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Analytical study of current density on a homogeneous conductor using relativistic and non-relativistic approach

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ABSTRACT

This paper presents analytical study of current density using nonrelativistic in parallel with relativistic concept. Each of these concepts was used to deduce the current density relating to the rate of flow of particle density. This represents steady equation $\frac{\partial \rho}{\partial t} = \nabla \cdot J$. The steady equation was solved by applying the divergence theorem to analyze the behaviour of charge density in a homogeneous conductor and dielectric materials.

Key words: current density, relativistic equation, continuity equation, charge density Klein-Gordon equation, Schrodinger equation.

INTRODUCTION

The study of current density has been of interest in the past as many books have been written that treated it. Current density J is often considered as a measure of density of electric current whose magnitude is the electric current per cross-sectional area and may be specified as a mean current density or as current density at a point. Many authors such as Jackson, 1974, Jones 1989 etc had discussed current density in cause of field equation involving Maxwell's equations. In quantum mechanics current density has been considered as probability current sometimes known as probability flux [cohen, 1976] and some case has been extended to the study of relativistic fermions [Bjorken and Drell, 1964] using quantum mechanical

and relativistic quantum idea. How in our study we used non relativistic and relativistic idea parallel to deduce an expression for steady equation and subsequently used it to analyze current density in a homogeneous and non homogeneous conductor.

The methods used here in the study involves non-relativistic.

Quantum mechanical

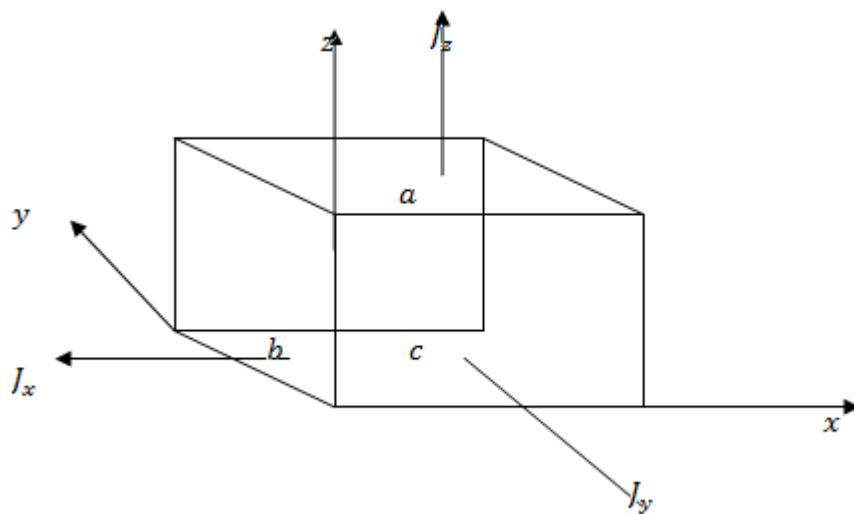
Relativistic approach

NON RELATIVISTIC CONSIDERATION OF CURRENT DENSITY

In this case we considered a volume, \mathbf{v} of particles moving in all directions and assuming no sources or sinks. That is particles are neither being created nor destroyed at any point of the volume $d\mathbf{v}$. The density of the particles at any given point is

$$P(x) = \Psi(x) \quad \Psi(x) = |\Psi(x)|^2 \quad (1)$$

Now considering an elemental volume $d\mathbf{v}$ around the point $X(x,y,z)$ with faces $a = dx dy$, $b = dy dz$ and $c = dz dx$



Fig; 1. Flow of current, J into and out of a volume, $d\mathbf{v}$. (Maduemezia and Coker, 2003).

If the current density $J = (J_x, J_y, J_z)$ flows normally into and out of the volume to the faces respectively as shown in fig 1, then x – direction, the current flowing into the volume, $d\mathbf{v}$ is $J_x h$ and the current out of $d\mathbf{v}$

$$\left[J_x + \frac{\partial J_x}{\partial x} dx \right] b$$

Thus the net current flowing into $d\mathbf{v}$ is

$$-\frac{\partial J_x}{\partial x} b \, dx = -\frac{\partial J_x}{\partial x} dx \, dy \, dz \quad (2)$$

In the same manner, the current flowing in the y – and z – direction into dv is

$$-\frac{\partial J_y}{\partial y} dx \, dy \, dz \text{ and } -\frac{\partial J_z}{\partial z} dx \, dy \, dz \text{ respectively.}$$

Based on this, the total current flowing into the elemental volume dv is

$$-\left[\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right] dv = -\nabla \cdot J \, dv \quad (3)$$

