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Slow steady motion of a thermo-viscous fluid in a porous slab bounded between two impermeable parallel plates

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ABSTRACT

The problem of slow steady motion of a second order thermo-viscous incompressible fluid through a porous slab bounded by two non-permeable parallel plates is examined in this paper. The two plates are kept at two different temperatures and the flow is generated by a constant pressure gradient. Explicit calculations have been made to obtain the velocity and temperature distributions with appropriate boundary conditions. The flow in the absence of pressure gradient and porosity of the medium has been deduced as the special cases. The flow rate, Shear stress, heat transfer coefficient and the transverse force perpendicular to the flow direction are calculated.

Key Words: Darcy's flux, thermo stress coefficient, strain thermal conductivity coefficient, porosity parameter.

INTRODUCTION

Koh and Eringin [5] introduced the concept of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion due to external influences. For such a class of fluids, the stress-tensor 't' and heat flux bivector 'h' are postulated as polynomial functions of the kinematic tensor,viz.,the rate of deformation tensor 'd':

$$d_{ij} = (u_{i,j} + u_{j,i})/2$$

and thermal gradient bivector 'b'

$$b_{ij} = \in_{ijk} \theta_k$$

where u_i is the i^{th} component of velocity and θ is the temperature field.

A second order theory of thermo-viscous fluids is characterized by the pair of thermo-mechanical constitutive relations:

$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd) \quad \text{and}$$

$$h = \beta_1 b + \beta_3 (bd + db)$$

with the constitutive parameters α_i , β_i being polynomials in the invariants of d and b in which the coefficients depend on density(ρ) and temperature(θ) only. The fluid is Stokesian when the stress tensor depends only on the rate of deformation tensor and Fourier-heat-conducting when the heat flux bivector depends only on the temperature gradient-vector, the constitutive coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient of viscosity respectively and α_5 as that of cross-viscosity.

Fluid flows through porous media have been a subject of both experimental and theoretical research for over one and half centuries. Darcy, based on the findings of a large number of flows through porous media, proposed the empirical law known as Darcy's law $Q = -\frac{k^*}{\mu} A.\nabla P$, where Q is the total discharge of the fluid, k^* is the permeability of the medium, A is the cross-sectional area to flow the fluid, μ is the cross-viscosity of the fluid and ∇P is the pressure gradient in the direction of the fluid flow. Dividing both sides of the equation by the area then the above equation becomes $q = -\frac{k^*}{\mu}\nabla P$, where q is known as Darcy's fluid flux and we know that the fluid velocity(u) is proportional to the fluid flux(q) by the porosity(k^*) , then $\nabla P = -\frac{\mu}{k^*} u$. The negative sign indicates that fluids flows from the region of high pressure to low pressure.

The flow of incompressible thermo-viscous fluids satisfies the usual conservation equations.

Equation of Continuity

$$v_{i,i} = 0$$

Equation of Momentum

$$\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho F_k + t_{ji,i} - \frac{\mu}{k^*} v_i$$

and the Energy equation

$$\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho v - \frac{\mu}{k^*} v_i^2$$

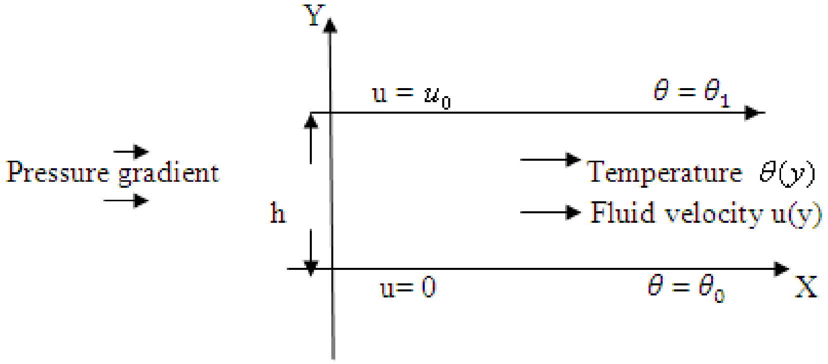
where

$F_k = k^{th}$ Component of external force per unit mass,
$c =$ Specific heat,
$v =$ Thermal energy source per unit mass and
$q_i = i^{th}$ Component of heat flux bivector = $\in_{ijk} h_{jk} / 2$

MATHEMATICAL FORMATION AND SOLUTION OF THE PROBLEM

Consider the slow steady flow of a second order thermo-viscous fluid through a porous medium bounded between two non-permeable parallel plates. The flow is generated by a constant pressure gradient in a direction parallel to the plates and the motion of the upper plate, moving with a given velocity u_0 relative to the lower plate along the direction of the pressure gradient.

With reference to a coordinate system O(XYZ) with origin on the fixed plate, the x-axis in the direction of the plate movement, y-axis perpendicular to plates, the plates are represented by y = 0 and y = h . Further the two plates are maintained at constant temperatures θ_0 and θ_1 respectively.



Let the steady flow between the two plates is characterized by the velocity field [u(y), 0, 0] and temperature field θ(y). This choice of the velocity evidently satisfies the continuity equation.

The Basic equations characterizing the flow :
in the X-direction :

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \alpha_6 \frac{\partial \theta}{\partial x}\frac{\partial^2 \theta}{\partial y^2} + \rho F_x - \frac{\mu}{k^*} u \tag{1}$$

in the Y-direction :

$$0= \mu_c \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2 + \rho F_y \tag{2}$$

in the Z- direction :

$$0= \alpha_8 \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y}\right) + \rho F_z \tag{3}$$

and the energy equation

$$\rho c u \frac{\partial \theta}{\partial x} = \mu\left(\frac{\partial u}{\partial y}\right)^2 - \alpha_6 \frac{\partial \theta}{\partial x}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} + k\frac{\partial^2 \theta}{\partial y^2} + \beta_3 \frac{\partial \theta}{\partial x}\frac{\partial^2 u}{\partial y^2} + \rho\gamma - \frac{\mu}{k^*}u^2 \tag{4}$$

together with the boundary conditions : u=0 , $\theta = \theta_0$ at y=0 and

$$u = u_0, \theta = \theta_1 \text{ at } y = h \tag{5}$$

The following non-dimensional quantities are introduced y = hY , u =($\frac{\mu}{\rho h}$) U , u_0 =($\frac{\mu}{\rho h}$)U_0, $T = \frac{\theta - \theta_0}{\theta_1 - \theta_0}$, $\frac{\partial \theta}{\partial x} = \frac{\theta_1 - \theta_0}{h} C_2$, $\frac{\partial p}{\partial x} = \frac{\mu^2}{\rho h^3} C_1$, S= $\frac{h^2}{k^*}$,

$p_r = \frac{\mu c}{k}$, $b_3 = \frac{\beta_3}{\rho h^2 c}$

where C_1 and C_2 are non- dimensional pressure and temperature gradients respectively

and S is the porosity parameter.

