Evaluation of Dynamic Performance in Terms of Effluent COD and Biomass Concentrations of UASB Reactor Treating Low Strength Wastewater

Vidya Singh*, R. P. Singh and N. D. Pandey

1Department of Chemistry, M.N.N.I.T Allahabad -211004, U.P, India
2Department of Civil Engineering, M.N.N.I.T Allahabad -211004, U.P, India

ABSTRACT

The present paper devoted to explore the suitability of using a simple CSTR model for evaluating the dynamic performance of UASB reactor treating low strength wastewater. Depending upon the idealization of UASB reactor as a single CSTR, the dynamic state model equations available in the literature. The simultaneous dynamic equations for substrate and biomass mass were used to assess the UASB reactor performance of low strength wastewater. The dynamic model equations were solved by using a m.file in MATLAB2011a command window and dynamic equations for substrate and biomass. The objectives of this paper are to evaluate the dynamic performances of UASB reactor treating low strength wastewater using the experimental results of Singh and Viraraghavan 1998 research.

INTRODUCTION

The first study on a comprehensive mathematical modelling of UASB reactor appears to done by (Van Der Meer and Heertjes 1983). Certain silent features of the study are as follows: (1) The sludge bed and the sludge blanket are treated as two separate well mixed regions, (2) Microbial growth kinetics has been described by Monod kinetics (3) Growth and wash-out of sludge are neglected at stationary situations (4) No conversion of substrates takes place in the settler zone, (5) The rates of biomass growth are same in the bed as well as blanket. Low strength wastewater such as domestic sewage (COD concentration 500-1000 mg/L), are at present being treated anaerobically employing high rate anaerobic treatment system like UASB reactor. UASB reactor system is facing a challenge in the treatment of low-strength wastewater. The production and quality of granular sludge influenced by the composition of wastewater, reactor design and operating conditions. Moreover, the formation of granular sludge with good settling characteristics and activity is a critical factor in dealing with low-strength wastewater (Singh and Viraraghavan 1998). It is apparent that, although much attention has given to the biochemistry and physical characteristics of anaerobic digestion, no systematic study are available in literature regarding the mathematical modelling of the granule size variation in UASB reactor treating low strength wastewater.
wastewaters. Further, not much effort made in the literature towards the evaluation of UASB reactor performance treating different low strength wastewaters using mathematical modelling approach. Therefore in the present work, a simple CSTR model has been applied and tested for its suitability to assess the dynamic performance of UASB reactor treating low strength wastewaters. To date, a large number of experimental studies have been conducted at laboratory, pilot plants and full-scale levels to study the treatability of a variety of wastes using UASB reactor. However, very few of these have been subjected to mathematical modelling and simulation. Most of the simulation efforts made so far have been concentrated towards the simplest type of effluents such as acetic acid or mixed volatile fatty acids (mixture of acetic, propionic and butyric acids) or lumping of all the volatile fatty acids into equivalent acetic acids. Little or no efforts are made till date to model the performance of UASB reactors treating low strength or municipal wastewaters, where granulation is difficult or achieved after a prolonged start-up. It is imperative that data pertaining to UASB reactor should be modeled so that a better insight can be obtained into the performance of UASB reactors treating low strength wastewaters. A low-strength wastewater such as municipal wastewater or domestic wastewater sewage has COD concentration in the range of 500-1000 mg/L. UASB reactor has been worldwide applied recently for treatment of low strength wastewaters during past 2 to 3 decades (Ligero and Soto 2002; Álveraz et al. 2006; Singh and Viraraghavan 1998; Das and Chaudhari 2009; Turkdogan-Aydinol et al. 2011; Lohani et al. 2015)\(^{2,5,6,7,12,13,16,19}\). Several attempts have been made in the past to the accelerate the granulation phenomenon in treatment of low strength wastewaters (Jeong et al. 2005; Sondhi et al. 2010)\(^{10,17}\). Some excellent experimental works on acceleration of the start-up period in treatment of low strength wastewater by UASB reactor are well reported in the literature (Jeong et al. 2005)\(^{10}\). However, very few efforts are made towards the modelling and assessment of dynamic performances of UASB reactor treating low strength wastewaters (Agrawal et al. 1997; Alveraz et al. 2008; Kalyuzhnyi et al. 2006; Singh and Viraraghavan 1998; Turkdogan-Aydinol et al. 2011)\(^{1,3,11,16,19}\).

