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Peristaltic transport of a power-law fluid in an asymmetric channel bounded by permeable walls

Y. V. K. Ravi Kumar[1*], S. V. H. N. Krishna Kumari. P[2], M. V. Ramana Murthy[2] and S. Sreenadh[3]

[1]*Department of Mathematics, Stanley College of Engineering and Technology for Women, Hyderabad, A.P., INDIA*
[2]*Department of Mathematics, Osmania University, Hyderabad, A.P., INDIA*
[3]*Department of Mathematics, Sri Venkateswara University, Tirupati, A.P, INDIA*

ABSTRACT

In this paper peristaltic transport of a power law fluid in an asymmetric channel bounded by permeable beds is studied under long wavelength and low Reynolds number assumptions. The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitude and phase due to the variation of channel width, wave amplitudes and phase differences. Expressions for the velocity, pressure rise and frictional force are obtained. The effect of various parameters on these is discussed through graphs.

Keywords: Peristaltic transport; power law fluid; asymmetric channel; permeable walls.

INTRODUCTION

Peristalsis is a form of fluid transport in a tube when a progressive wave of contraction or expansion propagates along its length. Peristalsis is now well known to the physiologists to be one of the major mechanisms for fluid transport in many biological systems like urine transport from kidney to bladder, movement of ovum in the fallopian tubes and in the vasomotion of small blood vessels. Roller and finger pumps also use this mechanism to transport blood, slurries, corrosive fluids, foods etc, whenever it is desirable to prevent the transported fluid from coming in contact with the mechanical parts of the pump.

Several authors [Tang and Fung [20]] considered blood and other bio-fluids to behave like a Newtonian fluid for physiological peristalsis. Although this approach provides a satisfactory understanding of peristalsis mechanism in ureter, it fails to give a better understanding when the peristaltic mechanism is involved in small blood vessels, lymphatic vessels and intestine. It is now accepted that most of the bio-fluids behave like non-Newtonian fluids. [Raju and Devanathan [12], Patel et al. [10],Srivastava and Srivastava [15], Lou and Yang [6], Dutta and Tarbell [2]].

Raju and Devanathan [12] have first studied the peristaltic transport of power-law fluids and they have obtained the stream function as a perturbation series in the amplitude ratio. Radahakrishnamacharya [11] studied the peristaltic motion of a power-law fluid under long wave length approximation. J.B.Shukla et al. [14] studied the peristaltic transport of a power-law fluid with variable viscosity.

Krishna Kumari et.al [18] studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field. Krishna Kumari et.al [19] studied the peristaltic pumping of a Jeffrey fluid in a porous tube. Srivastava and Srivastava [16] studied the peristaltic transport of a power-law fluid in uniform and non-uniform two dimensional channels. The applications of these results to the rats observed in the ductus efferent of the male reproductive tract is discussed. Assuming chyme as a non-Newtonian fluid, transport of chyme in small intestine and oesophagus has been studied by Srivastava and Srivastava [17], Misra and Pandey [7,8],Usha and RamachandraRao[21] have discussed the peristaltic flow of two layered power-law fluids.

Tang and Fung [20], Gopalan [5],Chaturani and Ranganatha [1] in their discussion described the walls to be permeable in various ducts of living bodies. Ravi Kumar et.al [13] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. The importance of the study of peristaltic transport in an asymmetric channel has been brought out by Eytan et al Elad [3] with an application in intra uterine fluid flow in a non-pregnant uterus. Some of the physiological systems in a human body can not be approximated as a symmetric channel (for example sagittal cross section of the uterus). Eytan et al.[4], Mishra and Ramachandra Rao [7] studied the peristaltic flow in asymmetric channels.

In view of this, in this chapter, we consider the peristaltic transport of a power-law fluid in an asymmetric channel bounded by permeable walls. The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitude and phase due to the variation of channel width, wave amplitudes and phase differences.

