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Modeling of compaction process of metallic powders using dimensionless analysis and singular value decomposition

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ABSTRACT

A novel approach of modeling using input-output experimental data pairs is presented for compaction energy and compact density percentage of powder. In this way, singular value decomposition (SVD) method is used in conjunction with dimensionless parameters incorporated in such complex process. The obtained model shows very good agreement with the testing experimental data pairs which have been unforeseen during the training process. The approach of this paper can be generally applied to model very complex real-world processes using appropriate experimental data.

Keywords: Modelling, compaction, powder, Dimensionless Analysis, SVD

INTRODUCTION

Shock consolidation is a technique that shows considerable promise for producing bulk material from powders. Rapid solidification technology is a rapidly advancing field by which unique

Microstructures of metallic alloys and ceramics are created. Shock consolidation is a process by which the particle surfaces are highly deformed producing inter-particle bonding in a one-step-process. Powders that cannot be conventionally compacted by powder metallurgical process, due to their high strength, and powders in which post compaction sintering has a deteriorating effect on the mechanical properties, can be compacted by shock pressures [1-2].

The detonation of a highly explosive charge can give rise to induced shock pressures as high as 900 Gpa, lasting for several microseconds [3-4]. The exact amplitude and duration of the explosive pressure pulse depend on the type and size of the explosive charge and the means by which the energy of the explosive is transmitted to the work piece. In fact, explosively induced shock waves have been successfully utilized to change the state and properties of materials and to weld, cut, form metals, in particular in the compaction of powder metals. The most common form of direct compaction apparatus is the collapsing cylinder press as shown in figure (1). The powder is placed inside a metal tube, which is plugged at both ends and surrounded by a uniform layer of explosive.

The explosive is detonated at one end, producing a converging cylindrical shock wave which collapses the metal tube and consolidates the powder. There are parameters of interest that affect the performance of explosive metallic powder compaction in terms of compaction energy (E) and compact density percentage (D_c). Such parameters are, namely, powder packing density (ρ), explosive charge thickness (T_e), wall thickness of the cylinder (T_w), mass ratio (R, explosive charge mass to cylinder mass), initial powder density percentage (D_t), explosive detonation wave velocity (V_d), and cylinder diameter (d). However, only some of these parameters as input variables have a significant effect on the output variables in the performance of the powder compaction process.

In this paper, it is shown that Dimensionless Analysis and Singular Value Decomposition can effectively model and predict the compaction energy and the compaction density percentage, each as a function of important input parameters in explosive compaction of metallic powder process.

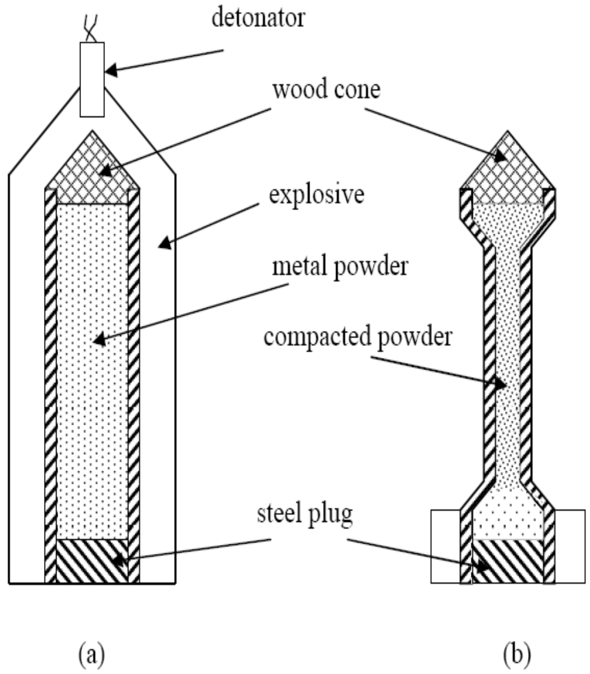

Figure 1: Explosive compaction assembly
(a) before detonation (b) during detonation