The motion is assumed to be slow, the non-linear terms in the above equations could be neglected. In terms of the above non-dimensional and in the absence of external forces and internal energy source , the equations (1) and (4) can be written as

$$0 = -C_1 + \frac{d^2U}{dY^2} - a_6 C_2 \frac{d^2T}{dY^2} - SU \quad (6)$$

$$UC_2 = b_3 C_2 \frac{d^2U}{dY^2} + \frac{1}{p_r}\frac{d^2T}{dY^2} \quad (7)$$

and the boundary conditions : U(0) = 0 , U(1) = U_0 (8)

T(0) = 0 , T(1) = 1 (9)

From the equations (6) and (7) and using the boundary conditions in (8) the velocity distribution is obtained as

$$U(Y) = \frac{1}{m^2 \sinh m}\left\{m_1 C_1 [\sinh m(1-Y) - \sinh m] + [m_1 C_1 + m^2 U_0]\sinh mY\right\} \quad (10)$$

Using the equation(10) in (7) with the boundary conditions in (9) the temperature distribution is obtained as

$$T(Y) = Y + \frac{p_r C_2}{2m^2}\left\{Y(1-Y)m_1 C_1 + 2(1 + m^2 b_3)[YU_0 - U(Y)]\right\} \quad (11)$$

The flow rate Q is:

$$Q = \int_0^1 U(Y)dY = \frac{1}{m^3}\left\{2mm_1 C_1 + (2m_1 C_1 + m^2 U_0)\tanh\frac{m}{2}\right\}$$

The Shear Stress is :

$$\frac{dU}{dY} = \frac{C_1}{m \sinh m}\left\{(m_1 C_1 + m^2 U_0)\cosh mY - m_1 C_1 \cosh m(1-Y)]\right\}$$

The Shear Stress on the Lower plate is:

$$\frac{dU}{dY}/(Y=0) = \frac{m_1 C_1}{m}\tanh\frac{m}{2} + mU_0 \cos eshm$$

The Shear Stress On the upper plate is:

$$\frac{dU}{dY}/(Y=1)=\frac{m_1C_1}{m}\tanh\frac{m}{2}+mU_0\coth m$$

The heat transfer coefficient characterized by the Nussult number is given by:

$$\frac{dT}{dY}=\frac{p_rC_2\left(1+b_3m^2\right)}{m^3\sinh m}\left\{m_1C_1\cosh m(1-Y)-\left(m_1C_1+m^2U_0\right)\cosh mY+mU_0\sinh m\right\}+\frac{p_rC_2m_1C_1}{2m^2}[1-2Y]$$

The heat transfer coefficient characterized by the Nussult number on the lower plate is:

$$\frac{dT}{dY}/(Y=0)=\frac{p_rC_2\left(1+b_3m^2\right)}{m^3\sinh m}\left\{m_1C_1\sec h\frac{m}{2}+mU_0\left(1-m\cos echm\right)\right\}+\frac{p_rC_2m_1C_1}{2m^2}$$

The heat transfer coefficient characterized by the Nussult number on the upper plate is:

$$\frac{dT}{dY}/(Y=1)=\frac{p_rC_2\left(1+b_3m^2\right)}{m^3}\left\{mU_0\left(1-m\cos echm\right)-\left(m_1C_1+m^2U_0\right)\tanh\frac{m}{2}\right\}-\frac{p_rC_2m_1C_1}{2m^2}$$

The cross-viscous stress generated in the Y-direction perpendicular to the flow is

$$\rho F_Y=\frac{\mu_c\mu^2}{\rho h^5 m\sin^2 hm}\left\{\left(m_1C_1+m^2U_0\right)^2\sinh 2mY-m_1{}^2C_1{}^2\sinh 2m(1-Y)+2m_1C_1\left(m_1C_1+m^2U_0\right)\sinh m\right\}$$

From this we observed that the viscous stress generated depends on Riner-Rivilin cross viscosity coefficient (i.e. μ_c).

The cross-viscous stress generated on the lower plate in the Y-direction is

$$\rho F_Y/(Y=0)=\frac{2\mu_c\mu^2m_1C_1}{\rho h^5 m}\left\{m^2U_0\cos echm-m_1C_1\tanh\frac{m}{2}\right\}$$

The cross-viscous stress generated on the upper plate in the Y-direction is

$$\rho F_Y/(Y=1)=\frac{2\mu_c\mu^2}{\rho h^5 m}\left\{m^2U_0\coth m+m_1C_1\coth\frac{m}{2}\right\}$$

The cross-viscous stress generated in the Z-direction perpendicular to the flow is

$$\rho F_Z=\frac{-\alpha_8\mu(\theta-\theta_0)p_rC_2}{2\rho h^3m^3\sin^2 hm}\left\{\begin{array}{l}2\left(1+m^2b_3\right)\left\{\left(m_1C_1+m^2U_0\right)\left[2m_1C_1\sinh m(1-2Y)\right]-m_1C_1\sinh 2m(1-Y)\right\}\\+m\sinh m\left(m_1C_1(1-2Y)+2U_0(1+m^2b_3)\right)\left\{\begin{array}{l}(m_1C_1+m^2U_0)\sinh mY\\+m_1C_1\sinh m(1-Y)\end{array}\right\}\\-2m_1C_1\sinh m\left\{(m_1C_1+m^2U_0)\cosh mY-m_1C_1\cosh m(1-Y)\right\}\end{array}\right\}$$

From this we observed that the viscous stress generated depends on thermo-stress coefficient (i.e. α_8) .

The cross-viscous stress generated on the lower plate in the Z-direction is

$$\rho F_Z /(Y=0) = \frac{-\alpha_8 \mu(\theta-\theta_0) p_r C_2 m_1 C_1}{2\rho h^3 m^3 \sin^2 hm} \left\{ \begin{array}{l} 2\left(1+2m^2 b_3\right)\left\{m^2 U_0 - 2m_1 C_1 \sin^2 h\frac{m}{2}\right\} \\ + m \sinh m\left\{m_1 C_1 + 2U_0\left(1+m^2 b_3\right)\right\} \end{array} \right\}$$

The cross-viscous stress generated on the upper plate in the Z-direction is

$$\rho F_Z /(Y=1) = \frac{-\alpha_8 \mu(\theta-\theta_0) p_r C_2}{2\rho h^3 m^3 \sin^2 hm} \left\{ 2m_1{}^2 C_1{}^2 + \left(m_1 C_1 + m^2 U_0\right) \left\{ \begin{array}{l} 2\cosh m\left[m_1 C_1 + 2\left(1+m^2 b_3\right)\right] \\ -\sinh m\left[m m_1 C_1 - 2U_0\left(1+m^2 b_3\right)\right] \\ -4m_1 C_1\left(1+m^2 b_3\right) \end{array} \right\} \right\}$$

here

$$m = \sqrt{\frac{S + a_6 p_r C_2{}^2}{1 + b_3 a_6 p_r C_2{}^2}} \quad , \quad m_1 = \frac{1}{1 + b_3 a_6 p_r C_2{}^2}$$

RESULTS AND DISCUSSION

The influences of various parameters like thermo-stress coefficient(a_6),thermal conductivity coefficient(b_3) and porosity of the medium(S) on velocity and temperature distributions and also the effect of Reiner-Rivlin coefficient(μ_c) and thermo-stress viscosity coefficient(α_8) on the transverse force perpendicular to the flow direction have been illustrated graphically by taking C_1=1,C_2 =1, p_r =1.The effect of all these parameters on the flow field by comparing the plates in relative motion(i.e.$U_0 = 1$) with the plates fixed(i.e. $U_0 = 0$) are discussed with the help of the graphs.

The velocity profiles for different values of thermo-viscous parameters(a_6&b_3) and porosity parameter(S) when the plates are fixed are shown in figs.(1,2&3) and when the plates are in relative motion are shown in figs.(4,5&6).

It is observed from the figs.(1,2&3) that(i.e. when the plates are fixed), the velocity profiles are parabolic type and these are increases as the values of the thermo-stress coefficient(a_6),thermal conductivity coefficient(b_3) and porosity parameter(S) increases. But from the figs.(4,5&6) (i.e. plates are in relative motion), it is noticed that, the bending of the velocity profiles decreases as the porosity parameter(S) increases ,this effect is due to the porosity parameter(S) causing friction to the flow. It is also found from the figs.(4,5&6) that ,the velocity profiles increases slowly as the values of thermo-viscous parameters(a_6 & b_3) increases.

The temperature distributions for different values of thermo-viscous parameters(a_6&b_3) and porosity parameter(S) when the plates are fixed and when the plates are in relative motion are shown in figs.(7,8&9).

From the figs.(7,8&9), it is observed that ,the temperature of the fluid when the plates are in relative motion increases at faster rate when compared to the plates are fixed and as the values of the porosity parameter(S) increases the temperature of the fluid decreases slowly.

