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Effects of magnetic field and an endoscope on peristaltic motion

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ABSTRACT

The Problem of peristaltic transport of a fluid with magnetic fluid with variable viscosity through the gap between coaxial tubes where the outer tube is non-uniform with sinusoidal wave traveling down is wall and the inner tube is rigid. The relation between the pressure gradient, friction force on the inner and outer tube are obtained in terms of magnetic fluid and viscosity. The numerical solution of pressure gradient, outer friction and inner friction force and flow rate are shown graphically.

Keywords: Peristaltic transport, inner frictions force, outer friction force flow rate.

INTRODUCTION

The purpose of this paper is an attempt to understand the fluid mechanics in a physiological situation with the presence of an endoscope is placed concentrically. The pressure rise, peristaltic pumping, augmented pumping and friction force on the inner tube (endoscope) and outer tube is discussed by the Srivastava et.al [1], and Siddiqui and Schwarz [2]. Latham [3] investigated the fluid mechanics of peristaltic pump and since then, other work on the same subject has been followed by Burns and Parker [4]. Barton and Raynor [5] have studied the case of a vanishingly small Reynolds number. Lyokoudis and Roos [6] studied the fluid mechanics of the ureter from a lubrication theory point of view. Zien and Ostrach [7] have investigated a long wave approximation to peristaltic motion, and the analysis is aimed at the possible application to urine flow in human ureters. Rose Lykoudis [8] studied the effect of the presence of a catheter upon the pressure distribution inside the ureter. Ramachandra and Usha [9] studied the influence of an eccentrically inserted catheter on the peristaltic pumping in a tube under long wavelength and low Reynolds numbers approximation Abd El Naby and El Misery [10] studied the effect of an endoscope and generalized Newtonian fluid on peristaltic motion .Gupta and Seshadri [11] studied peristaltic transport of a Newtonian fluid in non- uniform geometries. Srivastava and Srivastava [12] have investigated the effect of power law fluid in uniform and non-uniform tube and channel under zero Reynolds number and long wavelength approximation. Provost and

Schwarz [13] have investigated a theoretical study of viscous effect in peristaltic pumping and assumed that the flow is free of inertial effect and that non-Newtonian normal stresses are negligible. Bohme and Friedrich [14] have investigated peristaltic flow of viscoelastic liquids and assumed that the relevant Reynolds number is small enough to neglect inertia forces, and that the ratio of the wavelength and channel height is large, which implies that the pressure is constant over the cross-section. El-Misery et .al [15] have investigated the effect of a Carreau fluid in peristaltic transport for uniform channel. Elshehaway et .al [16] studied peristaltic motion of generalized Newtonian fluid in a non-uniform channel under zero Reynolds number with long wavelength approximation. Most of studies on peristaltic motion, that assume physiological fluids behave like a Newtonian fluid with constant viscosity fail to give a better understanding when peristaltic mechanics involved in small blood vessel, lymphatic vessel, intestine, ducts efferent of the male reproductive tracts, and in transport of spermatozoa in the cervical canal. According to Haynes [17], Bugliarllo and Sevilla [18] and Goldsmith and Skalak [19] it is clear that in pre mentioned body organs, viscosity of the fluid varies across the thickness of the duct. Cotton and Williams [20] study the practical gastrointestinal endoscope. Rathod and Asha [21] studied the peristaltic transport of a couple stress fluids in uniform and non-uniform annulus moving with a constant velocity. Rathod and Asha [22] studied the effect of couple stress fluid and an endoscope on peristaltic motion.

The effect of magnetic field with variable viscosity through the gap between inner and outer tubes where the inner tube is an endoscope and the outer tube has a sinusoidal wave traveling down its wall is the aim of present investigation.

Formulation and analysis: Consider the two-dimensional flow of an incompressible Newtonian fluid with variable viscosity through the gap between inner and outer tubes where the inner tube is an endoscope and the outer tube has a sinusoidal wave traveling down its wall. The geometry of the two wall surface is given by the equations:

$$\bar{r}_1 = a_1, \tag{1}$$

$$\bar{r}_1 = a_{20} + b\sin\frac{2\pi}{\lambda}(\bar{z} - c\bar{t}) \tag{2}$$

Where a_1 is the radius of endoscope a_{20} is the radius of the small intestine at inlet, b is the amplitude of the wave, λ is the wavelength, $\bar{t}$ is time and c is the wave speed.

