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ABSTRACT

We analyse the combined effects of Hall currents and radiation on the convective heat and mass transfer of a viscous electrically conducting rotating fluid through a porous medium past a vertical porous plate, in the presence of heat generating sources. The equations governing the flow, heat and mass transfer are solved exactly to obtain the velocity, temperature and concentration and are analysed for different variations of the governing parameters G, M, m, N1, So and γ. The rate of heat and mass transfer are numerically evaluated for different sets of the parameters. The influence of chemical action, Hall currents, thermo-diffusion and radiation effects on the flow, heat and mass transfer characteristics are discussed in detail.

Keywords: Chemical reaction, Hall effect, Soret effect, Rotating fluid.

INTRODUCTION

Free convection and mass transfer flow in porous medium has received considerable attention due to its numerous applications in geophysics and energy related problems. Such types of applications include natural circulation in isothermal reservoirs, acquifires, porous insulation in heat storage bed, grain storage, extraction of thermal energy and thermal insulation design. Studies associated with flows through porous medium in rotating environment have some relevance in geophysical and geothermal applications. Many aspects of the motion in a rotating frame of reference of terrestrial and planetary atmosphere are influenced by the effects of rotation of the medium. Also buoyancy and rotational effects are often comparable in geophysical processes. Convective transport in a rotating atmosphere over a locally heated surface gives rise to dust storms (typhoons) and other atmosphere circulations.

The unsteady flow of a rotating viscous fluid in a channel maintained by non-torsional oscillations of one or both the boundaries has been studied by several authors to analyse the growth and development of boundary layers associated with geothermal flows for possible applications in geophysical fluid dynamics[4,7,8,12-15,17]. Later Singh et al.[19] studied free convection in MHD flow of a rotating viscous liquid in porous media. Singh et al.[20] have also studied free convective MHD flow of a rotating viscous fluid in a porous medium past an infinite vertical porous plate.

In all these investigations, the effects of Hall currents are not considered. It was emphasized by Cowling [5,6] that when the strength of the magnetic field is sufficiently large, Ohm’s law needs to be modified to include Hall currents. The Hall effect is due to merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the Hall effect is ignored. But if the strength of the magnetic field is high and the number of
density of electrons is small, the Hall effect can not be disregarded as it has a significant effect on the flow pattern of an ionized gas. Hall effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. The current is termed as Hall currents. Model studies on the effect of Hall current in MHD convection flows have been carried out by many authors due to application of such studies in the problems of MHD generators and Hall accelerators. Yamanishi[22] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. Debnath [7] has studied the effects of Hall currents on unsteady hydromagnetic flow past a porous plate in a rotating fluid system and the structure of the steady and unsteady flow is investigated. Alam et al [2] have studied unsteady free convective heat and mass transfer flow in a rotating system with Hall currents, viscous dissipation and Joule heating. Taking Hall effects into account Krishna et al[9] have investigated Hall effects on the unsteady hydromagnetic boundary layer flow. Rao et al[15] have analysed Hall effects on unsteady hydromagnetic flow. Sivaprasad et al[21] have studied Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel. Achaya et al[1] have discussed Hall current effect on magnetohydrodynamics free convection flow past a semi-infinite vertical plate with mass transfer. Malique et al[11] have studied the effects of variable properties and Hall current on steady MHD laminar convective fluid flow due to a porous rotating disk. Sharma et al[11a] have discussed the Hall effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical plate immersed in a porous medium with heat source/sink. Ahmed et al[1b] have investigated the unsteady three dimensional MHD flow of a viscous, electrically conducting fluid past a suddenly started infinite horizontal porous plate taking into account the effect of the Hall currents. Reddy et al[10] have discussed the effect of Hall currents on convective heat and mass transfer flow of a viscous, rotating fluid through a porous medium past a vertical porous plate. Recently several authors [7a, 16a, 16b, 16c, 16d, 21a] have discussed the flow in wavy channels under varied conditions.

In this paper we investigate the combined effects of Hall currents and radiation on the convective heat and mass transfer of a viscous electrically conducting rotating fluid through a porous medium past a vertical porous plate, in the presence of heat generating sources. The equations governing the flow, heat and mass transfer are solved exactly to obtain the velocity, temperature and concentration and are analysed for different variations of the governing parameters G, M, m, N, So and γ. The rate of heat and mass transfer are numerically evaluated for different sets of the parameters. The influence of chemical action, Hall currents, thermo-diffusion and radiation effects on the flow, heat and mass transfer characteristics are discussed in detail.