Based on the above mentioned facts, the main objectives of the present paper are: (1) to evaluate the kinetic constants for UASB reactor treating low strength wastewaters idealizing flow regime of UASB reactor as CSTR. (2) to evaluate the dynamic performance of the UASB reactor treating low strength wastewaters using Monod kinetics for microbial growth and MATLAB2011a, ode15s tool. The present paper devoted to explore the suitability of using a simple CSTR model for evaluating the dynamic performance of UASB reactor treating municipal wastewater. Depending upon the idealization of UASB reactor as a single CSTR, the dynamic state model equations available in the literature. In case of treatment of low strength wastewaters (Singh and Viraraghavan 1998)\(^{16}\) where the stoichiometric relationships are not very clearly known/ available from literature, the simple model equations are derived for effluent waste COD and biomass concentrations. Determination of kinetic constants for low strength wastewater treatment in UASB reactor is necessary to predict the dynamic performances of the UASB system. Therefore, the kinetic constants (k, Ks, \(\mu_{\text{max}}\), Y and \(K_d\)) were determined using experimental results of (Singh and Viraraghavan 1998)\(^{16}\) treating municipal wastewater in UASB reactor.

**MATERIAL AND METHODS**

The rate of change of substrate and biomass in the system can be expressed as follows: ([Lokshina et al. 2000; Sponza 2001; Işik and Sponza 2005; Huang et al., 2006; Basu and Asolekar 2012; Yetilmezsoy 2012; and Rodríguez-Gómez et al. 2014])\(^{4,8,9,14,15,18,21}\).

\[
\text{Net rate of accumulation of substrate within the reactor} = \text{input} - \text{output} - \text{rate of substrate consumption in the reactor.}
\]

Mathematically, the mass balance equations on substrate and microorganisms given as Eqs. 1 and 2 simultaneously.
\[
\frac{dS_e}{dT} = \frac{(S_o - S_e)}{\theta} - \frac{k.S_e.X_e}{(K_s + S_e)}
\]

(1)

\[
\frac{dX_e}{dT} = \frac{(X_o - X_e)}{\theta} + \left( \frac{Y.k.S_e.X_e}{(K_s + S_e)} - K_d.X_e \right)
\]

(2)

If \(X_o\) is treated negligible \((X_o=0)\), then Eq. (2) can be written as

\[
\frac{dX_e}{dT} = \left( \frac{-X_e}{\theta} \right) + \left( \frac{Y.k.S_e.X_e}{(K_s + S_e)} - K_d.X_e \right)
\]

(3)

The simultaneous dynamic equations for substrate and biomass were solved to assess the UASB reactor performance. The dynamic model equations were solved by developing a m.file in MATLAB2011a command window and writing the dynamic equations for substrate and biomass. Then, the experimental results of (Singh and Viraraghavan 1998)\(^\dagger\) were entered into Microsoft Excel Sheet and the file was imported by ‘xlsread’ tool in MATLAB2011a. By using, the initial conditions and the kinetic constants were programmed in m.file in MATLAB2011a. Programmed file, Excel sheet and equations of substrate and biomass m.file must be present in the same path of the system. Programmed m.file was then run by using ode15s tool of MATLAB2011a and the solutions were obtained in command window of MATLAB2011a.

RESULTS AND DISCUSSION

Determination of kinetic parameters

In order to obtain the kinetic constants to be used in assessment of UASB reactor performance, the linear equations (4) and (5) were used, which are given below when influent biomass concentration \((X_o)\) is negligible. The kinetic constants \(k\) and \(K_s\) were determined from the slope and intercept of the straight line plot between \(0.X_e/(S_o - S_e)\) and \(1/S_e\) as per Eq. (4). Other kinetic constants \(Y\) and \(K_d\) were determined from the slope and intercept of straight line plot between \((S_o - S_e)/ X_e \) and \(\theta\) as per Eq. (5).

