (.2,3) & (.5,1) \\ (.5,2) & (.3,4) & (.6,2) \\ (.7,2) & (.6,1) & (.5,3) \end{bmatrix} \] \quad \text{and} \quad \tilde{B} = \begin{bmatrix} (.5,3) & (.1,6) & (.7,1) \\ (.8,1) & (.4,3) & (.5,2) \\ (.2,5) & (.3,6) & (.4,5) \\ (.1,7) & (.2,5) & (.5,1) \end{bmatrix} \]

Then

\[ \tilde{A} \cup \tilde{B} = \begin{bmatrix} (.5,2) & (.5,4) & (.7,1) \\ (.8,1) & (.4,3) & (.5,1) \\ (.5,2) & (.3,4) & (.6,2) \\ (.7,2) & (.6,1) & (.5,3) \end{bmatrix} \] \quad \text{and} \quad \tilde{A} \cap \tilde{B} = \begin{bmatrix} (.1,3) & (.1,6) & (.3,6) \\ (.4,4) & (.2,3) & (.5,2) \\ (.2,5) & (.3,6) & (.4,5) \\ (.1,7) & (.2,5) & (.5,3) \end{bmatrix} \]

\[ \tilde{A}^* = \begin{bmatrix} (.2,1) & (.4,5) & (.6,3) \\ (.4,4) & (.3,2) & (.1,5) \\ (.2,5) & (.4,3) & (.2,6) \\ (.2,7) & (.1,6) & (.3,5) \end{bmatrix} \]

**Proposition 5.1**

Let \( \tilde{A} = [(\mu'_i, \nu'_i)], \tilde{B} = [(\mu''_i, \nu''_i)] \in \text{IFSM}_{m \times n} \). Then

(i) \( (\tilde{A} \cup \tilde{B})^* = \tilde{A}^* \cap \tilde{B}^* \)

(ii) \( (\tilde{A} \cap \tilde{B})^* = \tilde{A}^* \cup \tilde{B}^* \)

**Proof**: For all \( i \) and \( j \)

(i)

\[ (\tilde{A} \cup \tilde{B})^* = \{((\mu'_i, \nu'_i)) \cup ((\mu''_i, \nu''_i))\}^* \]

\[ = \{\max\{\mu'_i, \mu''_i\}, \min\{\nu'_i, \nu''_i\}\}^* \]

\[ = \{\min\{\nu'_i, \nu''_i\}, \max\{\mu'_i, \mu''_i\}\} \]

\[ = \{\nu''_i, \mu''_i\} \cap \{\nu'_i, \mu'_i\} \]

\[ = \tilde{A}^* \cap \tilde{B}^* \]

(ii) Similar to (i).
Proposition 5.2

Let \( \tilde{A} = [(\mu_{ij}, \nu_{ij})] \in \text{IFSM}_{m \times n} \). Then

(i) \( (\tilde{A}^*)^* = \tilde{A} \)

(ii) \( (\tilde{I})^* = \tilde{I} \)

(iii) \( (\tilde{I}^*)^* = \tilde{I} \)

(iv) \( \tilde{A} \subseteq \tilde{I} \)

(v) \( \tilde{A} \supseteq \tilde{I} \)

Proposition 5.3

Let \( \tilde{A} = [(\mu_{ij}, \nu_{ij})] \in \text{IFSM}_{m \times n} \). Then

(i) \( \tilde{A} \cup \tilde{A} = \tilde{A} \)

(ii) \( \tilde{A} \cup \tilde{I} = \tilde{I} \)

(iii) \( \tilde{A} \cup I = \tilde{A} \)

(iv) \( \tilde{A} \cap \tilde{I} = \tilde{A} \)

(v) \( \tilde{A} \cap I = I \)

Proposition 5.4

Let \( \tilde{A} = [(\mu_{ij}, \nu_{ij})], \tilde{B} = [(\mu'_{ij}, \nu'_{ij})] \) and \( \tilde{C} = [(\mu''_{ij}, \nu''_{ij})] \in \text{IFSM}_{m \times n} \)

Then

(i) \( \tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A} \)

(ii) \( \tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A} \)

(iii) \( (\tilde{A} \cap \tilde{B}) \cap \tilde{C} = \tilde{A} \cap (\tilde{B} \cap \tilde{C}) \)

(iv) \( (\tilde{A} \cup \tilde{B}) \cup \tilde{C} = \tilde{A} \cup (\tilde{B} \cup \tilde{C}) \)

(v) \( \tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cap (\tilde{A} \cup \tilde{C}) \)

(vi) \( \tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C}) \)

Proposition 5.5

Let \( \tilde{A}, \tilde{B} \in \text{IFSM}_{4 \times 3} \) as in the Example 4.2

