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Advances in Applied Science Research, 2013, 4(4):279-282

ISSN: 0976-8610
CODEN (USA): AASRFC

On the study of magneto-hydrodynamics boundary layer flow over a continuously moving flat plate

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ABSTRACT

The objective of the present study is to investigate the steady, two dimensional laminar boundary layer flow of an incompressible, electrically conducting fluid over a continuously flat plate in the presence of transverse magnetic field. The governing equations of continuity and momentum are transformed into ordinary differential equations using similarity transformation. The present flow situation is derived by the one parameter group of transformation.

Keywords: Similarity analysis, one parameter group of transformation, MHD boundary layer, magnetic field

INTRODUCTION

The boundary-layer flow over moving continuous surfaces is important in several technical and industrial applications such as in metallurgy and chemical processes, thermal and moisture treatment of materials. For examples, in the extrusion of polymer sheet from a die, the lamination and melt-spinning process in the extrusion of polymers or the cooling of a large metallic plate in a bath, glass blowing continuous casting and spinning of fibers.

Sakiadis [1] was the first who study the problem of forced convection along an isothermal moving plate. Sakiadis' theoretical predictions for Newtonian fluids were later corroborated experimentally by Tsou et al. [2], he studied flow and heat transfer in the boundary layer on a continuously moving surface. Pavlov [3] presented the MHD flow over a stretching surface and obtained exact solution of momentum equation. Thereafter, in literature it is found that many solutions have been obtained for different aspects of moving surfaces. Takhar [4] obtained numerical solution to MHD asymmetric flow over a semi infinite moving surface. The problem of viscous variation for a moving flat plate in an incompressible fluid has been investigated by Pop et al. [5]. Noor Afzal [6] studied Momentum and transport on a continuous flat surface moving in a parallel stream. An analysis of heat and mass transfer characteristics in an electrically conducting fluid over a linearly stretching sheet with variable wall temperature was investigated by Vajravelu and Rollins [7]. B. S. Reddy, N. Kishan and Rajeshwar [8] discussed the problem MHD boundary layer flow of a non-newtonian power-law fluid on a moving flat plate. Kumari and Nath [9] presented the problem of MHD boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream. The similarity solution of thermal boundary layer for a moving flat plate was studied by Fang [10]. Seethamahalakshmi and G. V. Ramana Reddy [11] examined the problem of unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation. Kayvan and Mehdi [12] considered the local similarity solution for the flow of a "second-grade" viscoelastic fluid above a moving plate. Satya Sagar Saxena and G. K. Dubey [13] derived the Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion.

Jean-David Hoernel [14] investigated the similarity solutions for the steady laminar incompressible boundary layer governing MHD flow near forward stagnation-point of two-dimensional and axisymmetric bodies.

Mahmoud and Mahmoud [15] derived the analytical solutions of hydromagnetic boundary-layer flow of a non-Newtonian power-law fluid past a continuously moving surface. Recently, Patel and Timol [16] investigated the class of similarity equations for MHD boundary layer flow of viscous fluids part a moving continuous flat plate.

In this paper the one parameter group of transformation method is applied to derive the similarity analysis of steady, two dimensional laminar boundary layer flows over a continuously moving plate.

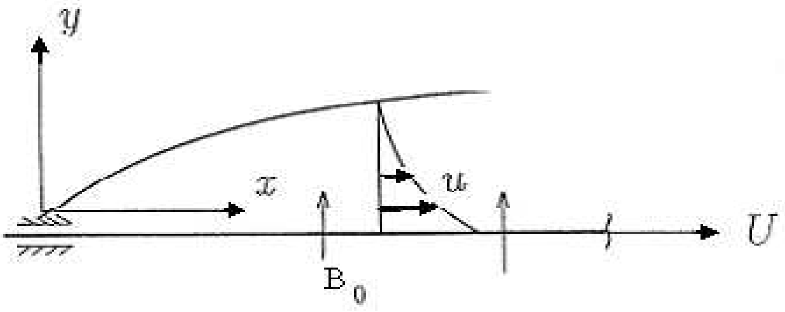

Figure 1: Physical Model of the problem

2. GOVERNING EQUATIONS:

Consider the steady, two-dimensional flow of an electrically conducting, viscous, incompressible fluid past a continuously moving surface in the presence of transverse magnetic field of strength B_0.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+S(x)u=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Where $S(x)=\dfrac{\sigma B_0(x)}{\rho}$

Together with the boundary conditions:

$$\begin{aligned} y=0: &\quad u=U(x),\ \ v=0 \\ y=\infty: &\quad u=0 \end{aligned} \tag{3}$$

Where,

X-Direction measured along the plate,

Y-Direction normal to the plate,

u : Velocity components of fluid in X direction

v : Velocity components of fluid in Y direction

ν -Magnetic viscosity

ρ - Fluid density

σ -Electrical conductivity

$B_0(x)$ – Magnetic field strength along the plate wall and acting perpendicular to it.