The kinetic parameters were determined using experimental results of (Singh and Viraraghavan 1998)\(^\dagger\) for treatment of low strength wastewaters. The experimental results of (Singh and Viraraghavan 1998)\(^\dagger\) for the treatment of municipal wastewater at eight different HRTs \((2, 1.66, 1.5, 1.33, 1.04, 0.83, 0.66 \text{ and } 0.41 \text{ days})\) for 280 days of the operation period in lab scale UASB reactor were used. During operation at HRT of 0.66 \(d\) and 0.41 \(d\), a large fluctuation in the influent and effluent COD concentrations were observed. Due to this reason, the last two data points corresponding to HRT of 0.66 \(d\) and 0.41 \(d\) were ignored in linear fitting of Eqs. (4) and (5) as shown in figures 1 and 2. In order to proceed with the simulation of UASB reactor performance data, it is necessary to evaluate the kinetic constants, i.e., maximum substrate utilization rate \((k)\) and half saturation constant \((K_s)\), biomass yield coefficient \((Y)\) and decay coefficient \((K_d)\). On the basis of the principles of ideal CSTR assumption without sludge recycle \((\text{HRT} = \text{SRT})\) and the following linear expressions can be obtained to evaluate the kinetic constants and re-written as (Matcalf and Eddy 1997).

\[
\frac{\theta.X_e}{(S_o - S_e)} = \frac{K_s}{k}.\frac{1}{S_e} + \frac{1}{k}
\]

(4)

\[
\frac{(S_o - S_e)}{X_e} = \frac{K_d}{Y}.\theta + \frac{1}{Y}
\]

(5)

Further, for the steady state condition when \(X_o\) taken into account the linear expressions represented by Eqs. (4) and linear Eq. (6) as given below were used to evaluate the kinetic constants.

\[
\frac{(X_o - X_e)}{\theta.X_e} = \frac{K_d}{Y}.\frac{(S_o - S_e)}{X_e} + K_d
\]

(6)

Where, \(\theta\) is the hydraulic retention time \((d)\) and SRT is the solid retention time \((d)\). Using linear regression of the experimental data
and using Eqs. (4) and (5), the kinetic parameters are determined. The kinetic constants $k$ and $K_s$ were determined from the slope and intercept of straight-line plot shown in figure 1 and the biomass yield coefficient ($Y$) and microorganism’s decay coefficient ($K_d$) were determined from the slope and intercept of straight line plot shown in figure 2.

The values of the kinetic constants are given in Table 1 later. When the influent biomass concentration ($X_o$) is taken into account, the values of kinetic constants $k$ and $K_s$ were obtained from Eq. (1) as shown in figure 1.

In order to evaluate the kinetic constants $Y$ and $K_d$ when influent biomass concentration ($X_o$) is accounted, the linear plot as per Eq. (6) is shown below in figure 3 with poor $R^2$-value of 0.681. The evaluated values of kinetic constants i.e., $Y$ and $K_d$ are presented in Table 1, when $X_o$ is accounted.

The kinetic constants were also reported by (Singh and Viraraghavan 1998) and are reproduced in Table 1.

The kinetic constants evaluated from all these figures are presented in Table 1.


(Singh and Viraraghavan 1998) studied the start up and operation of UASB reactor at 20 $^\circ$C treating low strength wastewater. Two polyacrylic laboratory scale UASB reactor of capacity 8L were used. Each reactor was seeded with digested sludge from an anaerobic digester operating at regina wastewater treatment plant. The soluble COD concentration of influent was in the range of 150-300 mg/L. Reactors were operated in a continuous mode and HRT was reduced in steps (2, 1.66, 1.5, 1.33, 1.04, 0.83, 0.66 and 0.41 days) for 280 days of the operation period. COD loading rate was varied from 0.30-1.05 kg COD/m$^3$.d and HRT was reduced from 1.66 days to 0.41 days. In experimental results of (Singh and Viraraghavan 1998), the dynamic period is about 140 days as evident from the results. The experimental results of (Singh and Viraraghavan 1998) used for evaluation of dynamic performance at 10 days time step of UASB reactors are given in Table 2.