Mathematical formulation of the problem

We consider the peristaltic transport of a power-law fluid in an asymmetric channel with flexible porous walls. The channel asymmetry is generated by propagation of waves on the channel walls traveling with different amplitudes, phases and with the same constant speed, c.

$$Y = H_1 = a_1 + b_1 Cos\frac{2\pi}{\lambda}(X - ct) \qquad \text{(upper wall)}$$

$$Y = H_2 = -a_2 - b_2 Cos(\frac{2\pi}{\lambda}(X - ct) + \theta) \qquad \text{(lower wall)} \qquad (1)$$

where b_1,b_2 are the amplitudes of the waves, $a_1 + a_2$ is the width of channel, λ is the wave length, θ is the phase difference which varies in the range $0 \leq \theta \leq \pi$ and t is the time . $\theta = 0$ corresponds to symmetric channel with waves out of phase and $\theta = \pi$ the waves are in phase.

Introducing the wave frame of reference (x, y) moving away with speed 'c' in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant. The transformation from the fixed frame of reference

(X, Y) to the wave frame of reference (x, y) is given by x = X-ct, y=Y, u= U-c, v=V and p(x) = P(X, t) (2)

where (u, v) and (U, V) are the velocity components, p and P are pressures in wave and fixed frame of references respectively.

We choose Ostwald-de Waele power-law model which is characterized by the constitutive equation [Bird et.al]

$$\tau = -\{m\left|\sqrt{\frac{1}{2}(\Delta:\Delta}\right|^{n-1}\}\Delta \tag{3}$$

Where τ is the stress tensor and Δ is the symmetric part of the velocity gradient tensor.

$$\sqrt{\frac{1}{2}(\Delta:\Delta)} = 2[\{(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2\} + (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2]^{\frac{1}{2}} \tag{4}$$

Where 'm' is the consistency parameter and 'n' is the fluid behaviour index parameter.

The fluid is shear thinning or pseudo plastic fluid if n < 1 and a shear thickening or dilatant fluid if n > 1. For n = 1 it becomes Newtonian fluid.

The equations of motion in a wave frame of reference are given by

$$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}$$

$$\rho(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \tag{5}$$

Where $\tau_{xx} = |\Phi|^{n-1} 2\frac{\partial u}{\partial x}$, $\tau_{xy} = |\Phi|^{n-1}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$

$\tau_{yy} = |\Phi|^{n-1} 2\frac{\partial v}{\partial y}$, ρ is the density. (6)

Introducing the non-dimensional variables

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{d}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c\delta}, \quad \delta = \frac{d}{\lambda},$$

$$\bar{t} = \frac{ct}{\lambda}, \quad h_1 = \frac{H_1}{d}, \quad h_2 = \frac{H_2}{d}, \quad \bar{p} = \frac{pd^{n+1}}{m\lambda c^n} \tag{7}$$

in equations (5) and (1) (dropping bars) , we get

$$\mathrm{Re}(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -(\frac{\partial p}{\partial x}) + 2\delta^2\frac{\partial}{\partial x}(\phi\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\varphi(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x})\}$$

$$\mathrm{Re}\,\delta^2\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\left(\frac{\partial p}{\partial y}\right)+\delta^2\frac{\partial}{\partial x}\left\{\left(\varphi\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)\right\}+2\delta^2\frac{\partial}{\partial y}\left(\varphi\left(\frac{\partial v}{\partial y}\right)\right\} \quad (8)$$

$$y=h_1=1+\phi_1 Cos 2\pi x\ , \quad y=h_2=-1-\phi_2 Cos(2\pi x+\theta) \quad (9)$$

Where $\varphi=\left|2\delta^2\left\{\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right\}+\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)^2\right|^{\frac{(n-1)}{2}}$

$\phi_1=\frac{a}{d}$, $\phi_2=\frac{b}{d}$, $\mathrm{Re}=\frac{\rho c^{2-n}d^n\delta}{m}$ is the Reynolds number for power-law fluid.

Under the lubrication approach neglecting the terms of order δ and Re the governing equations (8) becomes

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial p}{\partial y}=0 \quad (10)$$

The Beavers-Joseph boundary conditions for a power-law fluid are given by (in a wave frame of reference)

$u=-1+u_{B_1}$ at y = h_1 (11)

$u=-1+u_{B_2}$ at y = h_2

$\frac{\partial u}{\partial y}=\frac{-\alpha}{(Da)^{\frac{1}{n}+1}}(u_{B_1}-Q_1)$ at y = h_1 (12)

$\frac{\partial u}{\partial y}=\frac{\alpha}{(Da)^{\frac{1}{n}+1}}(u_{B_2}-Q_2)$ at y = h_2 (13)

$\frac{\partial u}{\partial y}=0$ at $y=\frac{h_1+h_2}{2}$ (14)

where $Q_i=-\left|\frac{\partial p}{\partial x}\right|^{\frac{1}{n}-1}\left(\frac{\partial p}{\partial x}\right)(Da)^{\frac{1}{n}}, i=1,2$

Solution of the problem

It is very difficult to choose a correct sign for the modulus which appears in the governing equation. A regularity condition on the axis is used to determine the axial velocity uniquely in a pipe flow. Two appropriate non – unique solutions of (10) satisfying the corresponding boundary conditions only are obtained.