FORMULATION and SOLUTION OF THE PROBLEM

We consider the steady flow of an incompressible, viscous, electrically conducting, rotating fluid past a semi-infinite vertical plate embedded in a porous medium. In the undisturbed state, both the plates and fluid rotate with same angular velocity \( \Omega \) and are maintained at constant temperature and concentration. Further the plates are cooled or heated by constant temperature gradient in some direction parallel to the plane of the plates. We choose a cartesian co-ordinate system O(x,y,z) such that the plate is at z=0 and the z-axis coincide with the axis of rotation of the plates. The steady hydromagnetic boundary layer equations of motion with respect to a rotating frame moving with angular velocity \( \Omega \) under Boussinesq approximations are
\[-W_0 \frac{\partial u}{\partial z} - 2\Omega v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} + (\mu_e H_0 J_z u) - (\frac{v}{k})u - \rho \bar{g} \quad (2.1)\]

\[-W_0 \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - (\mu_e H_0 J_z v) - (\frac{v}{k})v \quad (2.2)\]

The energy equation is

\[\rho_0 C_p (-W_0 \frac{\partial T}{\partial z}) = k_f \frac{\partial^2 T}{\partial z^2} + Q(T_{\infty} - T) - \frac{\partial (q_r)}{\partial z} + \mu (u_z^2 + v_z^2) + \frac{\sigma \mu_e^2 H^2}{\rho_0} (u^2 + v^2) \quad (2.3)\]

The diffusion equation is

\[\rho_0 C_p (-W_0 \frac{\partial C}{\partial z}) = D_1 \frac{\partial^2 C}{\partial z^2} - k_{11} C + k_{11} \frac{\partial^2 T}{\partial z^2} \quad (2.4)\]

Equation of State is

\[\rho - \rho_{\infty} = -\beta_0 \rho_{\infty} (T - T_{\infty}) - \beta^* \rho_{\infty} (C - C_{\infty}) \quad (2.5)\]

where \(u, v\) are the velocity components along \(x\) and \(y\) directions respectively, \(p\) is the pressure including the centrifugal force, \(\rho\) is the density, \(k\) is the permeability constant, \(\mu\) is the dynamic viscosity, \(k_e\) is the thermal diffusivity, \(D_1\) is the chemical molecular diffusivity, \(\beta_0\) is the coefficient of thermal expansion, \(\beta^*\) is the volumetric coefficient of expansion with mass fraction, \(Q\) is the strength of the heat source, \(k_{11}\) is the cross diffusivity and \(q_r\) is the radiative heat flux.

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm’s law is modified to

\[\bar{J} + \omega e \tau_e \bar{J} \times \bar{H} = \sigma (\bar{E} + \mu_e \bar{q} \bar{H}) \quad (2.6)\]

where \(q\) is the velocity vector, \(H\) is the magnetic field intensity vector, \(E\) is the electric field, \(J\) is the current density vector, \(\omega_e\) is the cyclotron frequency, \(\tau_e\) is the electron collision time, \(\sigma\) is the fluid conductivity and \(\mu_e\) is the magnetic permeability.

Neglecting the electron pressure gradient-slip and thermo-electric effects and assuming the electric field \(\bar{E} = 0\), equation (2.6) reduces

\[j_x + mj_y = \sigma \mu_e H_0 v \quad (2.7)\]

\[j_y - mj_x = -\sigma \mu_e H_0 u \quad (2.8)\]

where \(m = \omega_e \tau_e\) is the Hall parameter.

On solving equations (2.7)&(2.8) we obtain

\[j_x = \frac{\sigma \mu_e H_0}{1 + m^2} (v + mu) \quad (2.9)\]

\[j_y = \frac{\sigma \mu_e H_0}{1 + m^2} (mv - u) \quad (2.10)\]

By using Rosseland approximation the radiative heat flux is
It should be observed that by using Rosseland approximation the present analysis is limited to optically thick fluids. We assume that the temperature differences within the flow are sufficiently small so that \( T'^4 \) may be expressed as a linear function of the temperature. This is accompanied by expanding \( T'^4 \) in a Taylor series about \( T_\infty \) and neglecting higher order terms as

\[
T'^4 \equiv 4T_\infty^3 - 3T_\infty^4
\]  

(2.12)

where \( \sigma^* \) is Stefan-Boltzman constant and \( \beta_R \) is the mean absorption coefficient.