Then

\[
(\tilde{A} \cup \tilde{B})^* = \tilde{A}^* \cap \tilde{B}^* = \\
\begin{bmatrix}
(2,.5) & (4,.5) & (1,.7) \\
(1,.8) & (3,.4) & (1,.5) \\
(2,.5) & (4,.3) & (2,.6) \\
(2,.7) & (1,.6) & (1,.5)
\end{bmatrix}
\]

\[
(\tilde{A} \cap \tilde{B})^* = \tilde{A}^* \cup \tilde{B}^* = \\
\begin{bmatrix}
(3,.1) & (6,.1) & (6,.3) \\
(4,.4) & (3,.2) & (2,.5) \\
(5,.2) & (6,.3) & (5,.4) \\
(7,.1) & (5,.2) & (3,.5)
\end{bmatrix}
\]
Definition 5.4

If $\tilde{A} = [(\mu_{ij}', \nu_{ij}')]$, $\tilde{B} = [(\mu_{ij}'', \nu_{ij}'')] \in \text{IFSM}_{mon}$

Then the IFSM $\tilde{C} = [(\mu_{ij}, \nu_{ij})]$ is called

(i) the $'$ operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \tilde{B}$ if $\mu_{ij} = \mu_{ij}' \mu_{ij}''$ and

$$\nu_{ij} = \nu_{ij}' + \nu_{ij}'' - \nu_{ij}' \nu_{ij}''$$

for all $i$ and $j$.

(ii) the $+$ operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} + \tilde{B}$ if $\mu_{ij} = \mu_{ij}' + \mu_{ij}'' - \mu_{ij}' \mu_{ij}''$

$$\nu_{ij} = \nu_{ij}' \nu_{ij}''$$

for all $i$ and $j$.

(iii) the $@$ operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} @ \tilde{B}$

if $\mu_{ij} = \frac{\mu_{ij}' + \mu_{ij}''}{2}$, $\nu_{ij} = \frac{\nu_{ij}' + \nu_{ij}''}{2}$ for all $i$ and $j$.

(iv) the $'$ operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \tilde{B}$ if

$$\mu_{ij} = \sqrt{\mu_{ij}' \mu_{ij}''}, \nu_{ij} = \sqrt{\nu_{ij}' \nu_{ij}''}$$

for all $i$ and $j$.

Proposition 5.5

Let $\tilde{A}, \tilde{B}, \tilde{C} \in \text{IFSM}_{mon}$. Then

(i) $\tilde{A} \tilde{B} = \tilde{B} \tilde{A}$

(ii) $\tilde{A} + \tilde{B} = \tilde{B} + \tilde{A}$

(iii) $\tilde{A} @ \tilde{B} = \tilde{B} @ \tilde{A}$

(iv) $\tilde{A} \tilde{B} = \tilde{B} \tilde{A}$

(v) $(\tilde{A} + \tilde{B}) + \tilde{C} = (\tilde{A} + \tilde{B}) + \tilde{C}$

(vi) $(\tilde{A} \tilde{B}) \tilde{C} = \tilde{A}(\tilde{B} \tilde{C})$

(vii) $(\tilde{A} \cap \tilde{B}) + \tilde{C} = (\tilde{A} + \tilde{C}) \cap (\tilde{B} + \tilde{C})$

(Viii) $(\tilde{A} \cap \tilde{B}) \tilde{C} = (\tilde{A} \tilde{C}) \cap (\tilde{B} \tilde{C})$

(ix) $(\tilde{A} \cap \tilde{B}) @ \tilde{C} = (\tilde{A} @ \tilde{C}) \cap (\tilde{B} @ \tilde{C})$

(x) $(\tilde{A} \cup \tilde{B}) + \tilde{C} = (\tilde{A} + \tilde{C}) \cup (\tilde{B} + \tilde{C})$

(xi) $(\tilde{A} \cup \tilde{B}) \tilde{C} = (\tilde{A} \tilde{C}) \cup (\tilde{B} \tilde{C})$

Definition 5.5

If $\tilde{A} = [(\mu_{ij}, \nu_{ij})] \in \text{IFSM}_{mon}$. Then

(i) the necessity operation of $\tilde{A}$ denoted by $\tilde{A}$ if $\mu_{ij} = \mu_{ij}$, $\nu_{ij} = 1 - \nu_{ij}$ for all $i$ and $j$.

(ii) the possibility operation of $\tilde{A}$ denoted by $\tilde{A}$ if $\mu_{ij} = 1 - \nu_{ij}$, $\nu_{ij} = \nu_{ij}$ for all $i$ and $j$. 