Evaluation of dynamic performance using experimental results of Singh and Viraraghavan (1998) when influent biomass concentration ($X_o$) is negligible

Using the experimental results of (Singh and Viraraghavan 1998) on treatment of the low strength wastewater from Table 2 for a transient period of 140 days and kinetic constants from Table 1, the dynamic equations (1) and (3) were solved simultaneously for effluent soluble COD concentration ($S_e$) and effluent biomass concentration ($X_e$) using MATLAB2011a, ode15s tool with a time step of 10 days. A large step size of 10 days was taken due to large fluctuations in the experimental results of (Singh and Viraraghavan 1998) during initial phases of operation.

**Prediction of dynamic performance in terms of effluent COD concentration ($S_e$)**

The results of simulations at 10 days intervals are presented in Table 2 for 140 days of operation. The percentage error in predicted and experimental effluent COD concentrations ($S_e$) are also computed and presented in table 3, which varies from 6% to 55.73%. On an average, the percentage error in predictions lie in the range of 30% to 40%. From the reported results of experimental effluent COD concentration in first 60 days of operation, it is evident that effluent COD was continuously decreased from 0.199g COD/L to 0.048 g COD/L and thereafter slightly increased to a level of 0.095 g COD/L at 140th day. A more or less similar behaviour is seen in the predicted $S_e$ values with large margin of error. Variation of predicted effluent soluble COD concentration and experimental effluent soluble COD concentrations as a function of operation time are shown in figure 4. From figure 4, it is evident that the predicted and experimental effluent COD concentrations do not agree well and a large deviation seen between them. However, the predicted $S_e$ values are slightly lower in comparison to observed $S_e$ values as seen from figure 4.
Since the maximum error in predictions being quite large (about 56%) and the results of simulations don’t agree well with experimental results the application of dynamic equations in simulation of dynamic performance of UASB reactor seems to be inappropriate with limited accuracy. Therefore, dynamic simulation using simple CSTR model is not suitable to simulate the effluent COD concentration in UASB reactor in present case considered.

Prediction of dynamic performance in terms of effluent biomass concentration (Xe)

Eqs. (1) and (3) were solved simultaneously for $S_e$ and $X_e$ as per procedure described in previous section using experimental results of (Singh and Viraraghavan 1998) as presented in Table 2. The percentage error in predicted and experimental effluent biomass concentrations ($X_e$) are also computed and presented in Table 4, which varies from 11.75% to a maximum error of 403.8%. This may be due to the abrupt fluctuations in experimental conditions like influent COD concentration and abrupt changes in COD loading rates during the initial phases of operation.

This percentage error is not reasonable and not within the acceptable limit in such simulations. This shows that predictions of $X_e$ under dynamic phase using Eqs. (1) and (2) are also not in agreement with experimental $X_e$ values. Therefore a simple CSTR model is not suitable to predict $X_e$ values as per Eqs. (1) and (3). The variation of predicted and experimental effluent biomass concentrations ($X_e$) as a function of operation time is also shown in figure 5.

From figure 5, it is seen that the experimental effluent biomass concentration in first 40 days of dynamic period was continuously decreased from 0.182 g VSS/L to 0.022 g VSS/L and thereafter increased to 0.1 g VSS/L and thereafter increased to a level of 0.13 g VSS/L at 140th day. A similar behaviour of effluent biomass concentration was also reported by (Kalyuzhnyi et al. 2006) using dispersed plug flow model.

Statistical error estimates in predicted and effluent COD and biomass concentrations during dynamic simulations

Statistical error estimates are determined to judge the performance of model Eqs. (1) and (3) in prediction of effluent COD and biomass concentrations. Marquardt’s percent standard deviation (MPSD) and root mean square error (RMSE) were adopted for the purpose. These statistical error estimates between experimental and predicted effluent COD concentrations ($S_e$) and effluent biomass concentrations ($X_e$) during dynamic phase are presented below in Table 5.

From Table 5, it can reveal that marquardt’s percent standard deviation (MPSD) was found as 56.72 and 407.35 respectively for effluent COD and biomass concentrations. From MPSD values , it is evident that a relatively large geometrical mean error distribution and high sum of normalized error in the predicted biomass concentrations are clearly evident. For predicted effluent soluble COD concentrations, MPSD values are relatively lesser than the predicted biomass concentration. Root mean square error (RMSE) was determined for predicted effluent COD and predicted biomass concentrations and were found as $3.5 \times 10^{-2}$ and $7.4 \times 10^{-2}$ respectively. RMSE values are relatively lesser for predicted effluent soluble COD concentration of the order of $10^{-2}$ which indicate that predicted values are slightly nearer to the experimental values than those observed for predicted effluent biomass concentration.