Far away from the plate we have

\[
0 = -\frac{\partial \rho_\infty}{\partial x} - \rho_\infty g
\]  

(2.13)

Using the equations (2.5), (2.12) & (2.13) the equations of motions governing the flow through a porous medium with respect to a rotating frame are given by

\[
-W_0 \frac{\partial u}{\partial z} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} + \frac{(\sigma \mu^2 H_0^2)}{\rho_\infty (1 + m^2)} (mv - u) - \left(\frac{v}{k}\right) u + \beta g \rho_\infty (T - T_\infty) + \beta^* g \rho_\infty (C - C_\infty)
\]

(2.14)

\[
-W_0 \frac{\partial v}{\partial z} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{(\sigma \mu^2 H_0^2)}{\rho_\infty (1 + m^2)} (v + mu) - \left(\frac{v}{k}\right) v
\]

(2.15)

Combining (2.14) & (2.15) we get

\[
-W_0 \frac{\partial q}{\partial z} - 2i\Omega q = -\frac{(\sigma \mu^2 H_0^2)}{\rho_\infty (1 + m^2)} (1 - im) q - \left(\frac{v}{k}\right) q + \nu \frac{\partial^2 q}{\partial z^2}
\]  

(2.16)

where

\[ q = u + iv \]

The non-dimensional variables are

\[ z' = \frac{z}{(v/W_o)}, \quad q' = \frac{q}{W_o}, \quad \theta' = \frac{(T - T_\infty)}{T_w - T_\infty}, \quad C = \frac{(C - C_\infty)}{C_w - C_\infty} \]

The equations governing the flow, heat and mass transfer are (dropping the dashes)

\[
\frac{\partial^2 q}{\partial z^2} + \frac{\partial q}{\partial z} - \lambda^2 q = -G\theta + NC
\]

(2.17)

\[
\left(1 + \frac{4}{3N_c}\right) \frac{\partial^2 \theta}{\partial z^2} + \frac{P}{\partial z} - \alpha \theta 3N_c \theta
\]

(2.18)

\[
\frac{\partial^2 C}{\partial z^2} - Sc \frac{\partial C}{\partial z} - \gamma C = -\frac{Sc So}{N} \frac{\partial^2 \theta}{\partial z^2}
\]

(2.19)

where \( \lambda^2 = (D^{-1} + 2iE^{-1} + \frac{M^2(1-im)}{1+m^2}), \quad E = \frac{V}{L^2 \Omega} \) (Ekman number), \( D^{-1} = \frac{L^2}{k} \) (Darcy parameter).
\[
G = \frac{\beta g L^4 (T_w - T_e)}{\nu^2} \quad \text{(Grashof number)}, \quad N = \frac{\beta^* (C_w - C_\infty)}{\beta (T_w - T_e)} \quad \text{(Buoyancy ratio)}, \quad N_1 = \frac{\beta \kappa f}{4\sigma T_e^3} \quad \text{(Radiation parameter)}, \quad P = \frac{V}{k_f} \quad \text{(Prandtl number)}, \quad \alpha = \frac{QL^2}{k_1} \quad \text{(Heat source parameter)}, \quad Sc = \frac{V}{D_1} \quad \text{(Schmidt number)},
\]

\[
So = \frac{k_1 \beta^*}{\beta \nu} \quad \text{(Soret Number)}, \quad \gamma = k_1 L^2 / D_1 \quad \text{(Chemical reaction parameter)}
\]

\[
P_1 = \frac{3N_i P}{3N_1 + 4} \quad \alpha_i = \frac{3N_i \alpha}{3N_1 + 4}
\]

The boundary conditions are

\[
q = 0 \quad \text{on } z=0 \\
\theta = 1 \quad \text{, } \quad C = 1 \quad \text{on } z=0 \\
q \to 0, \quad \theta \to 0 \quad , \quad C \to 0 \quad \text{as } \quad z \to \infty \quad (2.20)
\]