The axial velocity attains a maximum value on the axis and the similar regularity condition extends to asymmetric channel geometry also .

Thus choose $\frac{\partial u}{\partial y}=0$ for $y=\frac{h_1+h_2}{2}$.

As u is maximum on the middle line, we have

$$\frac{\partial u}{\partial y} < 0 \text{ for } y = \frac{h_1 + h_2}{2}. \quad (15)$$

and $$\frac{\partial u}{\partial y} > 0 \text{ for } y = \frac{h_1 + h_2}{2}. \quad (16)$$

Solution of the equation (10) using the corresponding the corresponding boundary conditions (11), (14) and (15) is

$$u = \frac{n}{n+1}\left|\frac{\partial p}{\partial x}\right|^{\frac{1}{n}-1}\left(\frac{\partial p}{\partial x}\right)\left[\left(y - \frac{h_1+h_2}{2}\right)^{\frac{1}{n}+1} - \left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+1}\right] + u_{B_1} - 1, y \geq \frac{h_1+h_2}{2} \quad (17)$$

where the slip velocity u_{B1} is given by

$$u_{B1} = -\left|\frac{\partial p}{\partial x}\right|^{\frac{1}{n}-1}\left(\frac{\partial p}{\partial x}\right)\left[\frac{(Da)^{\frac{1}{n+1}}}{\alpha}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}} + (Da)^{\frac{1}{n}}\right] \quad (18)$$

In this solution nth root always to be taken for a positive quantity as the negative quantities lead to complex values which are not physically meaningful.

The solution (17) can not be used for $y < \frac{h_1 + h_2}{2}$ as it does not satisfy the boundary condition at the lower wall y = h_2.A physically meaningful solution in this region is obtained by solving (10) with boundary conditions (12), (14) and (16) as

$$u = \frac{n}{n+1}\left|\frac{\partial p}{\partial x}\right|^{\frac{1}{n}-1}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{h_1+h_2}{2} - y\right)^{\frac{1}{n}+1} - \left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+1}\right] + u_{B_2} - 1, y \leq \frac{h_1+h_2}{2} \quad (19)$$

where the slip velocity u_{B2} is given by

$$u_{B2} = -\left|\frac{\partial p}{\partial x}\right|^{\frac{1}{n}-1}\left(\frac{\partial p}{\partial x}\right)\left[\frac{(Da)^{\frac{1}{n+1}}}{\alpha}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}} - (Da)^{\frac{1}{n}}\right]$$

The volume flow rate ‘q’ in the wave frame of reference is given by

$$q = \int_{h_2}^{h_1} u\,dy = -\left|\frac{\partial p}{\partial x}\right|^{\frac{1}{n}-1}\frac{\partial p}{\partial x}\left[\frac{2n}{2n+1}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+2} + \frac{2(Da)^{\frac{1}{n}+1}}{\alpha}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+1}\right] - (h_1 - h_2) \quad (20)$$

From this, we have

$$\frac{\partial p}{\partial x} = -\left|-\left(\frac{q+h_1-h_2}{D}\right)\right|^{n-1}\left(\frac{q+h_1-h_2}{D}\right) \quad (21)$$

Where $D = \frac{2n}{2n+1}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+2} + \frac{2(Da)^{\frac{1}{n}+1}}{\alpha}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+1}$

The instantaneous flux Q(x,t) in the laboratory frame is

$$Q(x,t) = \int_{h_2}^{h_1} (u+1)dy = q + h_1 - h_2 \qquad (22)$$

The average volume flow rate over one period ($T = \frac{\lambda}{c}$)) of the peristaltic wave is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Q dt = \frac{1}{T}\int_0^T (q + h_1 - h_2)dt = q + 1 + d \qquad (23)$$

Pumping characteristics

Integrating the equation (21) with respect to x over one wave length, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^L \frac{dp}{dx}dx = \int_0^1 -\left|-\left(\frac{\overline{Q}-1-d+h_1-h_2}{D}\right)\right|^{(n-1)}\left(\frac{\overline{Q}-1-d+h_1-h_2}{D}\right) \qquad (24)$$

The pressure rise required to produce zero average flow rate is denoted by Δp_0 .