In the fixed coordinates $(\bar{r}, \bar{z})$ the flow in the gap between inner and outer tubes is unsteady but if we choose moving coordinates $(\bar{r}, \bar{z})$ which travel in the $\bar{z}$ - direction with the same speed as the wave, then the flow can be treated as steady. The coordinate's frames are related through:

$$\bar{z} = \bar{Z} - c\bar{t}, \qquad \bar{r} = \bar{R}, \tag{3}$$

$$\bar{w} = \bar{W} - c, \qquad \bar{u} = \bar{U}, \tag{4}$$

Where $\bar{U}, \bar{W}$ and $\bar{u}, \bar{w}$ are the velocity components in the radial and axial direction in the fixed and moving coordinates respectively.

Equations of boundary condition in the moving coordinates are: Continuity equation:

$$\frac{1}{\bar{r}}\frac{\delta(\bar{r}\,\bar{u})}{\delta\bar{r}}+\frac{\delta\bar{w}}{\delta\bar{z}}=0, \tag{5}$$

And the Navier Stokes equation:

$$\rho(\bar{u}\frac{\partial\bar{u}}{\partial\bar{r}}+\bar{w}\frac{\partial\bar{u}}{\partial\bar{z}})=-\frac{\partial\bar{p}}{\partial\bar{r}}+\frac{\partial}{\partial\bar{r}}[2\bar{\mu}(\bar{r})\frac{\partial\bar{u}}{\partial\bar{r}}]+\frac{2\bar{\mu}(\bar{r})}{\bar{r}}(\frac{\partial\bar{u}}{\partial\bar{r}}-\frac{\bar{u}}{\bar{r}})$$
$$+\frac{\partial}{\partial\bar{z}}[\bar{\mu}(\bar{r})(\frac{\partial\bar{u}}{\partial\bar{z}}+\frac{\partial\bar{w}}{\partial\bar{r}})]-\sigma B_0^2(\bar{u}) \tag{6}$$

$$\rho(\bar{u}\frac{\partial\bar{w}}{\partial\bar{r}}+\bar{w}\frac{\partial\bar{w}}{\partial\bar{z}})=-\frac{\partial\bar{p}}{\partial\bar{z}}+\frac{\partial}{\partial\bar{z}}[2\bar{\mu}(\bar{r})\frac{\partial\bar{w}}{\partial\bar{z}}]+$$
$$\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}[\bar{\mu}(\bar{r})\bar{r}(\frac{\partial\bar{u}}{\partial\bar{z}}+\frac{\partial\bar{w}}{\partial\bar{r}})]-\sigma B_0^2(\bar{w}) \tag{7}$$

$\bar{p}$ is the pressure , $\bar{\mu}(\bar{r})$ is the viscosity function, σ is Electric conductivity and B_0 is applied magnetic field. The boundary condition are

$$\bar{w}=-c \quad at \quad \bar{r}=\bar{r}_1 \quad \bar{r}=\bar{r}_2 \tag{8a}$$
$$\bar{u}=0 \quad at \quad \bar{r}=\bar{r}_1 \tag{2.8b}$$

The following are the non dimensional variables, the Reynolds number (Re) and the wave number (δ) introduced:

$$r=\frac{\bar{r}}{a_{20}},\quad R=\frac{\bar{R}}{a_{20}},\ r_1=\frac{\bar{r}_1}{a_{20}}=\frac{a_{10}}{a_{20}}=\varepsilon<1, z=\frac{\bar{z}}{\lambda},\quad Z=\frac{\bar{Z}}{\lambda},\ \mu(r)=\frac{\bar{\mu}(\bar{r})}{\mu_0},$$
$$u=\frac{\lambda\bar{u}}{a_{20}c}, U=\frac{\lambda\bar{U}}{a_{20}c},\ w=\frac{\bar{w}}{c}, W=\frac{\bar{W}}{c}, \delta=\frac{a_{20}}{\lambda}<<1,$$
$$\mathrm{Re}=\frac{ca_{20}\rho}{\mu_0}, p=\frac{a_{20}^2\bar{p}}{c\lambda\mu_0},\ t=\frac{c\bar{t}}{\lambda},\ \phi=\frac{b}{a_{20}}<1,$$
$$r_2=\frac{\bar{r}_2}{a_{20}}=1+\phi\sin 2\pi z \tag{9}$$