Solving equations (2.17)-(2.19) subject to the boundary conditions (2.20) we obtain

\[
q = (a_0) \exp(-m_1 z) - a_4 \exp(-m_2 z) + a_4 \exp(-m_3 z) \\
C = \exp(-m_1 z) \\
\theta = (1 + a_3) \exp(-m_2 z) - a_3 \exp(-m_1 z)
\]

where

\[
m_1 = 1 + \frac{\sqrt{1 + 4 \gamma^2}}{2} \\
m_2 = \frac{P_1 + \sqrt{P_1^2 + 4 \alpha_i}}{2} \\
a_2 = 1 + a_3 \\
\]

\[
a_3 = \frac{Q_i}{m_1^2 - P_1 m_1 - \alpha_i} \\
a_4 = a_5 - a_6 + a_7 \\
a_5 = \frac{G(1 + a_3)}{m_2^2 - m_2 - \lambda^2} \\
a_6 = \frac{G a_3}{m_2^2 - m_2 - \lambda^2} \\
a_7 = \frac{GN}{m_1^2 - m_1 - \lambda^2}
\]

**NuSSELT NUMBER AND SHERWOOD NUMBER**

Local rate of heat transfer across the walls (Nusselt Number) is given by

\[
(Nu)_{z=0} = \left( \frac{d\theta}{dz} \right)_{z=0} = -m_2 (1 + a_3) + a_3 m_1
\]

Rate of mass transfer (Sherwood Number) is given by

\[
(Sh)_{z=0} = \left( \frac{dC}{dz} \right)_{z=0} = -m_1
\]

**DISCUSSION OF THE NUMERICAL RESULTS**

The effect of Hall currents and thermo-diffusion on convective Heat and Mass transfer flow of a viscous fluid through a porous medium past a vertical plate is investigated. We take \( P = 0.71 \) in this analysis.
The velocity component \( u \) is shown in figs 1-7 for different values of \( G, M, m, E^{-1}, S_0, N_1, \gamma \). Fig. 1 represents the velocity with Grashof number \( G \). It is found that \( u > 0 \) for \( G > 0 \) and \( u < 0 \) for \( G < 0 \). \( |u| \) experiences an enhancement with increase in \( |G| (\geq 0) \). The variation of \( u \) with Hartmann number \( M \) shows that higher the Lorentz force larger \( u \) in the entire flow region (fig.2). An increase in the Hall current parameter \( (m) \) enhances \( |u| \) in the region (fig.3). \( |u| \) reduces with increase in the radiation parameter \( N_1 \) (fig.4). Thus higher the radiative heat flux smaller the velocity in the flow region. With respect to sorot parameter \( S_0 \) we observe that \( u \) is positive for all values of \( S_0 \). \( |u| \) reduces with increase in \( |S_0| (<0) \) and enhances with \( S_0 (>0) \) (fig.5). From fig.6 we find that \( u \) reduces with increase in the Ekman number \( E^{-1} \) in the entire flow region. The variation of \( u \) with chemical reaction parameter \( \gamma \) shows that \( u \) depreciates with increase in \( \gamma \leq 1.54 \) and enhances with higher \( \gamma \geq 2.5 \) in the entire flow region (fig.7).
The transverse velocity ($v$) is shown in figs. 8-13 for different parametric values. Maximum attained in the vicinity of the plate. The variation of $v$ with $M$ shows that higher the Lorentz force smaller $|v|$ in the entire flow region (figs. 8). From fig. 9, we find that higher the Hall parameter $m$ larger $|v|$ in the flow region. From fig. 10, we find that higher the radiative heat flux the change in $|v|$ is marginal in the vicinity of the plate and is appreciable large far away from the boundary. Fig. 11 represents $v$ with Soret parameter $S_0$. It is found that $|v|$ decreases with increase in $S_0>0$ and enhances with $|S_0|<0$. The variation of $v$ with $E^{-1}$ shows that $|v|$ depreciates with increase in $E^{-1}\leq 0.3$ and enhances
with $E^{-1} \geq 0.5$ in the flow region (fig. 12). An increase in the chemical reaction parameter $\gamma \leq 1.5$ results in a depreciation in $|v|$ and for higher $\gamma \geq 2.5$ we notice an enhancement in $|v|$ (fig. 13).