2-Mechanism of Explosive Compaction

In order to successfully compact powders using explosives, it is necessary to achieve a desired compact density without introducing some of the defects, such as melt holes and gross density variations. It has been suggested [2] that the compaction mechanism involves the transmission of a pressure pulse from an explosive charge causing densification of the powder mass. The experimental results obtained [4][5] have shown that the mechanism which depends only on the explosive detonation pressure and ignores the influence of the powder container tubes does not explain the densification of powders. On the basis of the experimental results, it is proposed that the sequence of events occurring, upon detonation of the explosive charge, is as follows. As the detonation front moves along the tube an oblique compression wave front is generated at the outer surface of the tube wall. The peak pressure of this pulse is determined by the explosive detonation velocity and the sonic impedance of the tube material. The length of the pulse depends on the thickness of the explosive charge. The pressure pulse will move to the inside surface of the tube wall where most of the energy of the pulse will be reflected back into the tube wall as a tensile pulse. The reflected tensile pulse will continue to be reflected from the tube surfaces, alternately in compression and in tension, until it is attenuated to zero. Thus, the shock pulse from the explosive charge will not be transmitted to the powder mass but will, instead, cause the rapid acceleration of the tube wall towards its axis. Those powder particles in contact with the tube wall will be accelerated inwards, causing impacts with adjacent particles. This will give rise to inter particle shearing, resulting in particles being broken up and oxide layers being ruptured. The cleaned particle surfaces will, thus, be capable of welding together, resulting in a dense, coherent mass of compact. The moving mass of container tube, compacted powder, and compacting powder particles decelerate from the initial high velocity of the container tube for several reasons. Plastic deformation of the container tube and the compacted ring of powder absorb energy, in addition to the energy absorption associated with void collapse and particle welding. The increasing mass of the moving compact also serves to reduce the velocity. Compaction will be completed when the velocity of the collapsing tube and powder is reduced to zero. The density achieved during this sequence of the events will depend on the energy initially possessed by the collapsing container tube, a fully dense compact resulting from the correct amount of energy. Too low a compaction energy results in compacts exhibiting central porosity, while an excess of energy gives rise to melting of the compact centre.

3- Experimental Assembly for Compaction of Cylindrical Specimens

Most of the explosive compaction experiments were carried out using the collapsing cylinder press arrangement. The tests made use of mild steel tubes of 26 mm internal diameter and machined from seamless tube. The wall thickness of the tubes was varied from 1mm to 4 mm. The length of the tube was 75 mm and closed at one end using a mild steel plug. The tube filled with metal powder and vibratory packed into the tube to a constant density. This density is referred to as the powder packing density. The value of such powder packing density is divided by the value of parent metal density which is known as the initial density percentage (a dimensionless parameter) in this work. The top plug was then inserted and the container tube assembly was centrally located inside a PVC cylinder. The annulus between the container tube and the PVC cylinder was filled with explosive. To obtain a uniform detonation front a wooden cone was glued to the top plug of the container tube, as shown in figure (1). Detonation of the charge took place in air with the assembly resting on soft sand. The detonation front, traveling along the length of the tube caused the collapse of the container tube, resulting in densification of the metal powder. The details of compact density measurements and detonation velocity measurements together with the procedure of computing the uncorrected compaction energy and the corrected compaction energy in association with the amount of absorbed energy of the tube during the plastic deformation have been comprehensively discussed in references [2][4][5].

4- Dimensionless modelling of compaction energy and compact density percentage using singular value decomposition (SVD)

The formal definition of modeling is to find a function $\hat{f}$ so that can be approximately used instead of actual one, f, in order to predict output $\hat{y}$ for a given input vector $X = (x_1, x_2, x_3, ..., x_n)$ as close as possible to its actual output *y*. Therefore given *M* observation of multi-input-single-output data pairs so that

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \; i = 1,2,...,M \, , \tag{1}$$

it is now possible to obtain $\hat{f}$ to predict the output values $\hat{y}_i$ for any give input vector

$$X_i = (x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \, , \tag{2}$$

such that

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \; i = 1,2,...,M \, . \tag{3}$$

The problem is now to determine $\hat{f}$ so that the square of different between the actual output and the predicted one is minimized, i.e.

$$\sum_{i=1}^{M} [\hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) - y_i]^2 \rightarrow Min. \tag{4}$$

In dimensionless modeling, however, a dimensionless set, $\pi = \{\pi_1, \pi_2, \pi_3, ..., \pi_k\}$, rather than the set of real physical variable $\{y, X\} = \{y, x_1, x_2, x_3, ..., x_n\}$, is used to obtain $\hat{f}$, i.e.