Hence $$\Delta p_0 = \int_0^1 -\left|\left(\frac{-1-d+h_1-h_2}{D}\right)\right|^{(n-1)}\left(\frac{-1-d+h_1-h_2}{D}\right) \qquad (25)$$

Where $D = \frac{2n}{2n+1}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+2} + \frac{2(Da)^{\frac{1}{n}+1}}{\alpha}\left(\frac{h_1-h_2}{2}\right)^{\frac{1}{n}+1}$

The dimensionless frictional force F at the wall across one wave length in the channel is given by

$$F = \int_{h_2}^{h_1} h\left(\frac{-dp}{dx}\right)dx \qquad (26)$$

RESULTS AND DISCUSSION

From the equation (24), we have calculated the pressure difference Δp for a power law fluid as a function of time averaged flux $\overline{Q}$ for different values of Darcy number 'Da' for a fixed $\phi_1 = 0.7, \phi_2 = 1.2$ and is shown in Figure .1.It is observed that for a given time averaged flux $\overline{Q}$,the pressure difference increases with an increase in Da. In the similar way the relation between Δp and $\overline{Q}$ is shown in Figure. 2 for the phase shift $\theta = \frac{\pi}{4}$.The similar phenomenon is observed .And it is also observed that Δp decreases with the increase in the phase shift θ.

The variation of the time averaged flux $\overline{Q}$ with Δp for fixed $\alpha, \phi_1 = 0.7, \phi_2 = 1.2, \theta$ and n for different values of α is shown in Figures .3 and 4.It is observed that ,increasing the values of α decreases the pumping and also free pumping.

Figures 5 and 6 are drawn for the variation of Δp with $\overline{Q}$ for fixed α, ϕ_1, ϕ_2 and n for different values of phase shift θ . It is observed that for a given time averaged flux $\overline{Q}$,the pressure difference decreases with an increase in the phase shift θ. And also it is clear that for a given Δp, $\overline{Q}$ decreases with an increase in θ.

Figure .1. The variation of Δp with $\overline{Q}$ for different values of Da with $\alpha = 4, \phi_1 = 0.7, \phi_2 = 1.2, d = 2, \theta = 0, n = 1.$