The non-dimensional temperature ($\theta$) is shown in figs 14 & 15 for different values of $\alpha$, $N_1$. The temperature gradually reduces from its prescribed value 1 on $z = 0$ and attains the value ‘0’ for away from the wall. An increase in the heat source parameter $\alpha$ reduces the temperature (fig. 15). Also the temperature experiences a depreciation with increase in the radiation parameter $N_1$ (fig. 15).

The concentration distribution ($C$) is exhibited in fig 16-18 for different $S_0$, $\gamma$ and $N_1$. The variation of $C$ with radiation parameter $N_1$ shows that higher the radiative heat flux smaller the concentration in the entire region (fig. 16). With respect to $S_0$ we find that the concentration depreciates with increase in $S_0 > 0$ and enhances with $|S_0| < 0$ (fig. 17). An increase in the chemical reaction parameter $\gamma$ reduces the concentration in the vicinity of the plate and its variation is marginal far away from the boundary (fig. 18).

The shear stress at $z = 0$ is shown in tables 1 - 6 for different $G$, $D^{-1}$, $M$, $m$, $N$, $Sc$, $So$, $\gamma$ and $N_1$. It is found that the shear stress enhances with increase in $G$. Higher the Lorentz force/lesser the permeability of the porous medium smaller $|\tau|$ at $z = 0$. An increase in the Hall parameter $m$ enhances $|\tau|$ at the wall (table.1). When the molecular buoyancy force dominates over the thermal buoyancy force the stress enhances when the buoyancy forces act in the same direction and for the forces acting in opposite directions it depreciates at $z=0$. It reduces with increase in the radiation parameter $N_1 \leq 5$ and enhances with $N_1 \geq 10$ (tables.2). From table 3, we find that lesser the molecular diffusivity smaller $|\tau|$ at $z = 0$ and for further lowering of the molecular diffusivity larger $|\tau|$. The variation of $\tau$ with soret parameter $S_0$ shows that $|\tau|$ enhances with $|S_0|$ ($S > 0$) (table.4). An increase in the Ekman number $E^{-1}$ or the chemical reaction parameter $\gamma$ results in a depreciation in the stress at $\gamma=0$. (table.5). The variation of $\tau$ with $P$ shows
that higher the thermal conductivity (P\leq 10) larger the stress and for further higher P\geq 15 smaller the stress at z=0(table.6)

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{N} & \textbf{I} & \textbf{II} & \textbf{III} & \textbf{IV} \\
\hline
\textbf{2} & -0.34169 & -1.23396 & -1.56770 & -1.67809 \\
\textbf{4} & -1.14576 & -1.65524 & -2.07927 & -2.21784 \\
\textbf{6} & -1.37968 & -1.98009 & -2.47466 & -2.63535 \\
\hline
\end{tabular}
\caption{Nusselt Number (Nu) at z = 0}
\end{table}

The rate of heat transfer (Nu) at z=0 is shown in table.1. It is found that the rate of heat transfer enhances with increase in the heat sources parameter \(\alpha\). Higher the radiation parameter \(N_1\) larger the rate of heat transfer.

The Sherwood number (Sh) at z = 0 is shown in table 2&3 for different values of Sc, So, N, N1, \(\gamma\) and P. Lesser the molecular diffusivity larger |Sh| at the wall. Also it enhances with increase in the soret parameter So>0 and reduces with increase in So<0. When the molecular buoyancy force dominates over the thermal buoyancy force the rate of mass transfer experiences a depreciation when the buoyancy forces act in the same direction and for the forces acting in opposite directions it enhances at the boundary(table.2). The rate of mass transfer enhances with increase in the radiation parameter N1. Also it reduces with \(\gamma\leq 1.5\) and Sc\leq 0.6 and enhances with Sc>1.3 and for higher values of \(\gamma>2.5\) it enhances at the boundary(table.3).
### Table-2 Sherwood Number (Sh) at \( z = 0 \)

<table>
<thead>
<tr>
<th>Sc</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
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</tbody>
</table>

### Table-3 Sherwood Number (Sh) at \( z = 0 \)

<table>
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<tr>
<th>Sc</th>
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<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
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### REFERENCES