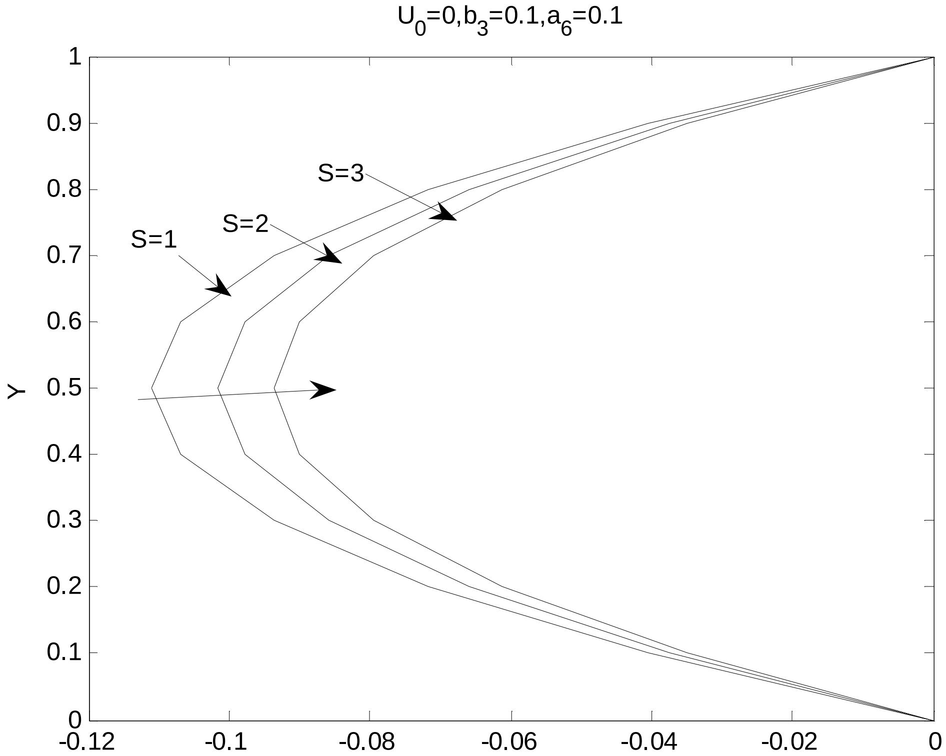


Fig.(1):Variations of the velocity profiles U(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

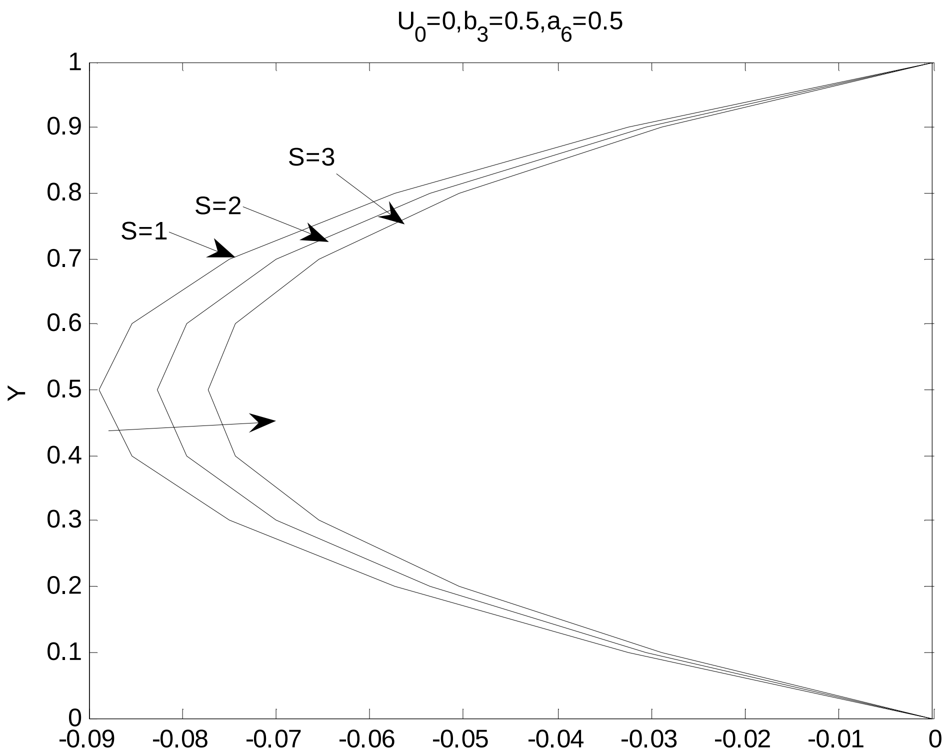


Fig.(2):Variations of the velocity profiles U(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

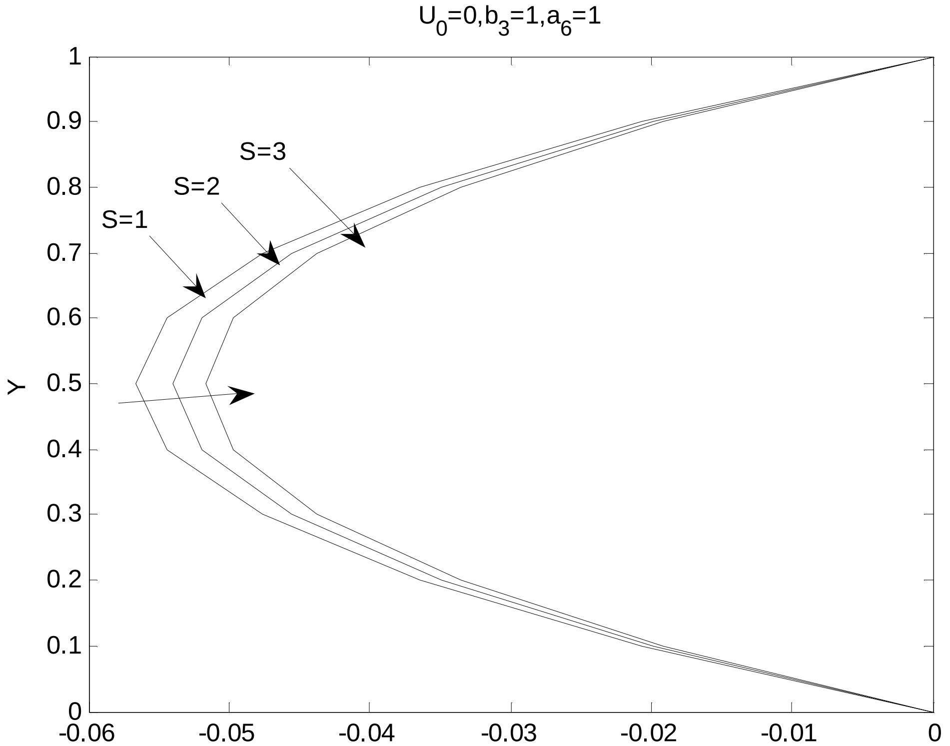


Fig.(3):Variations of the velocity profiles U(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