Where ε is the radius ratio, ϕ is the amplitude ratio and μ_0 is the viscosity on the endoscope. Then the equation of motion and boundary conditions in the dimensionless form become:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \tag{10}$$

$$\mathrm{Re}\,\delta^3(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial r}+\delta^2\frac{\partial}{\partial r}(2\mu(r)\frac{\partial u}{\partial r})+\delta^2\frac{\partial}{\partial z}[\mu(r)(\delta^2\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r})$$
$$+\frac{2\delta^2\mu(r)}{r}(\frac{\partial u}{\partial r}-\frac{u}{r})-\delta^2M^2(u) \tag{11}$$

$$\operatorname{Re}\delta\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left[\mu(r)r\left(\delta^2\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right] + \delta^2\frac{\partial}{\partial z}\left(2\mu(r)\frac{\partial w}{\partial z}\right)-M^2(w) \tag{12}$$

$M=\sqrt{\dfrac{\sigma}{\mu}}B_0a_{20}$ is the Hartmann number , σ is Electric conductivity

The dimensionless boundary conditions are:

$$w=-1 \quad at \quad r=r_1, \qquad r=r_2 \tag{13a}$$

$$u=0 \quad \text{at} \quad r=r_1. \tag{13b}$$

Using the long wavelength approximation and neglecting the wave number ($\delta=0$), one can reduce Navier-Stokes equation:

$$\frac{\partial p}{\partial r}=0 \tag{14}$$

$$\frac{\partial p}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu(r)r\frac{\partial w}{r}\right)-M^2(w) \tag{15}$$

The instantaneous volume flow rate in the fixed coordinate system is given by:

$$\bar{Q}=2\pi\int_{\bar{r}_1}^{\bar{r}_2}\bar{W}\bar{R}d\bar{R} \tag{16}$$

Where $\bar{r}_1$ is a constant and $\bar{r}_2$ is a function of $\bar{Z}$ and $\bar{t}$. On substituting equations (3) and (4) into (16) and then integrating, one obtains:

$$\bar{Q}=\bar{q}+\pi c(\bar{r}_2^{\,2}-\bar{r}_1^{\,2}) \tag{17}$$

Where $\bar{q}=2\pi\int_{\bar{r}_1}^{\bar{r}_2}\bar{w}\bar{r}d\bar{r}$ (18)

is the volume flow rate in the moving coordinate system and is independent of time. Here, $\bar{r}_2$ is a function of $\bar{z}$ alone and is defined through equation (2). Using the dimensionless variable, we find equation (18) becomes:

$$F=\frac{q}{2\pi a_2^2c}=\int_{r_1}^{r_2}wrdr \tag{19}$$

The time- mean flow over a period $T=\dfrac{\lambda}{c}$ at a fixed Z position is defined as:

$$Q=\frac{1}{T}\int_0^T\bar{Q}d\bar{t} \tag{20}$$

Using equations (17) and (18) in (20) and integrating we get:

$$\bar{Q} = \bar{q} + \pi c(a_2^2 - a_1^2 + \frac{b^2}{2})$$

Which may written as:

$$\frac{\bar{Q}}{2\pi a_{20}^2 c} = \frac{\bar{q}}{2\pi a_{20}^2 c} + \frac{1}{2}(1 - \varepsilon^2 + \frac{\phi^2}{2}) \tag{21}$$

On defining the dimensionless time-mean flow as:

$$\Theta = \frac{\bar{Q}}{2\pi a_{20}^2 c}$$

We write equation (21) as:

$$\Theta = F + \frac{1}{2}(1 - \varepsilon^2 + \frac{\phi^2}{2}) \tag{22}$$

Solving equations (13) - (15), we obtain:

$$W = \frac{1}{2}\frac{dp}{dz}[I_1(r) - \frac{I_2(r)\{I_1(r_2) - I_1(r_1)\} - I_1(r_1)I_2(r_2) - I_2(r_1)I_1(r_2)}{I_2(r_2) - I_2(r_1)}] - (1/2)M^2D - 1 \tag{23}$$