Thus, it is inferred that both $S_e$ and $X_e$ are not simulated well with experimental results of (Singh and Viraraghavan 1998) for dynamic phase when $X_o$ was not accounted.

Evaluation of dynamic performance using experimental results of (Singh and Viraraghavan 1998) when influent biomass concentration ($X_o$) is accounted

Using the experimental results of (Singh
and Viraraghavan (1998)\textsuperscript{16} on treatment of the low strength wastewater from Table 2 for a transient period of 140 days, the dynamic equations (1) and (2) were solved simultaneously for effluent soluble COD concentration ($S_\varepsilon$) and effluent biomass concentration ($X_\varepsilon$) using MATLAB2011a, ode15s tool with time step of 10 days.

Prediction of dynamic performance in terms of effluent COD concentration ($S_\varepsilon$) ($X_\varepsilon$ accounted)

Using the experimental results of (Singh and Viraraghavan 1998)\textsuperscript{16}, the results of dynamic simulations at 10 days intervals are presented in Table 6. The percentage error in predicted and experimental effluent COD concentrations ($S_\varepsilon$) are also computed and are shown by table 6, which varies from 50.35\% to 76.65\%.

From the reported results of experimental effluent COD concentrations in first 60 days of operation, it is evident that effluent COD was continuously decreased from 0.199g COD/L to 0.048 g COD/L and thereafter slightly increased to a level of 0.095 g COD/L. Due to large percentage error in predicted $S_\varepsilon$ values, the dynamic performance is not agreeable with the experimental $S_\varepsilon$ values and hence, the use of dynamic Eqs. (1) and (2) can be easily ruled out in prediction of effluent COD concentration. From Table 6, it is also evident that maximum error in predictions of $S_\varepsilon$ values is about 77\%, hence the results of simulations don’t agree well with experimental results and therefore, a simple CSTR model can’t be used to assess the performance of UASB reactor satisfactorily. Variation of predicted effluent biomass concentration and Experimental effluent biomass concentrations as a function of operation time is also shown in figure 7, where a large deviations in predicted $X_\varepsilon$ values can be seen from experimental $X_\varepsilon$ values. Thus, it is inferred that simple CSTR model Eqs. (1) and (2) can not predict well the performance of UASB reactor.

**CONCLUSION**

The kinetic constants required for prediction of performances in terms of effluent COD concentration ($S_\varepsilon$) and effluent biomass concentration ($X_\varepsilon$) are evaluated and presented using experimental result of (Singh and Viraraghavan 1998)\textsuperscript{16} treating low strength wastewater in UASB reactor. A simple CSTR model for evaluation of UASB reactor performance developed by considering the flow regime in UASB reactor as completely stirred tank reactor (CSTR) with or without consideration of influent biomass concentrations in the influent stream. Linear equations derived for the evaluation of kinetic constants for their use in model equations. The evaluation of dynamic performance of UASB reactor treating low
strength wastewater were carried out by using experimental results of (Singh and Viraraghavan 1998)\(^6\). From the results, it concluded that a simple CSTR model is inappropriate for the evaluation of dynamic performances of UASB reactors treating low strength wastewater as the errors in predictions were obtained too large with respect to their corresponding experimental values. From case study, it inferred that there are large amount of errors in predictions in comparison to their corresponding experimental values for both effluent COD and biomass concentrations. Therefore, a simple CSTR model not found suitable to evaluate the performance of UASB reactor in the treatment of low strength wastewater.