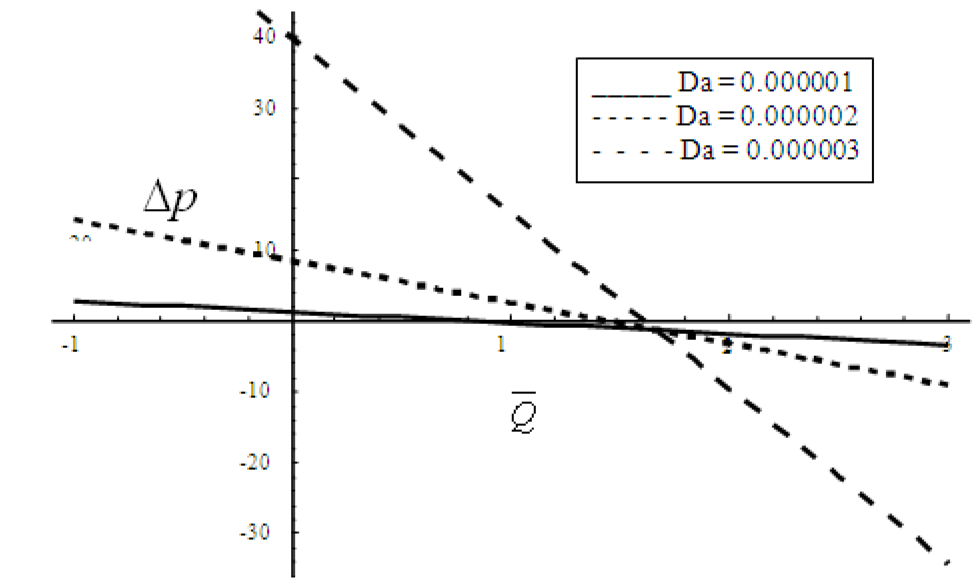
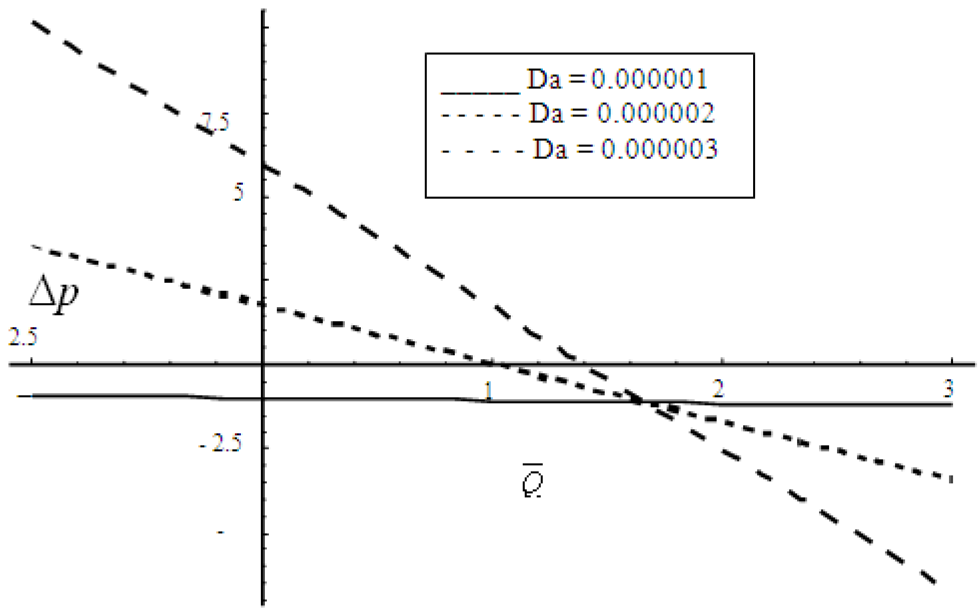


Figure2. The variation of Δp with $\overline{Q}$ for different values of Da with $\alpha = 4, \phi_1 = 0.7, \phi_2 = 1.2, d = 2, \theta = \frac{\pi}{4}, n = 1.$



The variation of axial velocity 'u' with 'y' for different values of Darcy number with fixed α, ϕ_1, ϕ_2 and for θ=0 is shown in Figure 7.It is observed that the velocity is maximum at the centerline of the channel. It is also observed that the velocity decreases with decrease in Darcy number for negative pressure gradient.

Figure 3. The variation of Δp with $\overline{Q}$ for different values of α with $\phi_1 = 0.7$, $\phi_2 = 1.2, d = 2, \theta = 0, Da = 0.000001, n = 1$.