Where D = $\frac{I_2(r)\{I_1(r_2) - I_1(r_1)\} - I_1(r_1)I_2(r_2) - I_2(r_1)I_1(r_2)}{I_2(r_2) - I_2(r_1)}$

$$I_1(r) = \int \frac{r}{\mu(r)} dr \tag{24}$$

$$I_2(r) = \int \frac{dr}{r\mu(r)} \tag{25}$$

Using equation (19), we obtain the relationship between $\frac{dp}{dz}$ and F as follows:

$$F = \frac{1}{4}\frac{dp}{dz}[\frac{(I_1(r_2) - I_1(r_1))^2}{I_2(r_2) - I_2(r_1)} - I_3] + \frac{1}{4}M^2[\frac{(I_1(r_2) - I_1(r_1))^2}{I_2(r_2) - I_2(r_1)} - I_3] - \frac{1}{2}(r_2^2 - r_1^2) \tag{26}$$

$$I_3 = \int_{r_1}^{r_2} \frac{r^3}{\mu(r)} dr \tag{27}$$

Solving equation (26) for $\frac{dp}{dz}$, we obtain:

$$\frac{dp}{dz} = \frac{4F + 2(r_2^2 - r_1^2) - (1/4)M^2(\frac{[I_1(r_2) - I_1(r_1)]^2}{I_2(r_2) - I_2(r_1)} - I_3)}{\frac{[I_1(r_2) - I_1(r_1)]^2}{I_2(r_2) - I_2(r_1)} - I_3} \tag{28}$$

The pressure rise ΔP_λ and friction force on inner and outer tubes $F_\lambda^{(i)}$ and $F_\lambda^{(o)}$, in their non-dimensional forms, are given by:

$$\Delta P_\lambda = \int_0^1 \left(\frac{dp}{dz}\right) dz \tag{29}$$

$$F_\lambda^{(i)} = \int_0^1 r_1^2 \left(-\frac{dp}{dz}\right) dz \tag{30}$$

$$F_\lambda^{(o)} = \int_0^1 r_2^2 \left(-\frac{dp}{dz}\right) dz \tag{31}$$

The effect of viscosity variation on peristaltic transport can be investigated through equation (29) - (31) for any given viscosity function $\mu(r)$.

For the present instigation, we assume viscosity variation in the dimensionless form following Srivastava et al. [1], as follows:

$$\mu(r) = e^{-ar} \tag{32}$$

Or

$$\mu(r) = 1 - \alpha r \qquad \text{for } \alpha << 1 \tag{33}$$

Where α is viscosity parameter. The assumption is reasonable for the following physiological reason. Since normal person or animal or similar size takes 1 to 2L of fluid every day. On the of that, another 6 to 7L of fluid received by the small intestine daily as secretion from salivary glands, stomach, pancreas, liver and the small intestine itself. This implies that concentration of fluid is dependent on the radial distance, hence the viscosity of the fluid adjacent to the wall of small intestine is less than the away from the wall. Therefore, the above choice of $\mu(r) = e^{-ar}$ is justified.

Substituting from equation (33) into equations (24), (25) and (27) and using equation (28), we obtain:

$$\frac{dp}{dz} = [[(\{16\Theta - 8(1-\varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\}] / \{\frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4)\}]$$

$$\times(1 - (4\alpha + M^2)\{\frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3\log(r_2/r_1)} - \frac{(r_2^2 - r_1^2)^2 (r_2 - r_1}{4(\ln(r_2/r_1))^2} - \frac{(r_2^5 - r_1^5)}{5}\} \qquad /\{\frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4)\})$$