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Proposition 5.6

Let $\tilde{A}, \tilde{B} \in \text{IFS}_{4\times3}$. Then

(i) $\tilde{\bigcup} (\tilde{A} \bigcup \tilde{B}) = \tilde{A} \bigcup \tilde{B}$

(ii) $\tilde{\bigcap} (\tilde{A} \bigcap \tilde{B}) = \tilde{A} \bigcap \tilde{B}$

(iii) $\tilde{\bigcap} \tilde{A} = \tilde{\bigcap} \tilde{A}$

(iv) $\tilde{\bigcup} (\tilde{A} \bigcup \tilde{B}) = \tilde{\bigcup} \tilde{A} \bigcup \tilde{B}$

(v) $\tilde{\bigcap} (\tilde{A} \bigcap \tilde{B}) = \tilde{\bigcap} \tilde{A} \bigcap \tilde{B}$

(vi) $\tilde{\bigcap} \tilde{A} = \tilde{\bigcap} \tilde{A}$

Example 5.5

Let $\tilde{A}, \tilde{B} \in \text{IFS}_{4\times3}$ as in the Example 4.2.

Then

\[
\tilde{\bigcup} (\tilde{A} \bigcup \tilde{B}) = \tilde{A} \bigcup \tilde{B} = \begin{bmatrix}
(.,5) & (.,5) & (.,7) \\
(.,2) & (.,4) & (.,5) \\
(.,5) & (.,3) & (.,6) \\
(.,7) & (.,6) & (.,5)
\end{bmatrix}
\]

and

\[
\tilde{\bigcap} (\tilde{A} \bigcap \tilde{B}) = \tilde{A} \bigcap \tilde{B} = \begin{bmatrix}
(.,8) & (.,6) & (.,9) \\
(.,9) & (.,7) & (.,9) \\
(.,8) & (.,6) & (.,8) \\
(.,8) & (.,9) & (.,9)
\end{bmatrix}
\]

6. Product of Intuitionistic Fuzzy Soft Matrices (IFSs)

In this section, five types of products of IFSM are defined in continuation to four special products of fuzzy soft matrices.

Definition 6.1

Let $\tilde{\tilde{A}} = [(\mu_{ij}', v_{ij}')], \tilde{\tilde{B}} = [(\mu_{ik}', v_{ik}')] \in \text{IFS}_{n\times m}$. Then ' $\times_1$ ' product of $\tilde{A}$ and $\tilde{B}$ is defined by

\[\times_1 : \text{IFS}_{n\times m} \times \text{IFS}_{m\times n} \rightarrow \text{IFS}_{n\times m}\]

such that $\tilde{\tilde{A}} \times_1 \tilde{\tilde{B}} = [(\mu_{ij}', v_{ij}')][[(\mu_{ik}', v_{ik}')] = [(\mu_{ip}', v_{ip}')]$

where $\mu_{ip} = \mu_{ij}' + \mu_{ik}' - \mu_{ij}' \mu_{ik}'$ and $v_{ip} = v_{ij}' v_{ik}'$ such that $p = n(j - i) + k$.

Definition 6.2

Let $\tilde{\tilde{A}} = [(\mu_{ij}', v_{ij}')], \tilde{\tilde{B}} = [(\mu_{ik}', v_{ik}')] \in \text{IFS}_{n\times m}$. Then ' $\times_2$ ' product of $\tilde{A}$ and $\tilde{B}$ is defined by

\[\times_2 : \text{IFS}_{n\times m} \times \text{IFS}_{m\times n} \rightarrow \text{IFS}_{n\times m}\]

such that $\tilde{\tilde{A}} \times_2 \tilde{\tilde{B}} = [(\mu_{ij}', v_{ij}')][[(\mu_{ik}', v_{ik}')] = [(\mu_{ip}', v_{ip}')]$

where $\mu_{ip} = \mu_{ij}' + \mu_{ik}' - \mu_{ij}' \mu_{ik}'$ and $v_{ip} = v_{ij}' v_{ik}'$ such that $p = n(j - i) - k$. 

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Definition 6.3
Let $A = [(\mu_{ij}, \nu_{ij})], B = [(\mu_{ik}, \nu_{ik})] \in IFSM_{mn}$. Then \( \times_3 \) product of $A$ and $B$ is defined by
$$
\times_3 : IFSM_{mn} \times IFSM_{mn} \rightarrow IFSM_{mn},
$$
such that $A \times_3 B = [(\mu_{ij}, \nu_{ij})] \times_3 [(\mu_{ik}, \nu_{ik})] = [(\mu_{ij}', \nu_{ij}')]
$$
where $\mu_{ij}' = \mu_{ij}' \cdot \mu_{ik}'$ and $\nu_{ij}' = \nu_{ij}' + \nu_{ij}' - \nu_{ij}' \cdot \nu_{ij}'$ such that $p = n(j - i) + k$.