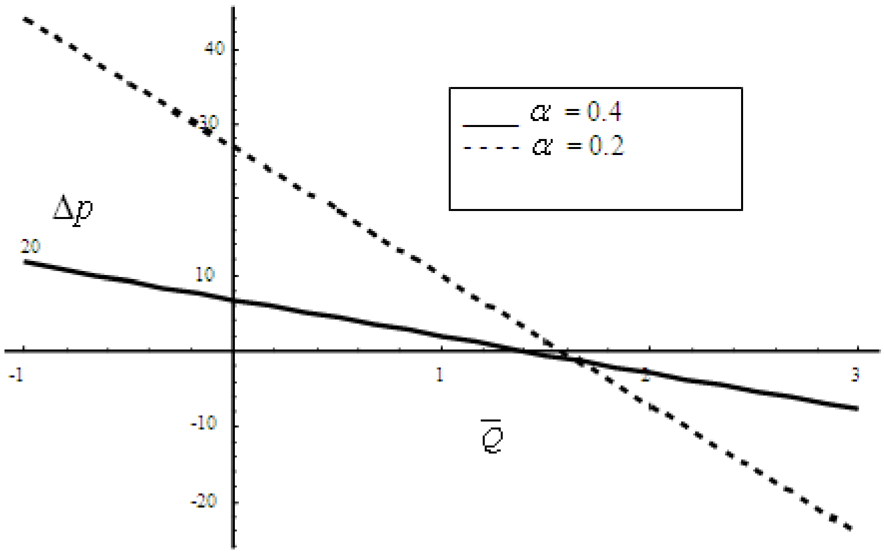
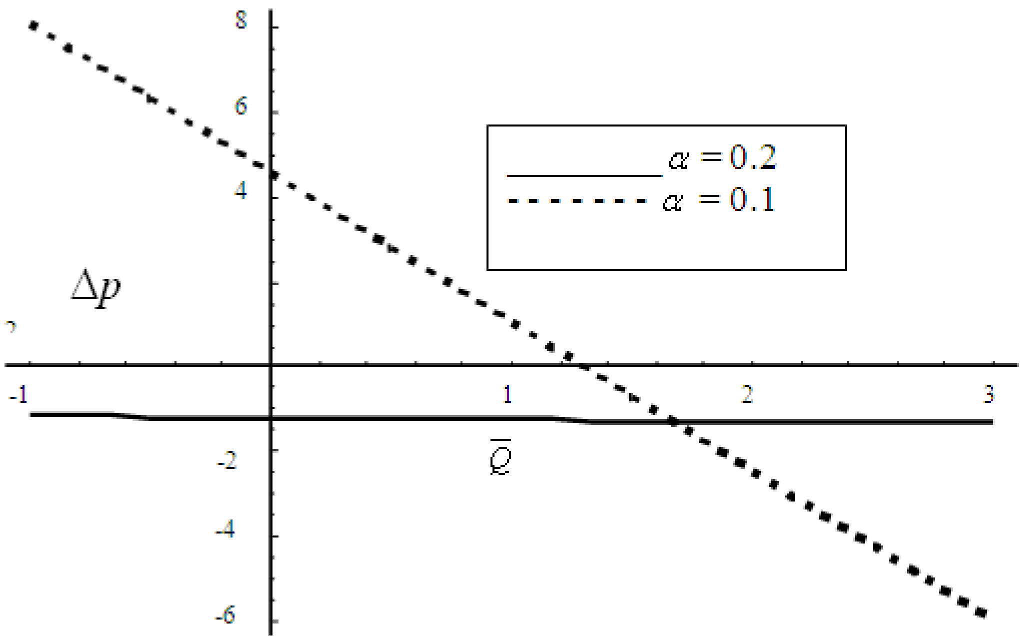


Figure 4. The variation of Δp with $\overline{Q}$ for different values of α with $\phi_1 = 0.7$, $\phi_2 = 1.2, d = 2, \theta = 0, Da = 0.000001, n = 1$.



The variation of axial velocity 'u' with 'y' for different values of Darcy number with fixed α, ϕ_1, ϕ_2 and for $\theta = \frac{\pi}{2}$ is shown in Figure 8.Here also it is observed that the velocity is maximum at the centerline of the channel and observed that the velocity decreases with decrease in Darcy number for negative pressure gradient. It is observed that the velocity decreases with an increase in the phase shift θ for fixed Darcy number.

Figure 5. The variation of Δp with $\overline{Q}$ for different values of θ with $\phi_1 = 0.7$, $\phi_2 = 1.2, d = 2, \alpha = 1, Da = 0.01, n = 1.$