(34)

substituting from equation (34) into (29)-(31) yield:

$$\Delta P_\lambda = \int_0^1 \ [(\{16\Theta - 8(1-\varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} / \{\frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4)\})]$$

$$\times(1-(4\alpha+M^2)\{\frac{(r_2^2-r_1^2)(r_2^3-r_1^3)}{3\log(r_2/r_1)}-\frac{(r_2^2-r_1^2)^2(r_2-r_1)}{4(\log(r_2/r_1))^2}-\frac{(r_2^5-r_1^5)}{5}\}$$

$$/\{\frac{(r_2^2-r_1^2)^2}{\log(r_2/r_1)}-(r_2^4-r_1^4))\}\}dz \quad (35)$$

$$F_\lambda^{(i)}=\int_0^1 -r_1^2[(\{16\Theta-8(1-\varepsilon^2+\frac{\phi^2}{2})+8(r_2^2-r_1^2)\}/\{\frac{(r_2^2-r_1^2)^2}{\log(r_2/r_1)}-(r_2^4-r_1^4)\})]$$

$$\times(1-(4\alpha+M)^2\{\frac{(r_2^2-r_1^2)(r_2^3-r_1^3)}{3\log(r_2/r_1)}-\frac{(r_2^2-r_1^2)^2(r_2-r_1}{4(\log(r_2/r_1))^2}-\frac{(r_2^5-r_1^5)}{5}\}$$

$$/\{\frac{(r_2^2-r_1^2)^2}{\log(r_2/r_1)}-(r_2^4-r_1^4))\}\ dz \quad (2.36)$$

$$F_\lambda^{(o)}=\int_0^1 -r_2^2[(\{16\Theta-8(1-\varepsilon^2+\frac{\phi^2}{2})+8(r_2^2-r_1^2)\}/\{\frac{(r_2^2-r_1^2)^2}{\log(r_2/r_1)}-(r_2^4-r_1^4)\})$$

$$\times(1-(4\alpha+M^2)\{\frac{(r_2^2-r_1^2)(r_2^3-r_1^3)}{3\log(r_2/r_1)}-\frac{(r_2^2-r_1^2)^2(r_2-r_1}{4(\log(r_2/r_1))^2}-\frac{(r_2^5-r_1^5)}{5}\}$$

$$/\{\frac{(r_2^2-r_1^2)^2}{\log(r_2/r_1)}-(r_2^4-r_1^4))\}]dz \quad (37)$$

RESULTS AND DISCUSSION

The dimensionless pressure rise (P_λ) and the friction forces on the inner and outer tube for different given values of the dimensionless flow rate Θ, amplitude ratio ϕ , radius ratio ε, magnetic field M and viscosity parameter α are computed using the equations (35), (36) and (37). As the integrals of equation (35) to (37) are not integrable in the closed form so they are evaluated using

a_{20}=1.25cm and λ=0.156

The values of viscosity parameter α as reported by Srivastava et.al [1] are α=0.0 and α=0.1 Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported by Cotton and Williams [20] and radius ratio 1.25cm reported by Srivastava and Srivastava [12].

In figure (1) the pressure against the flow rate is plotted, here it is observed that the pressure increases with the increase of flow rate for different values of radius ratio $\varepsilon=0.32, \varepsilon=0.38\ and\ \varepsilon=0.44$ and pressure decreases for the viscosity α=0.0 and α=0.1. Figure (2) shows that as the viscosity α increases the pressure is decreases and for the different values of amplitude ratio ϕ=0.0 and ϕ=0.4 the pressure is decreases.

In figures (3) and (4) the friction force on the outer tube for different values of radius ratio and amplitude ratio are plotted , here it is observed that as radius ratio ε increases the friction force also decreases and they are independent of radius ratio at certain values of the flow rate [for the values φ=0.4 and α=0.0 and α=0.1].In figures (5) and (6) it is noticed that the friction force on the inner tube (endoscope) and on outer tube is plotted against the flow rate for different values of amplitude ratio φ and for different values radius ratio $\varepsilon = 0.32, \varepsilon = 0.38\ and\ \varepsilon = 0.44$ and for the values of viscosity α=0.0 and α=0.1 Here it is noticed that as the amplitude ratio φ increases the friction force on the outer tube and inner tube decreases and as the viscosity increases the friction force on the outer tube and inner tube decreases.

From Figure (7) it is noticed that the pressure increases for different values of magnetic field M=1, 3 and 5. From figures (8) and (9) it is noticed that the friction force decreases on endoscope and on the outer tube as magnetic field increases.

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