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Optimization in portfolio using maximum downside deviation stochastic programming model

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ABSTRACT

Portfolio optimization has been one of the important research fields in financial decision making. The most important character within this optimization problem is the uncertainty of the future returns. To handle such problems, we utilize probabilistic methods alongside with optimization techniques. We develop single stage and two stage stochastic programming with recourse with the objective is to minimize the maximum downside semi deviation. We use the so-called "Here-and-Now" approach where the decision-maker makes decision"now" before observing the actual outcome for the stochastic parameter. We compare the optimal portfolios between the single stage and two stage models with the incorporation of the deviation measure. The models are applied to the optimal selection of stocks listed in Bursa Malaysia and the return of the optimal portfolio is compared between the two stochastic models. The results show that the two stage model outperforms the single stage model in the optimal and in-sample analysis

Keywords: Portfolio optimization, Maximum Semi deviation Measure, Downside risk, Stochastic Linear Programming.

INTRODUCTION

Portfolio optimization has been one of the important research fields in financial decision making. The most important character within this optimization problem is the uncertainty of the future returns. To handle such problems, we utilize probabilistic methods alongside with optimization techniques. Stochastic programming is our approach to deal with uncertainty. Stochastic Programming is a branch of mathematical programming where the parameters are random. The objective of stochastic programming is to find the optimum solution to problems with uncertain data. This approach can deal the management of portfolio risk and the identification of optimal portfolio simultaneously. Stochastic programming models explicitly consider uncertainty in

some of the model parameters, and provide optimal decisions which are hedged against such uncertainty

In the deterministic framework, a typical mathematical programming problem could be stated as

$$\begin{aligned} &\min_{x} \quad f(x) \\ &s.t \quad g_i(x) \leq 0, \qquad i = 1,...m, \end{aligned} \tag{1.1}$$

where x is from R^n or Z^n. Uncertainty, usually described by a random element $\xi(\omega)$, where ω is a random outcome from a space Ω, leads to situation where instead of just $f(x)$ and $g_i(x)$ one has to deal with $f(x,\xi(\omega))$ and $g_i(x,\xi(\omega))$. Traditionally, the probability distribution of ξ is assumed to known or can be estimated and is unaffected by the decision vector x. The problem becomes decision making under uncertainty where decision vector x has to be chosen before the outcome from the distribution of $\xi(\omega)$ can be observed.

Markowitz used the concept of risk into the problem and introduced mean-risk approach that identifies risk with the volatility (variance) of the random objective [5, 6]. Since 1952, mean-risk optimization paradigm received extensive development both theoretically and computationally. Konno and Yamazaki proposed mean absolute deviation from the mean as the risk measure to estimate the nonlinear variance-covariance of the stocks in the mean-variance model [4]. It transforms the portfolio selection problem from a quadratic programming into a linear programming problem. At the same time, the popularity of downside risk among investors is growing and mean-return-downside risk portfolio selection models seem to oppress the familiar mean-variance approach. The reason for the success of the former models is that they separate return fluctuations into downside risk and upside potential. This is especially relevant for asymmetrical return distributions, for which mean-variance model punish the upside potential in the same fashion as the downside risk. This led Markowitz to propose downside risk measures such as (downside) semi variance to replace variance as the risk measure [6]. Consequently, one observes growing popularity of downside risk models for portfolio selection [7].

Young [8] introduced another linear programming model which maximize the minimum return or minimize the maximum loss (minimax) over time periods and applied to the stock indices from eight countries, from January 1991 until December 1995. The analysis showed that the model performs similarly with the classical mean-variance model. In addition, Young argues that, when data is log-normally distributed or skewed, the minimax formulation might be more appropriate method, compared to the classical mean-variance formulation, which is optimal for normally distributed data.

Dantzig [5] and independently Beale [1] suggested an approach to stochastic programming and termed as stochastic programming with recourse. Recourse is the ability to take corrective action after a random event has taken place. The main innovation is to amend the problem to allow the decision maker the opportunity to make corrective actions after a random event has taken place. In the first stage a decision maker a here and now decision. In the second stage the decision maker sees a realization of the stochastic elements of the problem but he is allowed to make

further decisions to avoid the constraints of the problem becoming infeasible.