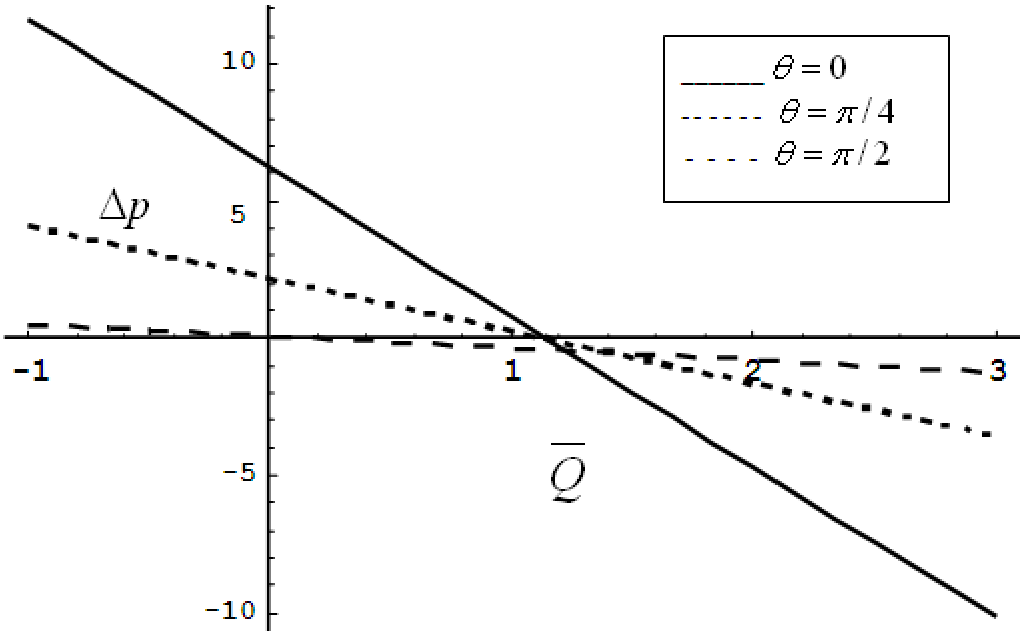
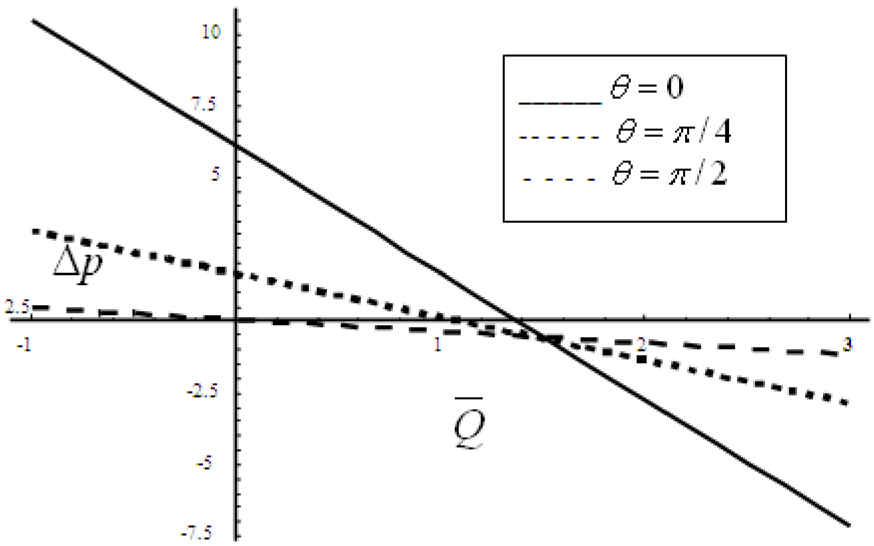


Figure 6. The variation of Δp with $\overline{Q}$ for different values of θ with $\phi_1 = 0.7$, $\phi_2 = 1.2, d = 2, \alpha = 3, Da = 0.000001, n = 1.$



The Figure 9 is drawn for the variation of the axial velocity u with y for varying α. It is observed that the velocity is maximum at the centerline of the channel. It is also observed that the velocity decreases with an increase in α.

Figure 7. The Velocity profiles for different values of Da with $\phi_1 = 0.5, \phi_2 = 0.8,$
$d = 2, \theta = 0, \alpha = 0.01, n = 1.$