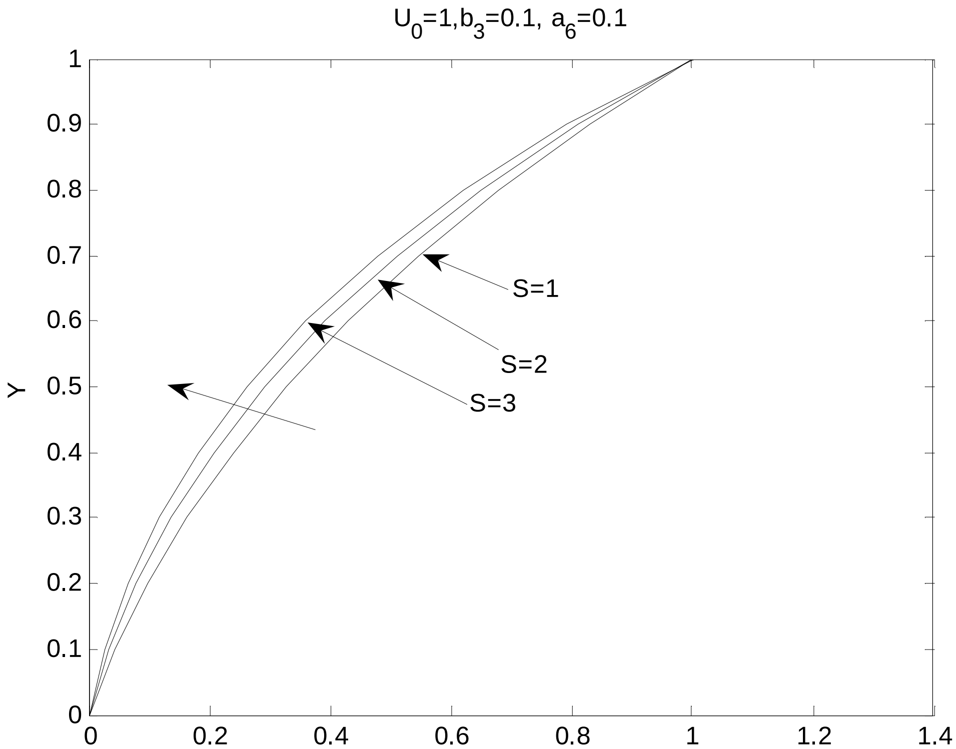


Fig.(4):Variations of the velocity profiles U(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

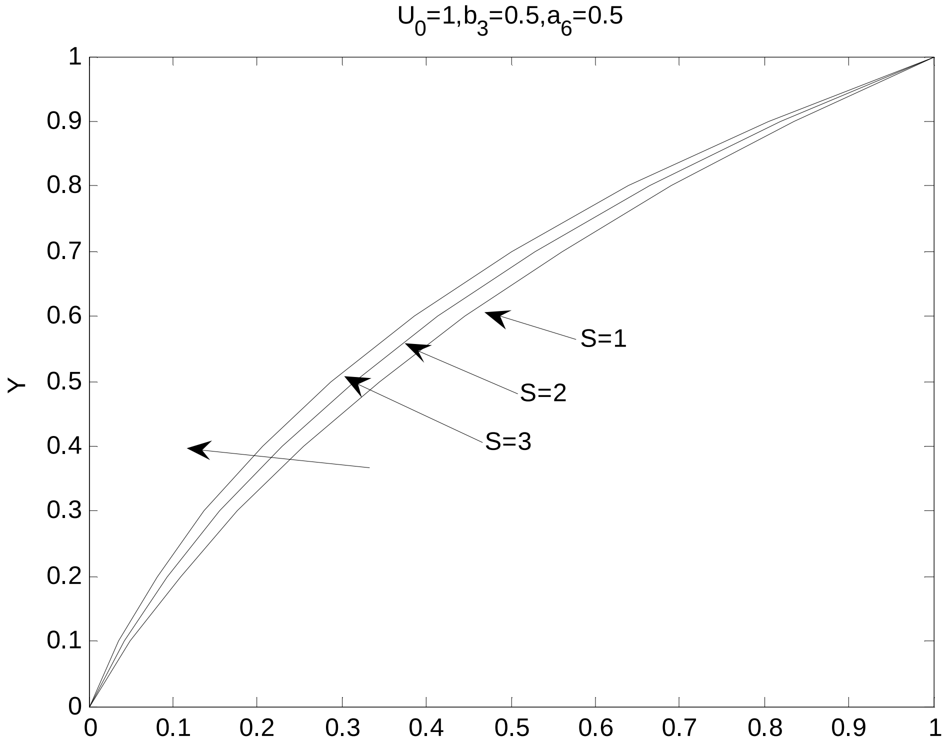


Fig.(5):Variations of the velocity profiles U(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

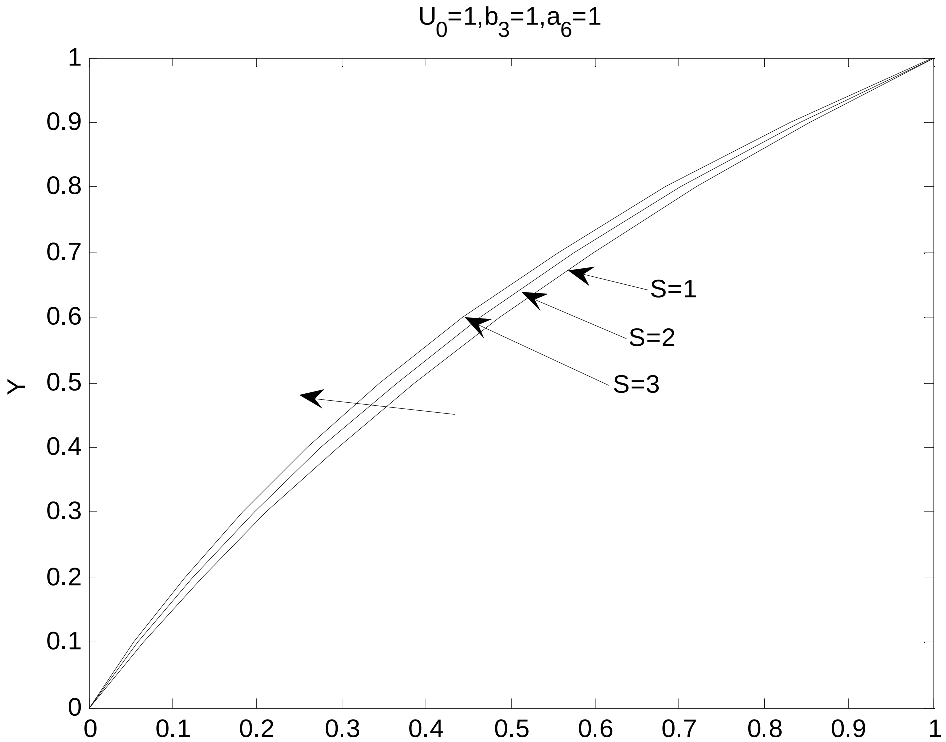


Fig.(6):Variations of the velocity profiles U(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

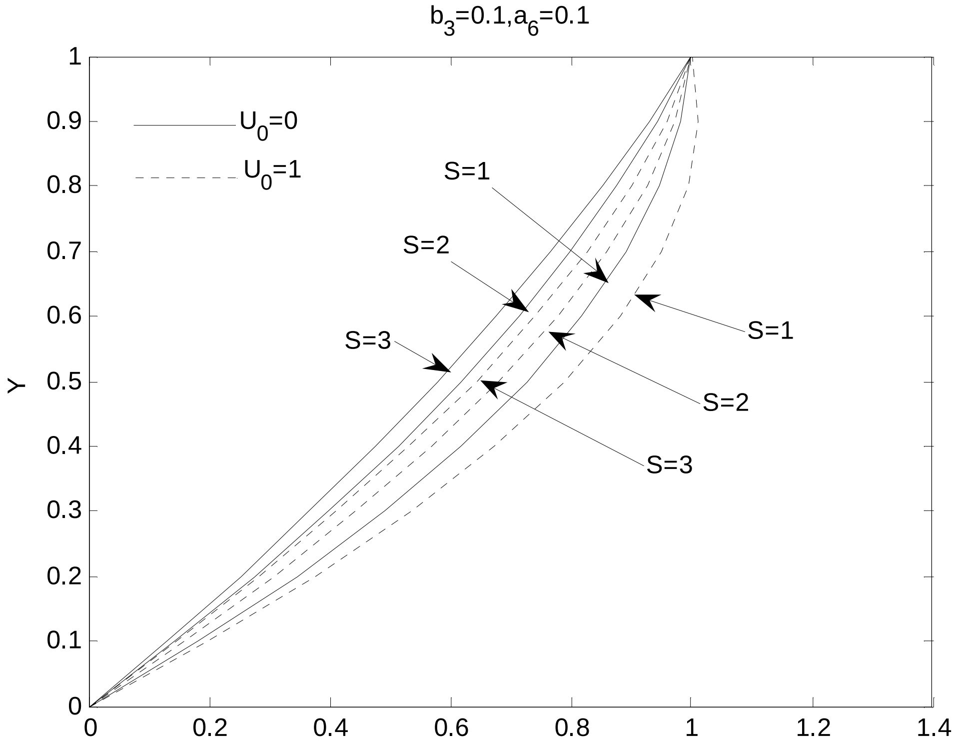


Fig.(7):Variations of the temperature distributions T(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

