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Pade approximation technique reduced model using stability equation method

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ABSTRACT

An algorithm is introduced for model reduction of linear time invariant system using combine advantage of pade approximation technique and stability equation method. The denominator of reduced order model is derived by stability equation method and numerator of reduced order transfer function is obtained by pade approximation technique. This method guarantees stability of reduced model if high order original system is stable

Key words: Model reduction, Stability equation method, Pade approximation technique, Transfer function.

INTRODUCTION

Development of reduced order model is important in analysis, synthesis and simulation of complex system. The design and analysis of high order system becomes complex and tedious. Numerous methods have been developed for approximation of original system of linear time invariant of dynamic system. Routh approximation method is good for stability and simplicity [1] but it is found that this method is unable to produce good reduced model. Several methods have been proposed based on pade approximation technique [2-6] stability equation method[7-8]continued fraction expansion [9-12] and error minimization technique [13-15] and time moment matching etc methods contain advantages and disadvantages. Continued fraction method is used as powerful tools for reducing the original system. This paper deals with both Pade approximation technique and stability equation method .A popular approach as pade approximate technique for deriving reduced order model has been based on matching of time moments of reduced order model and original system. This technique is very simple in computation and fitting of initial time moments and steady state value of output of reduced order model and original system in form of $\sum \alpha_i t^i$. A known drawback of this method is that unstable reduced order model arises from original stable system was improved by using stability equation method proposed by Pal[16] and Parthasarthy and Jayasimha. Some combined methods are also given for order reduction model [17-19].A proposed method for deriving reduced order model equivalent to original high order system is described. It combines Pade approximate technique and stability equation method.

MATERIALS AND METHODS

2.Methods Used

2.1 Statement of Approximation Method.

Let n^{th} order of time invariant dynamic system is given by transfer function

$$H(s) = \frac{D_0 + D_1 s + D_2 s^2 + \ldots\ldots \mathrm{D}_{n-1} s^{n-1}}{d_0 + d_1 s + d_2 + \ldots\ldots. \mathrm{d}_{n-1} s^{n-1} + d_n s^n} \qquad (1)$$

$$H(s) = \frac{\sum_{i=0}^{n-1} D_i s^i}{\sum_{j=0}^{n} d_j s^j}$$

Where D_0 ,D_1 ………..D_{n-1} and d_0,d_1 ……….d_n are constant.
Reduced order model is given by

$$H_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{E_o + E_1 s + E_2 s^2 + ... + E_{r\text{-}1} s^{r\text{-}1}}{e_0 + e_1 s + e_2 s^2 + ... + e_{r\text{-}1} s^{r\text{-}1} + e_r s^r} \qquad (2)$$

$$H_r(s) = \frac{\sum_{j=0}^{r-1} E_j s^j}{\sum_{j=0}^{r} D_j s^j}$$

Where E_0, E_1, E_2……E_n and e_0,e_1,e_2……e_n are constant.

2.2 Order Reduction Method.
This method consists of two types

Step 1
First numerator of reduced model is formulated in form of Pade approximation technique.
H(s) can be expanded into a power series about s=0

$$H(s) = p_0 + p_1 s + p_2 s^2 + \ldots\ldots \qquad (3)$$

Where

$$p_0 = \frac{D_0}{d_0}$$

Reduced order model $H_r(s)$ to be Pade approximate about s=0 of original high order system H(s), we have

$$E_0 = e_0 p_0 (4)$$

$$E_1 = e_0 p_1 + e_1 p_0 \qquad (4)$$

………………………..

$$E_{k-1} = e_0 p_{k-1} + e_1 p_{k-2} + \ldots + e_{k-1} p_1$$

E_j can be determined by solving the above k equation. Where
j=0,1,2…..k-1

Step 2
Determination of the denominator coefficients of reduced order model**:-**
From stable original system H(s), denominator of H(s) is separated in even parts and odd parts, as stability equation (5)

$$D_e(s) = d_0 \prod_{i=1}^{n_1} \left(1 + \frac{s^2}{z_i^2}\right) \qquad (5)$$

$$D_o(s) = d_1 s \prod_{i=1}^{n_2} \left(1 + \frac{s^2}{p_i^{\,2}}\right) \qquad (6)$$

Wheren_1 and n_2 are integer parts of k/2 and k−1/2 respectively and
$z_1^{\,2}<p_1^{\,2}< z_2^{\,2}< p_2^{\,2}$……………

by neglecting the factors with large magnitudes of z_i and p_i in eqn(5) and eqn (6) Stability equation for r^{th} order of reduced order model is obtained. as

$$D_e(s) = d_0 \prod_{i=1}^{n_3} \left(1 + \frac{s^2}{z_i^{\,2}}\right) \qquad (7)$$

$$D_o(s) = d_1 s \prod_{i=1}^{n_4} \left(1 + \frac{s^2}{p_i^{\,2}}\right) \qquad (8)$$

Where n_3 and n_4 are integer parts of r/2 and $\frac{r-1}{2}$.

