Hybrid approach for solving TSP by using DPX Cross-over operator

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ABSTRACT
The purpose of this paper is to present a new development in hybrid GA approach for solving TSP by using DPX cross-over operator. The strategy of algorithm is to implement and extend the successful results of genetic algorithm (GA) by using the concept of cross-over operator (DPX). The proposed algorithm is basically the combination of assignment algorithm and hybrid GA operator. This combination facilitates in finding quality solutions for TSP problems with lower solution complexity.

INTRODUCTION
Travelling salesman problem (TSP) is the most studied problem for finding optimal solution. Given a graph G with n number of cities, the objective is to find a close tour that visits each city once returning to the starting city. As far as the heuristic approach is concerned, TSP has provided many algorithms for finding near optimal solutions for symmetric as well as asymmetric TSP. The most common techniques are simulated annealing [8], threshold accepting [9], ant colonies [6], genetic algorithm [10], 2-opt, 3-opt and k-opt heuristic approach. These methods are very useful whenever applied individually. But the best results have been obtained from the hybrid approaches, especially for the large TSP problems. For example the tour of optimum or close to optimum results have been found when the local search techniques and genetic operators come across [1] [2]. This paper presents a hybrid GA approach for solving TSP by incorporating problem dependent knowledge into a genetic algorithm. Recall the Assignment problem (AP) is the problem of assigning n jobs against minimum cost, given a matrix C. Each
employee is only allowed to perform one job. The assignment of employee i to job j is denoted by the arc (i,j) and has the cost c(i, j). If A is the cost of the assignment, then the solution of the AP can be represented as a set of cycles, and the AP solution is also called the minimum cycle cover. Here the DPX cross over operator is applying on these minimum cycles. This combination of local search heuristics and genetic operators promises to be an approach that finds best quality of near optimum solution to the TSP while lowering the computational time required by the traditional approaches.

This paper organizes as follows. Section 1 gives the introduction, section 2 describes the hybrid GA approach, section 3 gives the main algorithm, section 4 gives the computational results, section 5 is the conclusion and section 6 is references.

Hybrid GA approach:
Mostly the hybrid GA approaches for TSP, produce near optimal solutions. Actually this approach provides some specific operators which improve the results of such problems [11]. Although the provided GA approach in this paper is based on much work done by previous researchers [1] [4] [11], this paper provides additionally a fittest criteria and the application of DPX crossover on the sub-tours of the assignment problem( cycles obtained after assignment procedure) and mutation operator to get lower computer complexity. Many researchers had proposed various techniques in hybrid GA to solve TSP, which starts with the configuration of n cities that develops the initial tour by using nearest neighbor-hood search technique. Then 2- opt local search method is applied to each individual to get the local optima. Although this 2- opt local search is not that much effective as compared to 3- opt, also Lin-Kernighar heuristic (LK) has been proven much more efficient for the larger cities [2].

The Main Algorithm:
The main steps of the proposed hybrid GA techniques are as follows:
• Generate the initial population as the random combination of the sub tours (cycles) that are obtained after applying the assignment procedure. It is the well known fact that the value of assignment problem is the lower bound of the optimal TSP and the optimal path is an arrangement of these sub tours at its least possible cost. Therefore, pattern of these sub tours (cycles) in the initial tour is very important.
• Apply the fittest criteria given by (1). If the value of the fitness function is close to 1 then the related parent population will be selected for applying the DPX cross-over operator, Also the specific mutation operator is applied until required fitness is achieved.

Generally, a solution to AP (Assignment procedure) produces a number of sub tours (cycles) [10]. Let the ith of these sub tours be called $S_i$

Then construct the initial population by the random combination of these sub tours (cycles) and calculate its fittest value by using the fitness function defined as.

$$F(t) = \frac{\text{value of the assignment of the given problem}}{\text{value of the string}}$$  \hspace{1cm} (1)
Select the parent tours with values close to 1, apply the DPX cross-over operator on the selected tours as,

According to the DPX cross-over operator, all the edges which are not common in the other parents are then removed. The off-spring is then left with different sized “chunks” or city segments, which are actually the assignment sub-tours that are common in both the parents, these broken edges are then recombined without replacing any edge originally broken. The original DPX operator reconnects remaining edges using a greedy procedure.