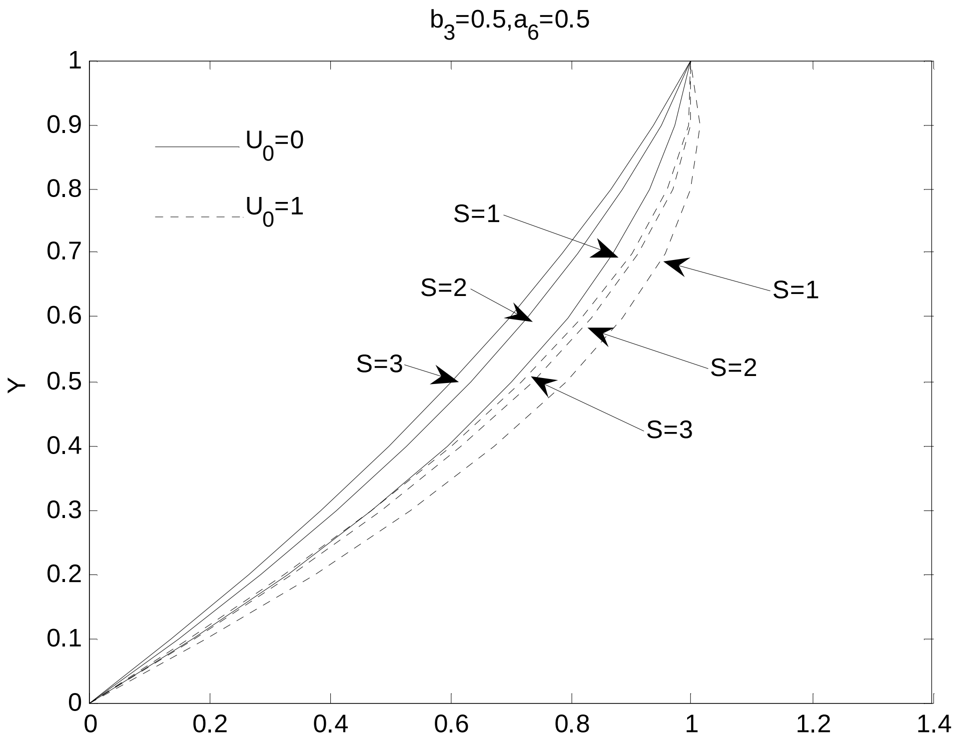


Fig.(8):Variations of the temperature distributions T(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

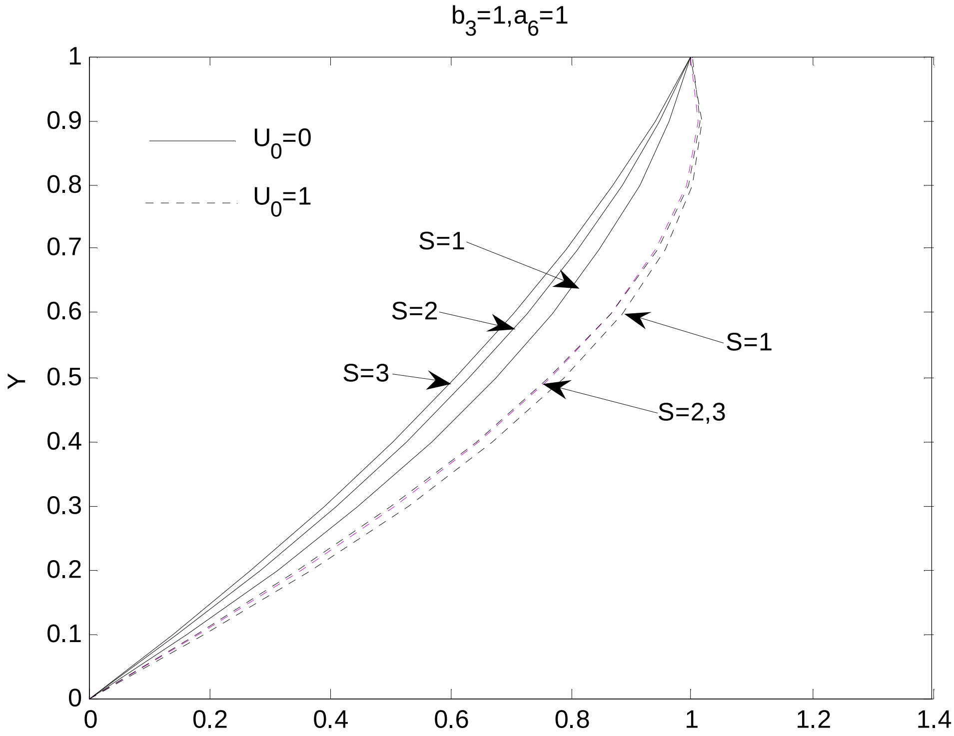


Fig.(9):Variations of the temperature distributions T(Y) vs porosity parameter(S) and thermo-viscous parameters (b_3,a_6)

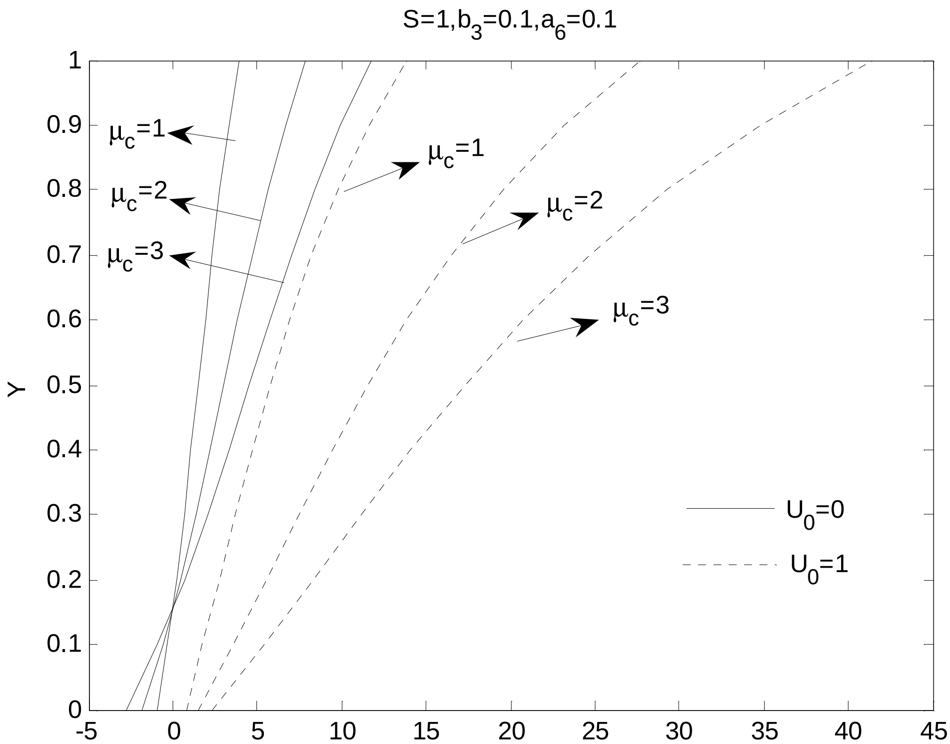


Fig.(10)-Force in Y-direction vs Riener Rivlin coeficient,porosity parameter(S) and thermo-viscous parameters(b_3,a_6)