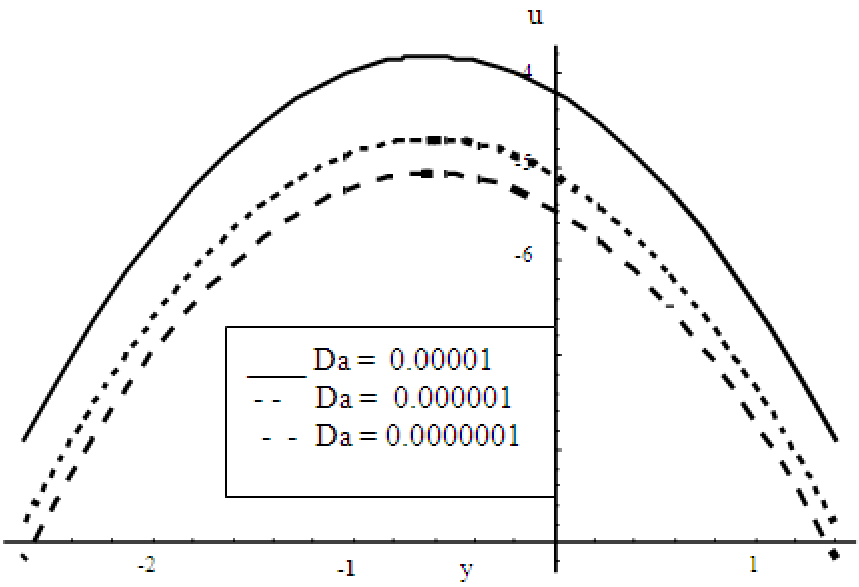


Figure 8.The Velocity profiles for different values of Da with $\phi_1 = 0.5, \phi_2 = 0.8,$
$d = 2, \theta = \pi/2, \alpha = 0.01, n = 1.$

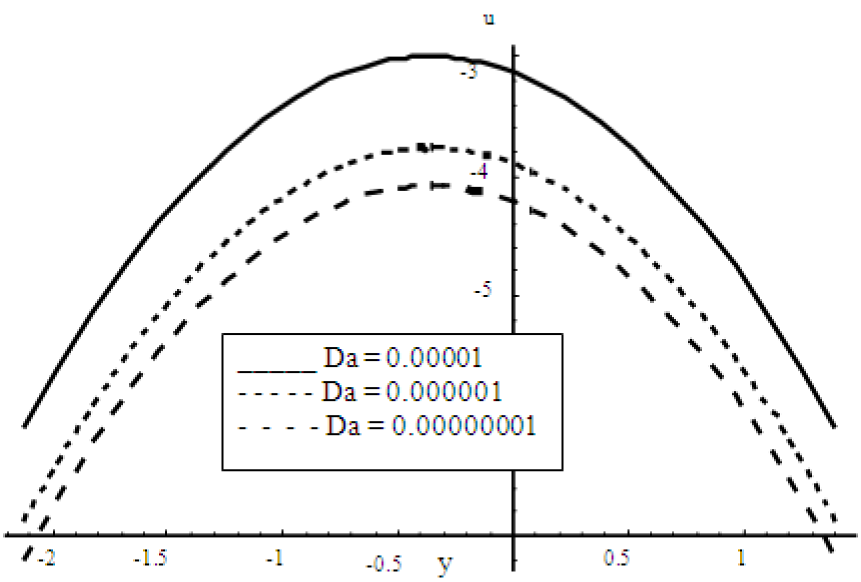
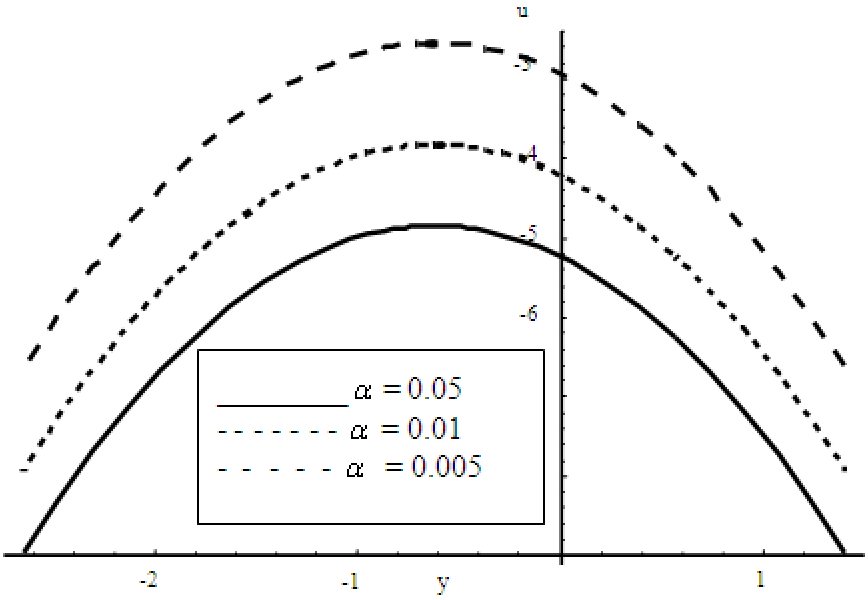


Figure 9. The Velocity profiles for different values of α with $\phi_1 = 0.5, \phi_2 = 0.8,$
$d = 2, \theta = 0, Da = 0.00001, n = 1.$



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