$$\hat{\pi}_{1i} = \hat{f}(\pi_{2i}, \pi_{3i}, \pi_{4i}, ..., \pi_{ki}) \; i = 1,2,...,M \tag{5}$$

such that

$$\sum_{i=1}^{M} [\hat{f}(\pi_{1i}, \pi_{2i}, \pi_{3i}, ..., \pi_{ki}) - \hat{\pi}_{oi}]^2 \rightarrow Min \, . \tag{6}$$

In order to construct such independent dimensionless parameters in the case of modeling of corrected compaction energy (E), powder packing density (ρ), explosive charge thickness (T_e), wall thickness of the cylinder (T_w), mass ratio (R, explosive charge mass to cylinder mass), initial powder density percentage (D_t), and detonation wave velocity (V_d) have been considered as input parameters [6]

In order to use SVD to obtain the model, that is[7,8]

$$E = f(\rho, T_e, T_w, R, D_t, V_d) \tag{7}$$

From this set of inputs-output parameters, 4 independent dimensionless parameters have been constructed according to 3 main dimensions (M, L, T), as follows

$$\pi_1 = \frac{E}{\rho V_d^2} \, , \tag{8 - a}$$

$$\pi_2 = \frac{T_e}{T_w} \, , \tag{8 - b}$$

$$\pi_3 = R \,, \tag{8 - c}$$

$$\pi_4 = D_t \,, \tag{8 - d}$$

so that

$$\pi_1 = f\,(\pi_2, \pi_3\, \pi_4) \tag{9}$$

Equation (9) can be represented as

$$\pi_1 = C(\pi_2)^{\alpha}(\pi_3)^{\beta}(\pi_4)^{\gamma} \tag{10}$$

Therefore, the problem of modeling is now to find coefficients *C*, α, β, and γ so that equation (6) is satisfied. By using natural logarithm, equation (10) can be represented as a linear relation with respect to the coefficients $(\eta = LnC)$, (α), (β) and (γ) as

$$Ln(\pi_1) = \eta + \alpha Ln(\pi_2) + \beta Ln(\pi_3) + \gamma Ln(\pi_4) \tag{11}$$

Consequently, a system of *M* Linear algebraic equation with *K*=4 unknown of the above mentioned coefficients is now constructed based on *M* input-output experimental data pairs as follows [9,10]

$$\begin{cases} \eta + \alpha\zeta_{12} + \beta\zeta_{13} + \gamma\zeta_{14} = \zeta_{11} \\ \eta + \alpha\zeta_{22} + \beta\zeta_{23} + \gamma\zeta_{24} = \zeta_{21} \\ \vdots \\ \eta + \alpha\zeta_{M2} + \beta\zeta_{M3} + \gamma\zeta_{M4} = \zeta_{M1} \end{cases} \tag{12}$$

where

$$\zeta_{ij} = Ln(\pi_{ij})\,,\; i = 1,2,\ldots,M \;, j=1,2,3 \tag{13}$$

and

$$\zeta_{i1} = Ln(\pi_{i1})\,.\; i = 1,2,\ldots,M \tag{14}$$

Such system of linear equations in which M>>*K*=4 can be represented as:[11]

$$AX = Y\,, \tag{15}$$

where

$$X = [\eta \;\; \alpha \;\; \beta \;\; \gamma]^T\,, \tag{16}$$

$$Y = [\zeta_{10} \;\; \zeta_{20} \;\; \cdots \;\; \zeta_{M0}]^T\,,$$

and

$$A = \begin{bmatrix} 1 & \zeta_{12} & \zeta_{13} & \zeta_{14} \\ 1 & \zeta_{22} & \zeta_{23} & \zeta_{24} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \zeta_{M2} & \zeta_{M3} & \zeta_{M4} \end{bmatrix} \tag{17}$$

The least-squares technique from multiple-regression analysis leads to the solution of the normal equation in the form of

$$X = (A^T A)^{-1} A^T Y\,, \tag{18}$$

which determines the vector of the best $k = 4$ unknown of equation (10) for the whole set of *M* experimental observation data. However, such solution directly form solving normal equations (18) is rather susceptible to round off error and, more importantly, to the possible singularity of these equations. Therefore, SVD is used to solve equation (15) which leads to better results in comparison with those of using equation (18).