Definition 6.4
Let $\tilde{A} = [(\mu_{ij}', \nu_{ij}')], \tilde{B} = [(\mu_{ik}', \nu_{ik}')] \in IFSM_{mn}$. Then \( \times_4 \) product of $\tilde{A}$ and $\tilde{B}$ is defined by
$$
\times_4 : IFSM_{mn} \times IFSM_{mn} \rightarrow IFSM_{mn},
$$
such that $\tilde{A} \times_4 \tilde{B} = [(\mu_{ij}', \nu_{ij}')] \times_4 [(\mu_{ik}', \nu_{ik}')] = [(\mu_{ij}, \nu_{ij})]
$$
where $\mu_{ij} = \min\{\mu_{ij}', \mu_{ik}'\}$ and $\nu_{ij} = \max\{\nu_{ij}', \nu_{ik}'\}$ such that $p = n(j - i) + k$.

Definition 6.5
Let $\tilde{A} = [(\mu_{ij}', \nu_{ij}')], \tilde{B} = [(\mu_{ik}', \nu_{ik}')] \in IFSM_{mn}$. Then \( \times_5 \) product of $\tilde{A}$ and $\tilde{B}$ is defined by
$$
\times_5 : IFSM_{mn} \times IFSM_{mn} \rightarrow IFSM_{mn},
$$
such that $\tilde{A} \times_5 \tilde{B} = [(\mu_{ij}', \nu_{ij}')] \times_5 [(\mu_{ik}', \nu_{ik}')] = [(\mu_{ij}, \nu_{ij})]
$$
where $\mu_{ij} = \max\{\mu_{ij}', \mu_{ik}'\}$ and $\nu_{ij} = \min\{\nu_{ij}', \nu_{ik}'\}$ such that $p = n(j - i) + k$.

Proposition 6.1
Let $A, B \in IFSM_{mn}$ as in the Example 4.2

(i) $(A \times B) \times \tilde{C} = A \times (B \times \tilde{C})$

(ii) $(A \cup B) \times \tilde{C} = (A \times \tilde{C}) \cup (B \times \tilde{C})$

(iii) $(A \cap B) \times \tilde{C} = (A \times \tilde{C}) \cap (B \times \tilde{C})$

Example 6.1
Let $A, B \in IFSM_{3,3}$ as in the Example 4.2

Then
$$
A \times_1 B = \begin{bmatrix}
(0.05, 0.06) & (0.01, 0.12) & (0.07, 0.02) & (0.25, 0.12) & (0.05, 0.24) & (0.35, 0.04) & (0.15, 0.18) & (0.03, 0.36) & (0.21, 0.06) \\
(0.32, 0.04) & (0.12, 0.16) & (0.20, 0.08) & (0.16, 0.03) & (0.08, 0.09) & (0.10, 0.06) & (0.40, 0.01) & (0.20, 0.03) & (0.25, 0.02) \\
(0.10, 0.10) & (0.15, 0.12) & (0.20, 0.10) & (0.06, 0.20) & (0.09, 0.24) & (0.12, 0.20) & (0.12, 0.10) & (0.18, 0.12) & (0.24, 0.10) \\
(0.07, 0.14) & (0.14, 0.10) & (0.35, 0.02) & (0.06, 0.07) & (0.12, 0.05) & (0.30, 0.01) & (0.05, 0.21) & (0.10, 0.15) & (0.25, 0.03)
\end{bmatrix}
$$

Then
$$
A \times_2 B = \begin{bmatrix}
(0.55, 0.06) & (0.19, 0.12) & (0.73, 0.02) & (0.75, 0.12) & (0.55, 0.24) & (0.85, 0.04) & (0.65, 0.18) & (0.37, 0.36) & (0.79, 0.06) \\
(0.88, 0.04) & (0.64, 0.12) & (0.70, 0.08) & (0.84, 0.03) & (0.52, 0.09) & (0.06, 0.06) & (0.90, 0.01) & (0.7, 0.03) & (0.75, 0.02) \\
(0.6, 0.10) & (0.65, 0.12) & (0.7, 0.10) & (0.44, 0.20) & (0.51, 0.24) & (0.58, 0.20) & (0.68, 0.10) & (0.72, 0.12) & (0.76, 0.10) \\
(0.73, 0.14) & (0.76, 0.10) & (0.85, 0.02) & (0.64, 0.07) & (0.68, 0.05) & (0.80, 0.01) & (0.55, 0.21) & (0.6, 0.15) & (0.75, 0.03)
\end{bmatrix}
$$

CONCLUSION

Molodtsov introduced the concept of soft sets, which is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parametrization tool of soft set theory enhance the flexibility of its applications. In this paper, we define intuitionistic fuzzy soft matrices some results are established in continuation to soft matrices introduced by Çağman et al.[1].
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REFERENCES