REFERENCES


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<th>K_s (g. COD/L)</th>
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Table 3. Percentage error between experimental and predicted effluent COD concentrations during dynamic phase, when influent biomass concentration is negligible

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Table 4. Percentage error between experimental and predicted effluent biomass concentrations during dynamic phase (Xₒ negligible)

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</table>
Table 5. Statistical error estimates between experimental and predicted effluent COD and effluent biomass concentrations during dynamic phase (X<sub>o</sub> negligible)

<table>
<thead>
<tr>
<th>Effluent Concentrations</th>
<th>MPSD</th>
<th>RMSE</th>
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</thead>
<tbody>
<tr>
<td>S&lt;sub&gt;e&lt;/sub&gt;</td>
<td>56.72</td>
<td>3.5E-02</td>
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<tr>
<td>X&lt;sub&gt;e&lt;/sub&gt;</td>
<td>407.35</td>
<td>7.4E-02</td>
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Table 6. Percentage error between experimental and predicted effluent COD concentrations during dynamic phase using the experimental results of Singh and Viraraghavan (1998) (X<sub>o</sub> accounted)

<table>
<thead>
<tr>
<th>Time</th>
<th>θ, (days)</th>
<th>S&lt;sub&gt;e&lt;/sub&gt;(Exp.)</th>
<th>S&lt;sub&gt;e&lt;/sub&gt;(Pred.)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
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<td>0.199</td>
<td>0.199</td>
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<td>2</td>
<td>0.121</td>
<td>0.048</td>
<td>59.52</td>
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</tr>
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<td>0.025</td>
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<td>1.66</td>
<td>0.048</td>
<td>0.023</td>
<td>50.68</td>
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<tr>
<td>70</td>
<td>1.66</td>
<td>0.082</td>
<td>0.022</td>
<td>73.13</td>
</tr>
<tr>
<td>80</td>
<td>1.5</td>
<td>0.058</td>
<td>0.023</td>
<td>58.68</td>
</tr>
<tr>
<td>90</td>
<td>1.5</td>
<td>0.068</td>
<td>0.023</td>
<td>65.51</td>
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<td>130</td>
<td>1.04</td>
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<td>64.15</td>
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<tr>
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<td>1.04</td>
<td>0.095</td>
<td>0.037</td>
<td>60.51</td>
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</tbody>
</table>

Table 7. Percentage error between experimental and predicted effluent biomass concentrations during dynamic phase (X<sub>o</sub> accounted)

<table>
<thead>
<tr>
<th>Time</th>
<th>θ, (days)</th>
<th>X&lt;sub&gt;e&lt;/sub&gt;(Exp.)</th>
<th>X&lt;sub&gt;e&lt;/sub&gt;(Pred.)</th>
<th>% Error</th>
</tr>
</thead>
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<td>0.182</td>
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<td>0.207</td>
<td>418.53</td>
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</table>
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<table>
<thead>
<tr>
<th>Substrate Utilization Rate (k)</th>
<th>Half Saturation Constant (K_s)</th>
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</thead>
<tbody>
<tr>
<td>100 1.33</td>
<td>0.039</td>
</tr>
<tr>
<td>110 1.33</td>
<td>0.029</td>
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<td>120 1.04</td>
<td>0.024</td>
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<tr>
<td>130 1.04</td>
<td>0.037</td>
</tr>
<tr>
<td>140 1.04</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Figure 1. Determination of maximum substrate utilization rate (k) and half saturation constant (K_s).
Figure 2. Determination of biomass yield coefficient ($Y$) and microorganism's decay coefficient ($K_d$).

$Y = 0.114 \text{ g VSS/g COD}$

$K_d = 0.49 \text{ d}^{-1}$

$y = -4.298x + 8.7916$

$R^2 = 0.93$

Figure 2. Determination of biomass yield coefficient ($Y$) and microorganism's decay coefficient ($K_d$), (when influent biomass concentration is accounted).

$Y = 0.234 \text{ g VSS/g COD}$

$K_d = 0.0018 \text{ d}^{-1}$

$y = 0.234x + 0.00188$

$R^2 = 0.681$
Figure 4. Agreement between the predicted effluent soluble COD and the experimental effluent soluble COD concentrations at different operation time during dynamic phase ($X_o$ negligible)

Figure 5. Agreement between the predicted effluent biomass and the experimental effluent biomass concentrations at different operation time during dynamic phase ($X_o$ negligible)
Figure 6. Agreement between the predicted effluent soluble COD concentrations and the experimental effluent soluble COD concentrations at different operation time during dynamic phase ($X_o$ accounted)

Figure 7. Agreement between the predicted effluent biomass and the experimental effluent biomass concentrations at different operation time ($X_o$ accounted)