This expression as obtained in equation(3) is considered equal to the rate of increase of particle number in the volume which is expressed as $\frac{\partial P}{\partial x} dv$. With this, it follows that

$$\frac{\partial P}{\partial t} + \nabla \cdot J \, dv = 0 \quad (4)$$

Equation (4) in its form is an expression representing continuity equation which involves probability and density.

i. Quantum Mechanically

We first write time dependent Schrodinger equation as

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi = -i\hbar \frac{\partial \Psi}{\partial t} \quad (5)$$

[Davydov, 1976]

Where V is a real Potential function? However the complex conjugate of equation (5) is

$$\frac{-\hbar^2}{2m} \nabla^2 \Psi^* + V \Psi^* = \frac{\hbar}{i} \frac{\partial \Psi^*}{\partial t} \quad (6)$$

No to obtain the required expression, we eliminate the term V from equations (5) from left with Ψ^* do the same to equation (6) but this time we use Ψ and then subtract equation (5) from equation (6) to obtain.

$$\frac{\partial(\Psi^* \Psi)}{\partial t} + \nabla \left[\frac{\hbar}{2im} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) \right] = 0 \quad (7)$$

Now relating the particle density

$$\rho = \Psi^* \Psi \quad (8a)$$

and particle current density.

$$J = \left[\frac{\hbar}{2im} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) \right] \quad (8b)$$

Equation (7) becomes

$$\frac{\partial \rho}{\partial t} = \nabla \cdot J \quad (9)$$

iii Relativistic Concept

Using relativistic energy

$$E^2 = C^2 P + m^2 C^4$$

[Pitaevskii, 1979]

Where E =energy, C =speed of light m =mass of the particle and P =monientum we obtain.

$$-h^2 \frac{\partial^2 \Psi}{\partial t^2} = -h^2 c^2 \nabla^2 \Psi + m^2 C^4 \Psi \quad (10)$$

After substituting the following expression

$$E \rightarrow ih \frac{\partial}{\partial t}; P = -ih \nabla \quad (11)$$

Equation (11) is known as Klein-Gordon equations which in other words describe an expression for probability and current density.

In order to do that, we hand the equation in the same manner we did to equations (5) and (6).

It will result into

$$P(r, t) = \frac{ih}{2mc^2} \left(\Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t} \right) \quad (12)$$

and

$$J(r, t) = \frac{h}{2im} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) \quad (13)$$

Equation (4), (9) can be related to equation (12) and (13) which describe also that both Ψ its derivative is continuous every where in the volume and clearly presents the concept of continuity equation. [Griffiths, 1999]

Since it is know that the motion of charge constitutes current which relates magnitudes of charge and its velocity, then current density at a point is

$J = Pv$ [Stratton 1941] and if it flows in the normal direction of the velocity, v the current 1 across the surface is

$$I = \int_S J \cdot n \, ds \quad (14)$$

(Jones 1986)

Applying the divergence theorem on the continuity equation gives us

$$\int_S J \cdot n \, ds = -\frac{\partial}{\partial t} \int_S \rho \, dx \quad (15)$$

For a perfect conductor in which the conductivity, σ is infinite,

$$J = \sigma E \quad [Batut, 1964] \quad (16)$$

Where E electric field with this consideration, the continuity equation becomes

$$\operatorname{div} \sigma E + \frac{\partial \rho}{\partial t} = dJ + \frac{\partial \rho}{\partial t} = 0 \quad (17)$$

Eliminating E , we obtain

$$\epsilon \frac{\partial \rho}{\partial t} + \sigma \rho = g r \operatorname{ad} (\epsilon/\sigma) \quad (18)$$

If the conductor is homogeneous, the right hand side vanishes and equation (18) is solved to obtain

$$\rho = \rho_0 \exp(-\sigma t/\epsilon) \quad (19)$$

This tells us that when the conductivity of the material is positivity the charge density decays exponentially.

If on the other hand the conductor is not homogeneous, the right hand side of equation does not vanish to zero unless (ϵ/σ) happens to be constant and if under this condition, a steady state is eventually attrition there is a charge density given by

$$\rho = J \cdot \operatorname{grad}(\epsilon/\sigma) \quad (20)$$

This signifies an expression that represents the occurrence of free charges on a given dielectric which is known as dielectric absorption.

CONCLUSION

Analytical study of current density was carried out using non relativistic and relativistic method was carried out from where an expression for current density was derived respectively Klein–Gordon equation which is a relativistic counter part of the Schrodinger equation was used to drive expression for continuity equation involving probability and density. The current flowing through a surface was obtained by applying divergence theorem that enabled us to obtain the current density for perfect homogeneous conductor with an infinite conductivity σ and a non homogeneous conductor with a finite conductivity or dielectric material.

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