In this paper we develop single stage and two stage stochastic programming with recourse for portfolio selection problem and the objective is to minimize the maximum downside deviation measure of portfolio returns from the expected return. We use the so-called "Here-and-Now" approach where the decision-maker makes decision "now" before observing the actual outcome for the stochastic parameter.

The main objective of this study is to solve portfolio optimization problem using two different stochastic programming models. We apply these models to the optimal selection of stocks listed in Bursa Malaysia and compare the optimal portfolios between the single stage and two stage models.

The remainder of the paper is organized as follows. In the next section we discuss the maximum downside semi deviation measure and formulate the equivalent single stage stochastic linear programming model for portfolio selection problem. Then we extend the single stage model to two stage stochastic programming with recourse model. Section 3 devoted to the experimental analysis on real-life data from Bursa Malaysia. Finally, some concluding remarks are given in section 4.

MATERIALS AND METHODS

Consider a set of securities $I=\{i: i=1,2,...,n\}$ for an investment. At the beginning of the holding period the investor wishes to apportion his budget to these assets by deciding on a specific allocation $\boldsymbol{x}=(x_1,x_2,...,x_n)^T$ such that $x_i \geq 0$ (i.e., short sales are not allowed) and $\sum_{i\in I} x_i = 1$ (budget constraint). At the end of a certain holding period the assets generate returns, $\tilde{\boldsymbol{r}}=(\tilde{r}_1,\tilde{r}_2,...,\tilde{r}_n)^T$. At the beginning of the holding period the returns are random. Suppose that $\tilde{\boldsymbol{r}}$ are represented by a finite set of discrete scenarios $\Omega=\{\omega: \omega=1,2,...,S\}$, whereby the returns under a particular scenario $\omega\in\Omega$ take the values $\boldsymbol{r}_\omega=(r_{1\omega},r_{2\omega},...,r_{n\omega})^T$ with associated probability $p_\omega>0$, $\sum_{\omega\in\Omega} p_\omega = 1$. The portfolio return under a particular realization of $\boldsymbol{r}_\omega$ is $R_\omega = R(\boldsymbol{x},\boldsymbol{r}_\omega)$ and the expected portfolio return is $\bar{R}(\boldsymbol{x},\boldsymbol{r}_\omega)=\sum_{\omega\in\Omega} p_\omega R(\boldsymbol{x},\boldsymbol{r}_\omega)$.

Let $M[R(\boldsymbol{x},\boldsymbol{r}_\omega)]$ be the minimum of the portfolio return. The *maximum (downside) semi deviation* measure is defined as

$$MM[R(\boldsymbol{x},\boldsymbol{r}_\omega)] = [\boldsymbol{E}[R(\boldsymbol{x},\boldsymbol{r}_\omega)] - Min[R(\boldsymbol{x},\boldsymbol{r}_\omega)] \quad (2.1)$$

$MM[R(\boldsymbol{x},\boldsymbol{r}_\omega)]$ is a very pessimistic risk measure related to the worst case analysis. It does not take into account the distribution of outcomes other than the worst one.

$\forall\omega\in\Omega$, let $\eta=\max_{\omega\in\Omega}[\bar{R}(\boldsymbol{x},\boldsymbol{r}_\omega) - R(\boldsymbol{x},\boldsymbol{r}_\omega)]$

Subject to $\eta \geq \max_{\omega \in \Omega} [\bar{R}(\boldsymbol{x},\boldsymbol{r}_\omega) - R(\boldsymbol{x},\boldsymbol{r}_\omega)]$ *for* $\forall \omega \in \Omega$

Then, we have $MM[R(\boldsymbol{x},\boldsymbol{r}_\omega)] = \eta$ (2.2)

Subject to $\eta \geq \max_{\omega \in \Omega} [\bar{R}(\boldsymbol{x},\boldsymbol{r}_\omega) - R(\boldsymbol{x},\boldsymbol{r}_\omega)]$ *for* $\forall \omega \in \Omega$