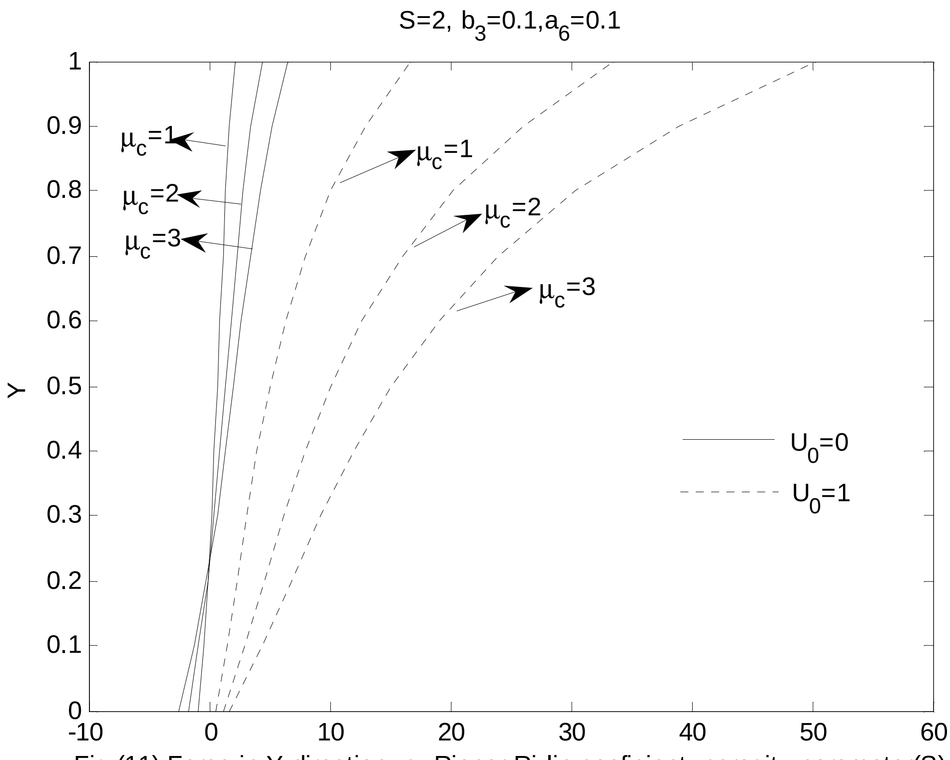


Fig.(11)-Force in Y-direction vs Riener Rivlin coeficient, porosity parameter(S) and thermo-viscous parameters(b_3,a_6)

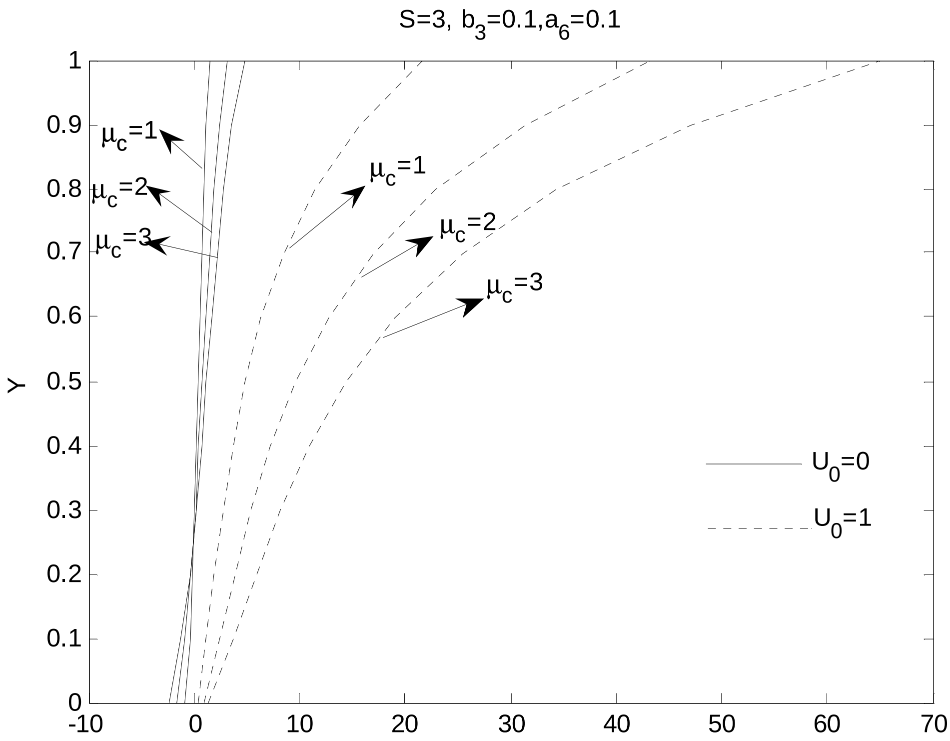


Fig.(12)-Force in Y-direction vs Riener Rivlin coeficient,porosity parameter(S) and thermo-viscous parameters(b_3,a_6)

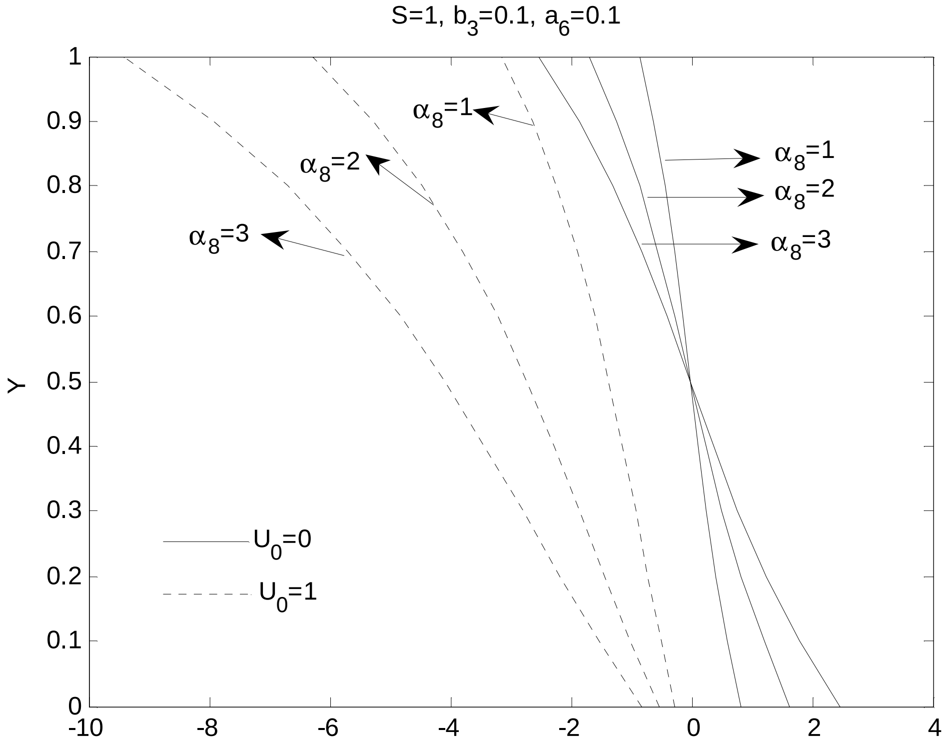


Fig.(13)-Force in Z-direction vs thermo-stress coeficient, porosity parameter(S) and thermo-viscous parameters(b_3,a_6)