SVD is the method for solving most linear least-squares problems that some singularities may exist in the normal equations. The SVD of a matrix, $\mathbf{A} \in \Re^{M \times K}$, is a factorization of the matrix into the product of three matrices, column orthogonal matrix $\mathbf{U} \in \Re^{M \times K}$, diagonal matrix $\mathbf{W} \in \Re^{K \times K}$ with non-negative elements (singular values), and orthogonal matrix $\mathbf{V} \in \Re^{K \times K}$ such that

$$\mathbf{A} = \mathbf{U}\mathbf{W}\mathbf{V}^T \tag{19}$$

The most popular technique for computing the SVD was originally proposed in [12]. The problem of optimal selection of vector of the coefficients in equation (15) and (18) is firstly reduced to the modified inversion of diagonal matrix **W** [13] in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal *X* are obtained using the following relation

$$X = \mathbf{V}\left[diag\left(\frac{1}{w_j} \right) \right] \mathbf{U}^{\mathbf{T}} \mathbf{Y}\,. \tag{20}$$

In order to demonstrate the prediction ability of SVD in such dimensionless modeling, the data have been divided into two different sets, namely, training and testing sets. The training set, which consists of randomly chosen N_t input-output data pairs, is used for training the $K = 4$ unknown coefficients involved in the dimensionless model of deflection-thickness ratio. The testing set, which consists of N_P unforeseen input-output data samples during the training process, is merely used for testing to show the prediction ability of the obtained simple model.

In order to model, based on experimental data presented in Table (1), the multi-input-single-output set of constructed dimensionless data according to equations (8-a) to (8-d) which are for corrected compaction energy.

The corresponding values of parameters are found as $C = 38.5$, $\alpha = 0.0765$, $\beta = 0.76$, and $\gamma = -0.423$. Hence, the model can now be given as

$$\left(\frac{E}{\rho {V_d}^2}\right) = 38.5\left(\frac{T_e}{T_W}\right)^{0.076} .(R)^{0.76} .(D_t)^{-0.423} \tag{21}$$

Figure (2) shows the comparison of (E) given by equation (21) with respect to the experimental values both for training and testing data sets. It is evident from this figure that equation (21) predicts the midpoint deflection-thickness ratio successfully for the testing data

Similarly, in order to construct such independent dimensionless parameters in the case of modeling of compact density percentage (D_c), cylinder diameter (d), explosive charge thickness (T_e), wall thickness of the cylinder (T_w), mass ratio (R, explosive charge mass to cylinder mass), initial powder density percentage (D_t), detonation wave velocity (V_d), and sound velocity in air have been considered as input parameters in neural network, that is

$$D_c = f\,(d, T_e, T_w, R, D_t, V_d, V_s) \tag{22}$$

From this set of inputs-output parameters, 5 independent dimensionless parameters have been constructed according to 3 main dimensions (M, L, T), as follows

$$\pi_1 = D_c\ , \tag{23-a}$$

$$\pi_2 = \frac{{T_e}^2}{T_w d}\ , \tag{23-b}$$

$$\pi_3 = R\ , \tag{23-c}$$

$$\pi_4 = \frac{V_d}{V_s}\,, \tag{23-d}$$

$$\pi_5 = D_t \tag{23-e}$$

so that

$$\pi_1 = f(\pi_2, \pi_3 \pi_4 \pi_5). \tag{24}$$

equation (24) can be represented as

$$\pi_1 = C(\pi_2)^{\alpha}(\pi_3)^{\beta}(\pi_4)^{\gamma}(\pi_5)^{\delta} \tag{25}$$

In order to model, based on experimental data presented in Table (1), the multi-input-single-output set of constructed dimensionless data according to equations (23-a) to (23-d) which are for compact density percentage.

Table 1: Inputs-outputs data of explosive compaction of metallic powders[5]