Combing these even and odd parts of reduced order denominator and reciprocating, we get

$$D_r(s) \; = D_e(s) \; + D_o(s) \qquad (9)$$

$$D_r(s) = d_0 \prod_{i=1}^{r/2} \left(1 + \frac{s^2}{z_i^{\,2}}\right) + d_1 s \prod_{i=1}^{r-1/2} \left(1 + \frac{s^2}{p_i^{\,2}}\right)$$

III. Results: - Numerical Analysis.

Let transfer function of 4^{th} order system [1] is given by

$$H_4(s) = \frac{28s^3 + 496s^2 + 1800s + 2400}{2s^4 + 36s^3 + 204s^2 + 360s + 240} \qquad (10)$$

From above original transfer function, second order reduced model is to be obtained.

Step 1:--
Power series of H(s) given as

H(s**)** =10-7.5s+………..(11)

Now
$E_0 = e_0 p_0 = 11.904$
$E_1 = e_0 p_1 + e_1 p_0 = 8.928$

Step 2:- Model of second order is obtained by Combing these reduced stability equations Separating the denominator of the above high order system (HOS) in even and odd parts, we get stability equations as

$D_e(s) = 240 + 204s^2 + 2s^4$

$$240\left(1 + \frac{s^2}{1.1904}\right)\left(1 + \frac{s^2}{100.8096}\right) \qquad (12)$$

$D_o(s) = 360s + 36s^3$

$$360s\left(1+\frac{s^2}{10}\right) \quad (13)$$

By discarding the factors with large magnitude of z_i^2 and p_i^2 in $D_e(s)$ and $D_o(s)$ respectively

The stability equations of 2nd order reduced model are given as

$$D_e(s) = 240\left(1+\frac{s^2}{1.1904}\right) \text{ and } D_o(s) = 360s$$

Combing these reduced order stability equations, we get denominator of 2nd order reduced model as
Finally 2nd order reduced model $H_2(s)$ is given by

$$H_2(s) = \frac{8.928s + 11.904}{s^2 + 1.7856s + 1.1904} \quad (14)$$

Fig-1.presents step response of original high order system H(s) and reduced order model $H_r(s)$ and fig-2 shows the frequency response of original high order system H(s) and reduced order model $H_r(s)$. It may be seen that steady state response of original system and reduced order model are exactly matching and transient response of reduced order model is also matching very close to the original system.

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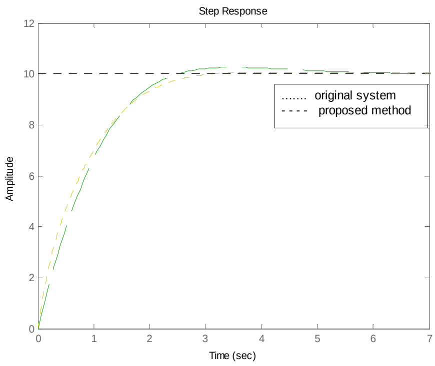

Fig-1: step Response

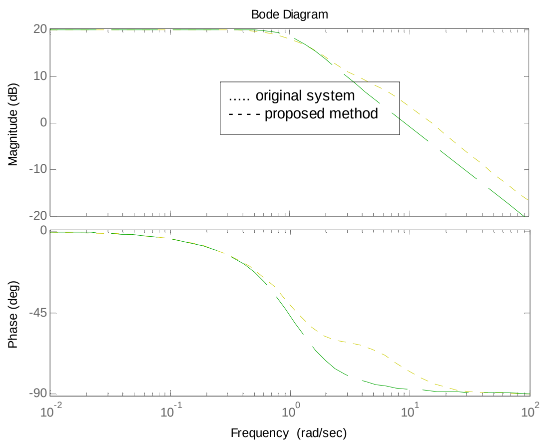

Fig-2: Frequency Response

Comparison of Reduced model with Original System
Original system
Rise Time: 1.6920
Settling Time: 2.5514
Settling min: 9.0181
Settling max: 10.0443
Overshoot: 0.4435
Undershoot: 0
Peak: 10.0443
Peak Time: 3.9460

Reduced Model
Rise Time: 1.6137
Settling Time: 4.2493
Settling min: 9.0700
Settling max: 10.2569
Over shoot: 2.5686
Under shoot: 0
Peak: 10.2569
Peak Time: 3.4667

DISCUSSION

In proposed model reduction method algorithm used which combines to take both advantages of stability equation method to derive the denominator of polynomial and pade approximation technique to determine the numerator of polynomial .It is observed that proposed method preserves the steady state and stability in reduced order model. This algorithm has been implemented in Metlab 7.0. Steady state and stability avoids any error in initial and final value of responses of original system and reduced order model. This method is extended for multivariable system. These methods are easily contribution to model reduction method due to flexibility of algorithm. This method is curiosity method to develop reduced order model for approximation of original system.

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