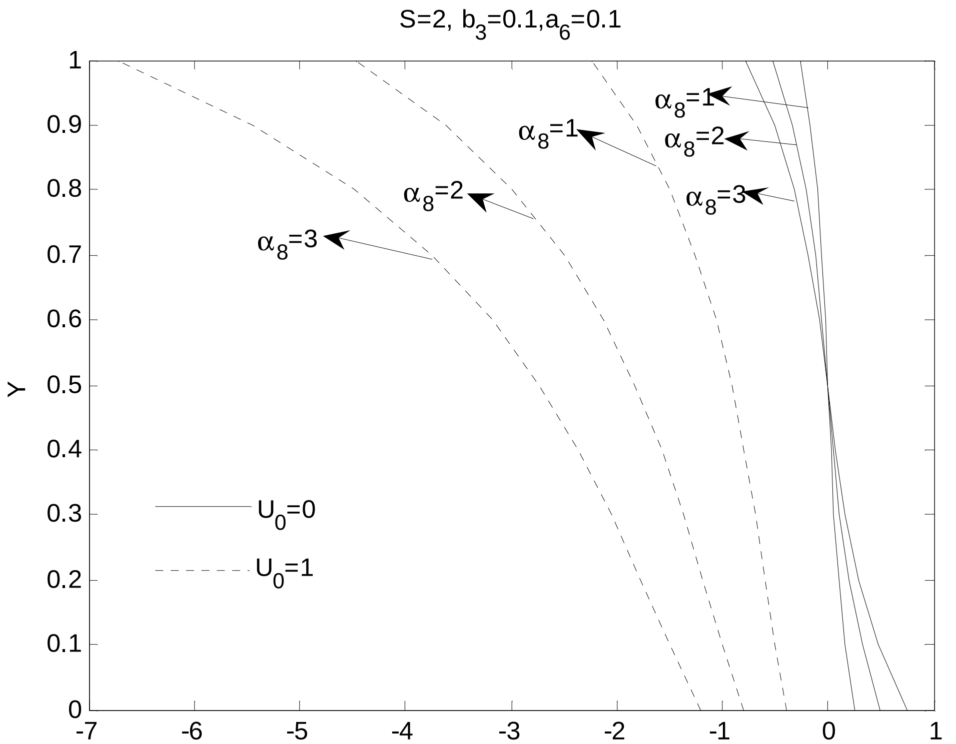


Fig.(14)-Force in Z-direction vs thermo-stress coeficient, porosity parameter(S) and thermo-viscous parameters(b_3,a_6)

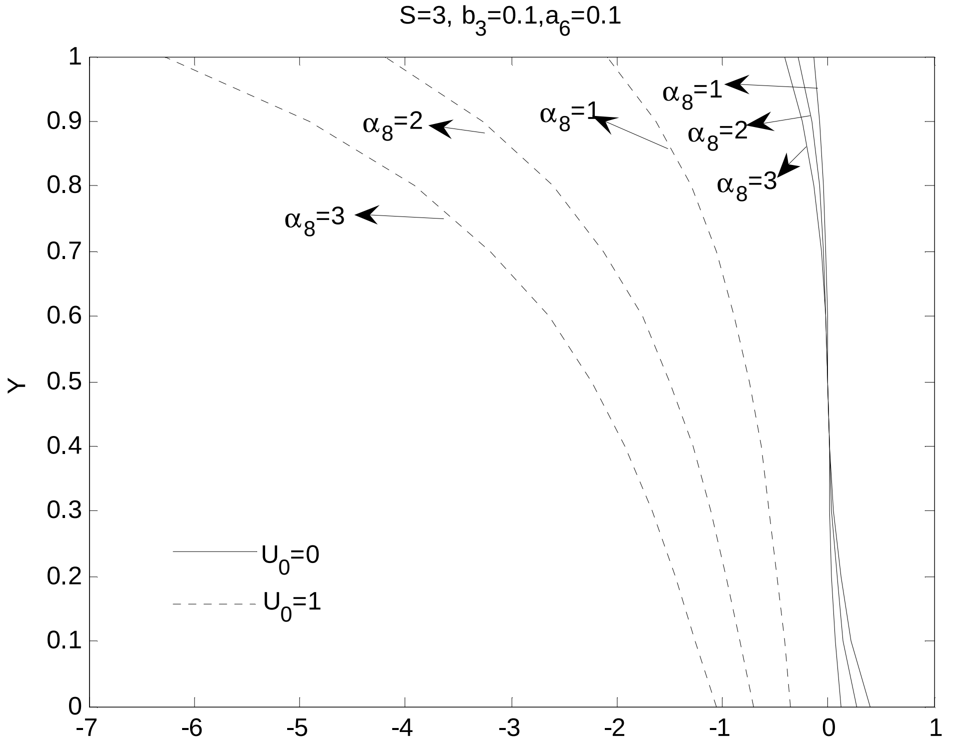


Fig.(15)-Force in Z-direction vs thermo-stress coeficient, porosity parameter(S) and thermo-viscous parameters(b_3,a_6)

The figs.(10,11&12) indicates that, the forces acting on a fluid in Y-direction when the plates are fixed is less when compared to the plates are in relative motion . This is due to the effect of Reiner-Rivlin coefficient(μ_c) .But reverse effect is observed in figs.(13,14&15), i.e. the forces acting on a fluid in Z-direction when the plates are fixed is more when compared to the plates are in relative motion .This effect is due to the thermo-stress viscosity coefficient(α_8).The force acting on a fluid in Y-direction when $U_0 = 0$ (i.e. when the plates are fixed) is slowly decreases near the cooler plate then it is increases as μ_c increases from (1-3).When the plates are in relative motion the force acting on a fluid in Y-direction is increases as μ_c increases from (1-3). The force acting on a fluid in Z-direction when $U_0 = 0$ (i.e. when the plates are fixed) is increases near the center of the channel then it is decreases as α_8 increases from (1-3).When the plates are in relative motion the force acting on a fluid in Z-direction is decreases as α_8 increases from (1-3).

4.SPECIAL CASES

SPECIAL CASE-I: If the medium is not porous (i.e. S=0)

In this case, the velocity distribution is obtained as

$$U(Y) = \frac{1}{{m_2}^2 \sinh m_2} \left\{ m_1 C_1 [\sinh m_2 (1-Y) - \sinh m_2] + [m_1 C_1 + {m_2}^2 U_0] \sinh m_2 Y \right\}$$

and the temperature distribution is obtained as

$$T(Y) = Y + \frac{p_r C_2}{2m_2^{\,2}}\left\{Y(1-Y)m_1 C_1 + 2\left(1 + m_2^{\,2} b_3\right)\left[YU_0 - U(Y)\right]\right\}$$

The flow rate Q is:

$$Q = \int_0^1 U(Y)dY = \frac{1}{m_2^{\,3}}\left\{2m_2 m_1 C_1 + (2m_1 C_1 + m_2^{\,2} U_0)\tanh\frac{m_2}{2}\right\}$$

The Shear Stress is :

$$\frac{dU}{dY} = \frac{C_1}{m_2 \sinh m_2}\left\{\left(m_1 C_1 + m_2^{\,2} U_0\right)\cosh m_2 Y - m_1 C_1 \cosh m_2 (1-Y)\right\}$$

The Shear Stress on the Lower plate is:

$$\frac{dU}{dY}/(Y=0) = \frac{m_1 C_1}{m_2}\tanh\frac{m_2}{2} + m_2 U_0 \cos eshm_2$$

The Shear Stress On the upper plate is:

$$\frac{dU}{dY}/(Y=1) = \frac{m_1 C_1}{m_2}\tanh\frac{m_2}{2} + m_2 U_0 \coth m_2$$

The heat transfer coefficient characterized by the Nussult number is given by:

$$\frac{dT}{dY} = \frac{p_r C_2\left(1 + b_3 m_2^{\,2}\right)}{m_2^{\,3}\sinh m_2}\left\{m_1 C_1 \cosh m_2 (1-Y) - \left(m_1 C_1 + m_2^{\,2} U_0\right)\cosh m_2 Y + m_2 U_0 \sinh m_2\right\} + \frac{p_r C_2 m_1 C_1}{2m_2^{\,2}}\left[1 - 2Y\right]$$