	INPUTS							OUTPUTS	
TEST NO.	Powder Packing Density kg/m^3	Explosive Wave Velocity km/s	Explosion Layer Thickness mm	Diameter of Cylinder mm	Cylindrical Wall Thickness mm	Explosive and Cylinder Mass Ratio	Initial Density Percentage	Corrected Compaction Energy Mj/m^3	Compact Density Percentage
1	3370	1.66	6	26	4	0.28	37.6	21.2	47.4
2	3370	1.48	4.75	26	2	0.42	37.6	32.2	55.5
3	3370	1.63	5.75	26	1	1	37.6	77	75
4	3370	2	8.45	26	2	0.81	37.6	94	86.9
5	3370	2.67	13.2	26	2	1.42	37.6	301.2	98.5
6	3568	1.48	4.75	26	2	0.4	45	28.5	59.8
7	3568	1.87	7.5	26	2	0.7	45	79.4	---
8	3568	2.26	10.25	26	2	1.05	45	168	95.5
9	3568	2.67	13.2	26	2	1.42	45	310	95.5
10	4410	1.66	6	26	2	0.54	56	73.5	69.8
11	4410	2.68	13.3	26	2	1.44	56	326	88.4
12	4410	3.84	21.5	26	2	2.76	56	982	96.4
13	4230	1.8	7	26	2	0.62	47.5	64.2	73.7
14	4230	1.87	7.5	26	1.5	0.86	47.5	87.9	81.8
15	4230	1.94	8	26	1	1.35	47.5	134.2	86
16	4230	2.71	13.5	26	1.5	1.81	47.5	356.6	96.7
17	4230	3.06	16	26	1.5	2.28	47.5	555.8	98.1
18	4560	1.48	4.75	26	2	0.4	51.2	40.5	67.6
19	4560	2.22	10	26	2	1	51.2	161.2	94
20	4560	2.92	15	26	2	1.69	51.2	421.4	98.1
21	3370	1.7	6.3	26	2	0.57	37.6	52.5	67
22	3370	2.3	10.5	26	2	1.06	37.6	167.2	98
23	3568	1.7	6.25	26	2	0.565	45	53.5	74.2
24	3568	2	9	26	2	0.87	45	119.3	92.6
25	4410	1.8	7	26	2	0.65	56	95.8	72.5
26	4410	2.26	10.25	26	2	1.05	56	197.5	82.2
27	4410	3.06	16	26	2	1.84	56	507	91
28	4230	2.3	10.5	26	2	1.01	47.5	160.6	91.3
29	4230	3	15.5	26	2	1.69	47.5	421.5	97.7
30	4560	2.22	10	26	2	1	51.2	160	92.8
31	3370	2.64	13	26	2	1.39	37.6	285.2	---
32	3568	1.81	7.1	26	2	0.66	45	72	82.2
33	4230	2.64	13	26	2	1.34	47.5	265.1	94.7
34	4560	1.8	7	26	2	0.646	51.2	72	81
35	4410	2.08	9	26	2	0.87	56	144.8	80.1

In order to model, based on experimental data presented in Table (1), the multi-input-single-output set of constructed dimensionless data according to equations (23-a) to (23-d) which are for corrected compaction energy.

The corresponding values of parameters are found as $C = 0.3758$, $\alpha = 0.5845$, $\beta = -0.815$, and $\gamma = -1.0823$. Hence, the model can now be given as

$$(D_C) = 0.2\left(\frac{T_e^{\,2}}{T_W d}\right)^{-1.12} .(R)^{1.7}.(\frac{V_d}{V_S})^{-0.58}(D_t)^{-0.04} \tag{26}$$

Figure (3) shows the comparison of (D_C) given by equation (26) with respect to the experimental values both for training and testing data sets. It is evident from this figure that equation (26) predicts the midpoint deflection-thickness ratio successfully for the testing data

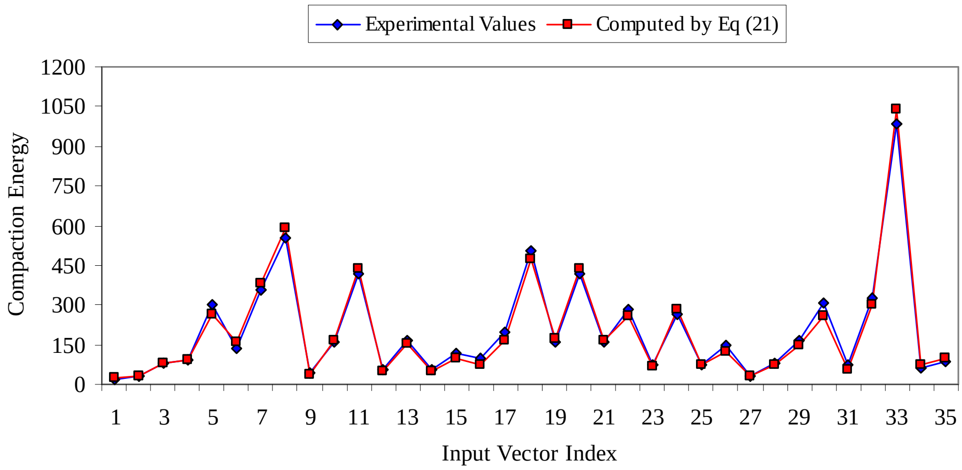

Figure 2: Variation of corrected compaction energy with input data samples: comparison of experimental values with computed values (Eq. 21)