For example consider two parent tours,
1-7-2-6-4-3-5-1
1-3-5-2-6-4-7-1

Fragments left after removal of non-common edges,
(1)(7)(2-6-4)(3-5)

After the DPX cross-over the off-spring will become,
1-2-6-4-5-3-7-1

This cross-over operator is very successful because it passes the most important information (cities' segment) from parent to child.

Mutation:
In order to get global optimum, mutation operator is usually applied. Several mutation operators are being used to improve the tour. According to the proposed algorithm the mutation is to be done by applying the procedure of rearrangement of the cities lying in the broken segments. Whichever combination has the shorter tour is kept, while the other is discarded as shown below,
1-7-5-3-2-6-4-1

After mutation the resultant tour will become,
1-7-3-5-2-6-4-1

Computational Results

<table>
<thead>
<tr>
<th>Problem</th>
<th>DPX-GA with LK best</th>
<th>DPX 2-opt best</th>
<th>Hybrid GA 2-opt best</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eli51.tsp</td>
<td>426 (0.0%)</td>
<td>430 (0.46%)</td>
<td>428 (0.46%)</td>
</tr>
<tr>
<td>kroA100.tsp</td>
<td>21282 (0.0%)</td>
<td>21285 (0.01%)</td>
<td>21285 (0.01%)</td>
</tr>
<tr>
<td>D198.tsp</td>
<td>15780 (0.0%)</td>
<td>15788 (0.0%)</td>
<td>15839.5 (0.1%)</td>
</tr>
<tr>
<td>Lin318</td>
<td>422029 (0.0%)</td>
<td>422463 (0.5%)</td>
<td>42605.3 (0.7%)</td>
</tr>
</tbody>
</table>
Table 3: Symmetric Travelling Salesman Problem Average Results

<table>
<thead>
<tr>
<th>Problem</th>
<th>DPX-GA with LK average</th>
<th>DPX 2-opt average</th>
<th>Hybrid GA 2-opt average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eli51.tsp</td>
<td>430</td>
<td>430</td>
<td>428.5</td>
</tr>
<tr>
<td>kroA100.tsp</td>
<td>21282</td>
<td>21285</td>
<td>21285</td>
</tr>
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<td>Lin318</td>
<td>422029</td>
<td>422463</td>
<td>42605.3</td>
</tr>
</tbody>
</table>

Table 4: Symmetric Travelling Salesman Problem using assignment cycles and DPX cross over

<table>
<thead>
<tr>
<th>Problem</th>
<th>Average Results</th>
<th>Best Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eli51.tsp</td>
<td>426</td>
<td>426</td>
</tr>
<tr>
<td>kroA100.tsp</td>
<td>21283</td>
<td>21282</td>
</tr>
<tr>
<td>D198.tsp</td>
<td>15782</td>
<td>15781</td>
</tr>
<tr>
<td>Lin318</td>
<td>422033</td>
<td>422030</td>
</tr>
</tbody>
</table>

Above Computational results shows the comparison with [1][2], [11] on the results taken from TSPLIB. According to the Table 1, it can be easily seen that the hybrid GA approach can produce quality results for small and medium cities. Table 2.3 shows the comparison between the DPX 2-opt cross-over GA and hybrid GA 2-opt-procedure and the third approach is the DPX-GA with LK procedure [11]. Two set of results are shown to note the difference. Table 2 and 3 shows the comparison between the best and average results of DPX with LK [11], DPX 2-opt and Hybrid GA with 2-opt. The comparison shows that the difference between the algorithms is the selection process for city recombination during cross-over. Table 4 shows the results that are obtained from the algorithm proposed in this paper that is the application of DPX cross-over on the assignment sub tour. These results are much better than the previous results except for DPX with LK. In this way it can be conclude that the proposed DPX can produce better results as compare to different approaches.

CONCLUSION

This paper has provided the new Hybrid approach with DPX cross-over operator applied on the cycles that are obtained from the assignment algorithm for solving the TSP problem. This method is the combination of assignment cycles and the hybrid GA DPX cross-over operator which has improved the efficiency of algorithm also, the mutation criteria is used to get the global optima. The comparison of computational result shows that the proposed algorithm can produce much improved quality results then other described techniques.

REFERENCES