2.1 Single Stage Stochastic Linear Programming Portfolio Optimization Model with MM deviation measure

Portfolio optimization problem where (2.1) is minimized constraining the expected portfolio return at the end of investment period can be formulated as a single stage stochastic linear programming model, S_MM below:

$$\begin{aligned} &\textit{Minimize } \eta && (2.3)\\ &\textit{Subject to :}\\ &\bar{R}(\boldsymbol{x},\boldsymbol{r}_\omega) \geq \alpha\\ &\bar{R}(\boldsymbol{x},\boldsymbol{r}_\omega) - R(\boldsymbol{x},\boldsymbol{r}_\omega) \leq \eta\\ &\sum_{i \in I} x_i = 1\\ &L_i \leq x_i \leq U_i \qquad \forall i \in I \end{aligned}$$

2.2 Two Stage Stochastic Linear Programming Model with recourse formulation for S_MM

We now introduce dynamic model where future changes, recourse, to the initial compositions are allowed. Assuming the investor can make corrective action after the realization of random values by changing the composition of the optimal portfolio, we formulate the single period stochastic linear programming model of S_MM as a two-stage stochastic programming problem with recourse. Consider the case when the investor is interested in a first stage decision $\boldsymbol{x}$ that hedges against the *risk* of the second-stage action. At the beginning of the investment period, the investor selects the initial composition of the portfolio, $\boldsymbol{x}$ assuming there is a known distribution of future returns. At the end of the planning horizon, once a particular scenario of return is realized, the investor rebalances the composition by either purchasing or selling the selected stocks. Let a set of second stage variables, $\boldsymbol{y}_{i,\omega}$ to represent the composition of stock *i* after rebalancing is done, i.e., $\boldsymbol{y}_{i,\omega} = \boldsymbol{x}_i + \boldsymbol{P}_{i,\omega}$ or $\boldsymbol{y}_{i,\omega} = \boldsymbol{x}_i - \boldsymbol{Q}_{i,\omega}$ where $\boldsymbol{P}_{i,\omega}$ and $\boldsymbol{Q}_{i,\omega}$ are the quantity purchased and sold respectively.

The maximum downside deviation of portfolio returns from the expected return in terms of the second stage variables $\boldsymbol{y}$ can be formulated as follows:.

$$MM[R(\boldsymbol{y}_\omega,\boldsymbol{r}_\omega)] = \max_{\omega \in \Xi} [\bar{R}(\boldsymbol{y}_\omega,\boldsymbol{r}_\omega) - R(\boldsymbol{y}_\omega,\boldsymbol{r}_\omega)] \quad (2.4)$$

For every scenario $\omega \in \Omega$, let the auxiliary variable $\eta = \max_{\omega \in \Omega} [\bar{R}(\boldsymbol{y}_\omega,\boldsymbol{r}_{,\omega}) - R(\boldsymbol{y}_\omega,\boldsymbol{r}_\omega)]$

$$\text{subject to } \eta \geq \max_{\omega \in \Omega} [\bar{R}(\mathbf{y}_\omega, \mathbf{r}_\omega) - R(\mathbf{y}_\omega, \mathbf{r}_\omega)] \text{ for } \forall \omega \in \Omega$$

Then, we have $MM[R(\mathbf{x}, \mathbf{r}_\omega)] = \eta$

$$\text{subject to } \eta \geq \max_{\omega \in \Omega} [\bar{R}(\mathbf{y}_\omega, \mathbf{r}_\omega) - R(\mathbf{y}_\omega, \mathbf{r}_\omega)] \text{ for } \forall \omega \in \Omega \quad (2.5)$$

We formulate the two stage stochastic linear programming model, 2S_MM, for portfolio optimization problem that minimizes second stage MM and constraining the expected portfolio return as follows:

$$\begin{aligned} &\textit{Minimize } \eta && && (2.6)\\ &\textit{Subject to } && \sum_{i \in I} x_i = 1 \\ & && \sum_{i \in I} y_{\omega i} = 1 && \forall \omega \in \Omega \\ & && \bar{R}(\mathbf{x}, \mathbf{r}_\omega) + R(\mathbf{y}_\omega, \mathbf{r}_\omega) \geq \alpha && \forall \omega \in \Omega \\ & && L_i \leq x_i \leq U_i && \forall i \in I \\ & && L_{\omega i} \leq y_{\omega i} \leq U_{\omega i} && \forall i \in I, \forall \omega \in \Omega \\ & && R(\mathbf{y}_\omega, \mathbf{r}_\omega) \geq \eta \quad \forall \omega \in \Omega \end{aligned}$$

3. NUMERICAL ANALYSIS

We tested our models on ten common stocks selected at random from a set of stocks that were already listed on the main board of Bursa Malaysia on December 1989 and still in the list on May 2004. The closing prices were obtained from Investors Digest. We use empirical distributions computed from past returns as equiprobable scenarios. Observations of returns over N_S overlapping periods of length Δt are considered as the N_S possible outcomes (or scenarios) of the future returns and a probability of $\frac{1}{N_s}$ is assigned to each of them. For each stock, we obtain 100 scenarios of the overlapping periods of length 1 month, i.e. N_S.