The heat transfer coefficient characterized by the Nussult number on the lower plate is:

$$\frac{dT}{dY}/(Y=0) = \frac{p_r C_2\left(1 + b_3 m_2^{\,2}\right)}{m_2^{\,3}\sinh m_2}\left\{m_1 C_1 \sec h\frac{m_2}{2} + m_2 U_0 \left(1 - m_2 \cos echm_2\right)\right\} + \frac{p_r C_2 m_1 C_1}{2m_2^{\,2}}$$

The heat transfer coefficient characterized by the Nussult number on the upper plate is:

$$\frac{dT}{dY}/(Y=1) = \frac{p_r C_2\left(1 + b_3 m_2^{\,2}\right)}{m_2^{\,3}}\left\{m_2 U_0 \left(1 - m_2 \cos echm_2\right) - \left(m_1 C_1 + m_2^{\,2} U_0\right)\tanh\frac{m_2}{2}\right\} - \frac{p_r C_2 m_1 C_1}{2m_2^{\,2}}$$

The cross-viscous stress generated in the Y-direction perpendicular to the flow is

$$\rho F_y = \frac{\mu_c \mu^2}{\rho h^5 m_2 \sin^2 hm_2}\left\{\begin{aligned}&\left(m_1 C_1 + m_2^{\,2} U_0\right)^2 \sinh 2m_2 Y - m_1^{\,2} C_1^{\,2} \sinh 2m_2 (1-Y)\\ &+ 2m_1 C_1\left(m_1 C_1 + m_2^{\,2} U_0\right)\sinh m_2\end{aligned}\right\}$$

The cross-viscous stress generated on the lower plate in the Y-direction is

$$\rho F_y /(Y=0)=\frac{2\mu_c\mu^2 m_1C_1}{\rho h^5 m_2}\left\{m_2{}^2U_0\cos ech m_2-m_1C_1\tanh\frac{m_2}{2}\right\}$$

The cross-viscous stress generated on the upper plate in the Y-direction is

$$\rho F_y /(Y=1)=\frac{2\mu_c\mu^2}{\rho h^5 m_2}\left\{m_2{}^2U_0\coth m_2+m_1C_1\coth\frac{m_2}{2}\right\}$$

The cross-viscous stress generated in the Z-direction perpendicular to the flow is

$$\rho F_Z=\frac{-\alpha_8\mu(\theta-\theta_0)p_rC_2}{2\rho h^3 m_2{}^3\sin^2 hm_2}\left\{\begin{array}{l}2\left(1+m_2{}^2b_3\right)\left\{\left(m_1C_1+m_2{}^2U_0\right)\left[2m_1C_1\sinh m_2(1-2Y)\right]-m_1C_1\sinh 2m_2(1-Y)\right\}\\+m_2\sinh m_2\left(m_1C_1(1-2Y)+2U_0(1+m_2{}^2b_3)\right)\left\{\begin{array}{l}(m_1C_1+m_2{}^2U_0)\sinh m_2Y\\+m_1C_1\sinh m_2(1-Y)\end{array}\right\}\\-2m_1C_1\sinh m_2\left\{(m_1C_1+m_2{}^2U_0)\cosh mY-m_1C_1\cosh m_2(1-Y)\right\}\end{array}\right\}$$

The cross-viscous stress generated on the lower plate in the Z-direction is

$$\rho F_Z /(Y=0)=\frac{-\alpha_8\mu(\theta-\theta_0)p_rC_2m_1C_1}{2\rho h^3 m_2{}^3\sin^2 hm_2}\left\{\begin{array}{l}2\left(1+2m_2{}^2b_3\right)\left\{m_2{}^2U_0-2m_1C_1\sin^2 h\frac{m_2}{2}\right\}\\+m_2\sinh m_2\left\{m_1C_1+2U_0\left(1+m_2{}^2b_3\right)\right\}\end{array}\right\}$$

The cross-viscous stress generated on the upper plate in the Z-direction is

$$\rho F_Z /(Y=1)=\frac{-\alpha_8\mu(\theta-\theta_0)p_rC_2}{2\rho h^3 m_2{}^3\sin^2 hm_2}\left\{2m_1{}^2C_1{}^2+\left(m_1C_1+m_2{}^2U_0\right)\left\{\begin{array}{l}2\cosh m_2\left[m_1C_1+2\left(1+m_2{}^2b_3\right)\right]\\-\sinh m_2\left[m_2m_1C_1-2U_0\left(1+m_2{}^2b_3\right)\right]\\-4m_1C_1\left(1+m_2{}^2b_3\right)\end{array}\right\}\right\}$$

$$m_2=\sqrt{\frac{a_6p_rC_2{}^2}{1+b_3a_6p_rC_2{}^2}}\qquad m_1=\frac{1}{1+b_3a_6p_rC_2{}^2}$$

SPECIAL CASE-II: In the absence of pressure gradient(i.e. $C_1=0$)

In this case, the velocity distribution is obtained as

$$U(Y)=U_0\frac{\sinh mY}{\sinh m}$$

and the temperature distribution is obtained as

$$T(Y)=\frac{p_rC_2\left(1+m^2b_3\right)U_0}{m^2}\left\{Y-\sinh mY\right\}$$

The flow rate Q is:

Q= $\int_0^1 U(Y)dY = \frac{U_0}{m}\tanh\frac{m}{2}$

The Shear Stress is :

$$\frac{dU}{dY} = mU_0 \frac{\cosh mY}{\sinh m}$$

The Shear Stress on the Lower plate is:

$$\frac{dU}{dY}/(Y=0) = mU_0 \cos echm$$

The Shear Stress On the upper plate is:

$$\frac{dU}{dY}/(Y=1) = mU_0 \coth m$$

The heat transfer coefficient characterized by the Nussult number is given by:

$$\frac{dT}{dY} = \frac{p_r C_2 \left(1+b_3 m^2\right) U_0}{m^2}\left\{1-\frac{m\cosh mY}{\sinh m}\right\}$$

The heat transfer coefficient characterized by the Nussult number on the lower plate is:

$$\frac{dT}{dY}/(Y=0) = \frac{p_r C_2 \left(1+b_3 m^2\right) U_0}{m^2}\{1-m\cos echm\}$$

The heat transfer coefficient characterized by the Nussult number on the upper plate is:

$$\frac{dT}{dY}/(Y=1) = \frac{p_r C_2 \left(1+b_3 m^2\right) U_0}{m^2}\{1-m\coth m\}$$

The cross-viscous stress generated in the Y-direction perpendicular to the flow is

$$\rho F_y = \frac{\mu_c \mu^2 m^3 U_0{}^2 \sinh 2mY}{\rho h^5 \sin^2 hm}$$

The cross-viscous stress generated on the lower plate in the Y-direction is

$$\rho F_y /(Y=0) = 0$$

The cross-viscous stress generated on the upper plate in the Y-direction is

$$\rho F_y /(Y=1) = \frac{2\mu_c \mu^2 m U_0}{\rho h^5}\coth m$$

The cross-viscous stress generated in the Z-direction perpendicular to the flow is

$$\rho F_Z = \frac{-\alpha_8 \mu (\theta - \theta_0) p_r C_2 U_0 (1 + m^2 b_3)}{2\rho h^3 m \sin^2 hm} \sinh mY [2\cosh mY + mU_0 \sinh m]$$

The cross-viscous stress generated on the lower plate in the Z-direction is

$$\rho F_Z /(Y = 0) = 0$$

The cross-viscous stress generated on the upper plate in the Z-direction is

$$\rho F_Z /(Y = 1) = \frac{-\alpha_8 \mu (\theta - \theta_0) p_r C_2 U_0 (1 + m^2 b_3)}{\rho h^3 m} [mU_0 + 2\coth m]$$

$$m = \sqrt{\frac{S + a_6 p_r C_2^{\,2}}{1 + b_3 a_6 p_r C_2^{\,2}}}$$

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