3. PROBLEM FORMULATION:

Introducing the stream function $\psi(x,y)$ such that $u=\dfrac{\partial\psi}{\partial y}$, $v=-\dfrac{\partial\psi}{\partial x}$ which satisfies the equation (1) identically.

Also the equations (2) and (3) becomes

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} + S(x)\frac{\partial \psi}{\partial y} = \nu \frac{\partial^3 \psi}{\partial y^3} \tag{4}$$

With the boundary conditions:

$$\begin{aligned} y=0: &\quad \frac{\partial \psi}{\partial y} = U(x), \quad \frac{\partial \psi}{\partial x} = 0 \\ y=\infty: &\quad \frac{\partial \psi}{\partial y} = 0 \end{aligned} \tag{5}$$

4. SIMILARITY ANALYSIS:

Similarity analysis by the group-theoretic method is derived from theory of continuous group transformations. Recently, this theory is found to give more adequate treatment of boundary layer equations (Refer Seshadri and Na [17]). Birkoff [18] was the first who introduced the basic concept of this method and later on number of authors like Hansen [19], Ames [20] and Moran [21] has contributed much to the development of the theory.

Our method of solution depends on the application of a one-parameter deductive group of transformation to the partial differential equations (4)-(5). Under this transformation the two independent variables (x, y) will be reduced by one i.e. η and the partial differential equation (4) along with the boundary conditions (5) will transforms into an ordinary differential equation.

Consider the one parameter group of transformation:

$$\begin{aligned} &x = A^{\alpha_1}\bar{x}, \quad y = A^{\alpha_2}\bar{y}, \quad \psi = A^{\alpha_3}\bar{\psi}, \\ &U = A^{\alpha_4}\bar{U}, \quad S = A^{\alpha_5}\bar{S} \end{aligned} \tag{6}$$

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ and A are constants.

Applying the above group transformation to equations (4)-(5) we get

$$A^{2\alpha_3-2\alpha_2-\alpha_1}\left(\frac{\partial \bar{\psi}}{\partial \bar{y}}.\frac{\partial^2 \bar{\psi}}{\partial \bar{y}\partial \bar{x}} - \frac{\partial \bar{\psi}}{\partial \bar{x}}.\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2}\right) + A^{\alpha_5+\alpha_3-\alpha_2} = A^{\alpha_3-3\alpha_2}\frac{\partial^3 \bar{\psi}}{\partial \bar{y}^3} \tag{7}$$

$$A^{\alpha_3-\alpha_2}\frac{\partial \bar{\psi}}{\partial \bar{y}} = A^{\alpha_4}\bar{U}(x), \quad A^{\alpha_3-\alpha_1}\frac{\partial \bar{\psi}}{\partial \bar{x}} = 0 \tag{8}$$

For equations (7)-(8) to be invariant, we must have

$$2\alpha_3 - 2\alpha_2 - \alpha_1 = \alpha_5 + \alpha_3 - \alpha_2 = \alpha_3 - 3\alpha_2$$

$$\alpha_3 - \alpha_2 = \alpha_4, \quad \alpha_3 = \alpha_1$$

Solving above equations we get $\alpha_1 = \alpha_3 = \alpha_4; \ \alpha_2 = \alpha_5 = 0$

The "absolute invariants" under the above group of transformation are

$$\eta = y; \ f(\eta) = \frac{\psi}{x}; \ \ U(x) = U_0 x; \ \ S(x) = S_0 .$$

Where U_0 and S_0 are constants.

Substituting these invariants in equation (4), we get the ordinary differential equation as follows

$$f'^2 - ff'' + S_0 f' - \nu f''' = 0 \tag{9}$$

With the transformed boundary conditions,

$$at \quad \eta = 0: \quad f' = U_0; \quad f = 0; \qquad at \quad \eta = \infty: \quad f' = 0 \tag{10}$$

CONCLUSION

Similarity analysis of MHD laminar boundary layer flow of viscous fluids over a continuously moving flat plate in the presence of transverse magnetic field is derived by one parameter group of transformation, which transformed the governing partial differential equations into ordinary differential equations. Present similarity equations are in well agreement to those available in literature.

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