To evaluate the performance of the two models, we examined the portfolio returns resulting from applying the two stochastic optimization models. We make comparison between S_MM and 2S_MM models by analyzing the optimal portfolio returns, in-sample portfolio returns and out-of-sample portfolio returns over 60-month period from to 06/1998 to 05/2004. At each month, we use the historical data from the previous 100 monthly observations as scenarios and solve the resulting optimization models using the minimum monthly required return α equals to one.

3.1 Comparison of Optimal Portfolio returns between S_MM and 2S_MM

Figures 1 presents the graphs of optimal portfolio returns resulting from solving the two models; S_MM and 2S_MM (see appendix). The optimal portfolio returns of the two models exhibit the same pattern. There is a decreasing trend in the optimal returns in both models. However, in

figure 1, it can be seen that the optimal portfolio returns from 2S_MM are higher than the optimal portfolio returns from S_MM in all testing periods. This shows that an investor can make a better decision regarding the selection of stocks in a portfolio when he takes into consideration both making decision facing the uncertainty and the ability of making correction actions when the uncertain returns are realized compared to considers only making decision facing the uncertainty alone.

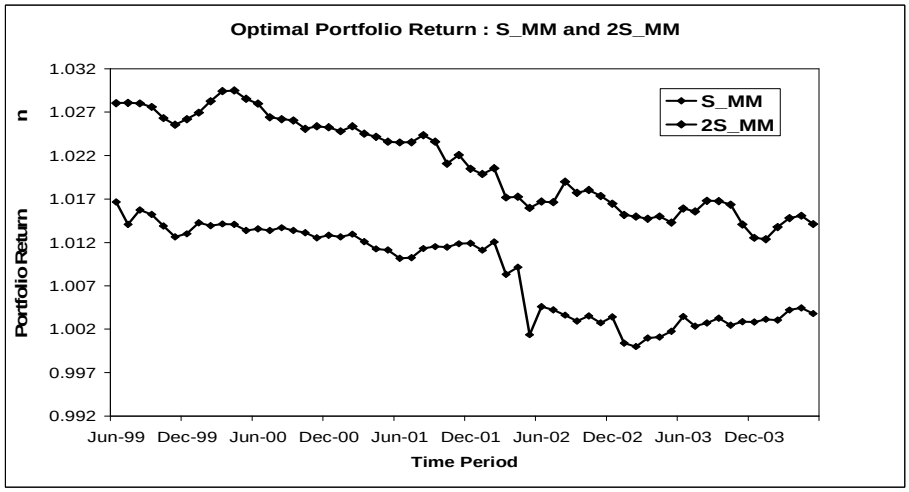

Figure 1: Comparison of Optimal portfolio Returns S_MM and 2S_MM models

3.2 Comparison of Average In-Sample Portfolio returns between S_MM and 2S_MM

We use average realized returns to comparison In-Sample portfolio returns between S_MM model and 2S_MM model and the results are presented in Figure 2. There is an increasing trend in the months from December 1999 until April 2000, then decreasing trend until June 2001. Starting from June 2001 until May 2004, both averages show an increasing trend.