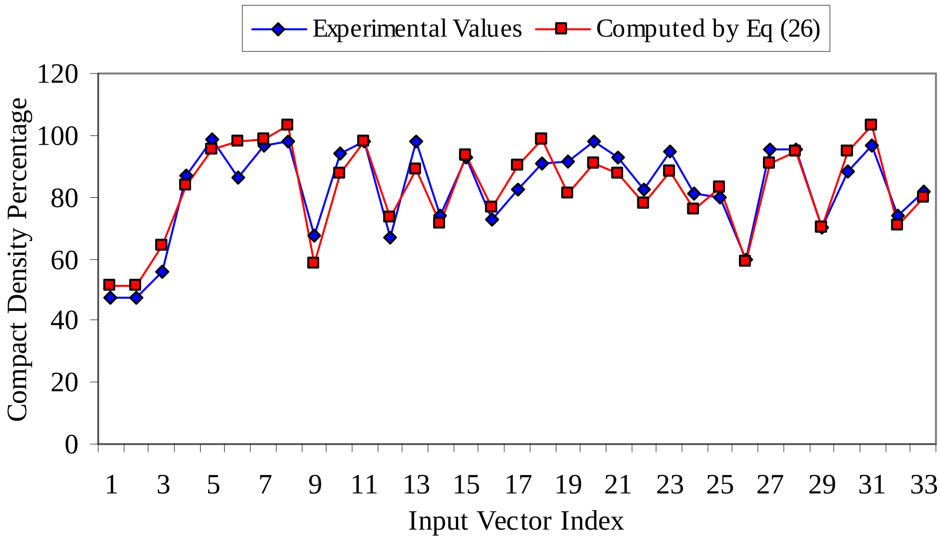

Figure 4: Variation of compact density percentage with input data samples: comparison of experimental values with computed values (Eq. 26)

CONCLUSION

Singular value decomposition and dimensionless analysis have been used to model the compaction powder using some experimental input-output data. It has been shown that the simple obtained models can successfully predict the compaction energy and compact density percentage compared with the actual experimental values. The methodology of this paper can be readily applied to find simple closed-form equations of complex real-world processes where some experimental input-output data pairs are available.

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REFERENCES

[1] Meyers, M.A., and Wang, S.L., *Acta metal*, **1988**, 36, 4.
[2 Shakeri, M., " Impact Mechanics", Vol. 3, (in Persian), Amir Kabir University Press, **2001**.

[3],Linse, D., Proc. 1st Workshop on Shock processing of Powders, New Mexico, **1988**.
[4] Jamei, M., MSc. Thesis, Tehran University, **2001**.
[5] Bierd, M., PhD Thesis, Queen's University of Belfast, **1979**.
[6] Gharib,M., Saba,H.R., Barazesh, A , *European Journal of Experimental Biology*, **2012**, 2 (4):1034-1038.
[7] Osei D. Y. and Jackson E.N. *Advances in Applied Science Research*, **2012**, 3 (6):3658-3662
[8] Ayub,S and Saini, J.P., *Advances in Applied Science Research*, **2010**, 1 (2): 76-83
[9] Olaniyi S Maliki, Anthony O Agbo, Adeola O Maliki, Lawrence M Ibeh and Chukwuemeka O Agwu, *Advances in Applied Science Research*, **2011**, 2 (5):329-339
[10] Arulmozhi, K.T. and Sheelarani, R. , *Advances in Applied Science Research*, **2011**, 2 (4):147-155.
[11] Majid Yazdani1, Ali Mohammad Borghaee, Shahin Rafiee, Saeid Minaei and Babak Beheshti, *European Journal of Experimental Biology*, **2013**, 3(3):712-721
[12] Golub, G.H. and Reinsch, C., *Singular value decomposition and least squares solutions, Numer. Math.*, **1970**, 14 (5), 403–420.
[13] Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., *Numerical Recipes in FORTRAN*, The Art of Scientific Computing, 2nd Edition, Cambridge University Press, Cambridge, **1992**.