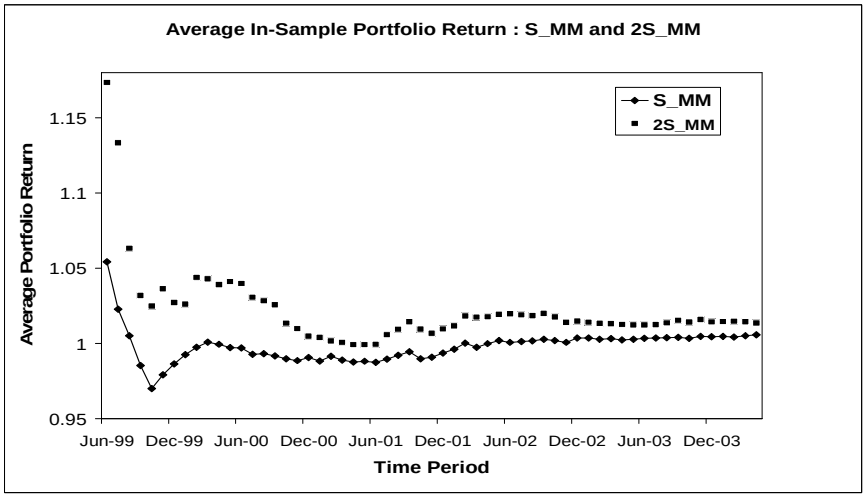

Figure 2: Comparison of Average In-Sample Portfolio Return between S_MM and 2S_MM models

The average in-sample portfolio returns of 2S_MM are higher than the average in-sample portfolio returns in all testing periods.

3.3 Comparison of Out-Of-Sample Portfolio returns between S_MM and 2S_MM models

The comparison of out-of-sample portfolio returns between S_MM and 2S_MM is also done using the average return. The results of Out-Of-Sample analysis are presented in Figure 3. Throughout the testing periods, the average returns from the two models show similar patterns. There is an increasing trend in the months from December 1999 until December 2000, then decreasing trend until June 2001. Starting from June 2001, both averages show an increasing trend. The average out-of-sample of the two-stage model, 2S_MM is higher than those of single stage model, S_MM. Certainly, the models have been applied directly to the original historical data treated as future returns scenarios thus loosing the trend information. Possible application of some forecasting procedures prior to the portfolio optimization models, we consider, seems to be an interesting direction for future research. For references on scenarios generation see [2].

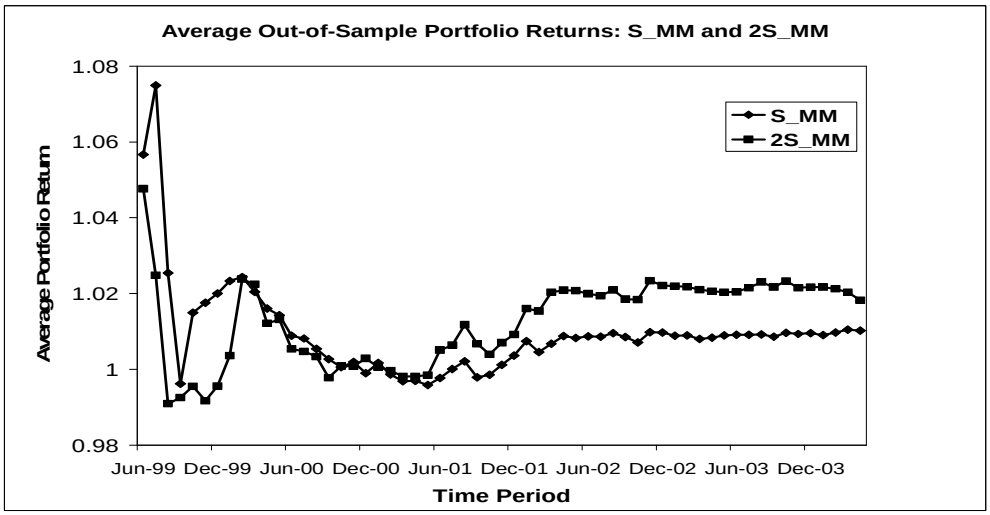

Figure 3: Comparison of Out-Of-Sample Analysis between single stage S_MM and two stage 2S_MM models

CONCLUSION

In this paper, a portfolio selection of stocks with maximum downside semi deviation measure is modeled as a single stage and a two stage stochastic programming models. Single stage model incorporates uncertainty in the model and in the two stage model the uncertainty is incorporated in the models and at the same considers rebalancing the portfolio composition at the end of investment period. The comparison of the optimal portfolio returns, the in-sample portfolio returns and the out-of-sample portfolio returns shows that the performance of the two stage model is better than that of the single stage model. Here, we use historical data as scenarios of future returns. In our future research we will generate scenarios of future asset returns using appropriate scenario generation method before applying to our developed models.

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