Alternate queuing system for tatkal railway reservation system

S. Vijay Prasad and V. H. Badshah

School of Studies in Mathematics, Vikram University, Ujjain, M. P, India

ABSTRACT

In the country like India where Railway is one of the most popular and cheapest means of transportation, it is always difficult to book confirmed tickets for the journey. The population that the country has it doesn't match up with number of trains running on various routes especially those connecting the metro cities. Indian Railway is trying to meet the ever increasing demand of over billion people. This paper presents the Railway Reservation System which is at Bapat Chourah, Indore, M.P, India, after studying advantages and disadvantages of system structure, this paper recommends changing the present queuing system to alternate queuing system, to avoid the inconvenience of passengers. It was proved that this model of the queuing system is feasible and the results are effective and practical.

Key words: M/M/1 queuing model, E_k/M/1 queuing model, Alternative queuing model, Tatkal railway reservation system

INTRODUCTION

Indian Railways, which had a modest beginning in 1853, has since then been an integral part of the nation - a network that has held together a population of one billion. A self-propelled social welfare system that has become the lifeline of a nation. Indian Railways has woven a sub-continent together and brought to life the concept of a united India. The railways in India are the largest rail web in Asia and the world’s second largest under one management. With a huge workforce of about 1.65 million, it runs some 11,000 trains every day, including 7,000 passenger trains. The tale of how railway communication gained foothold in India, where the locomotive was once considered as a “fire-spitting demon”, is indeed an interesting one. As matters started to gain momentum, the Bombay Great Eastern Railway locally prepared plans for constructing a railway line from Bombay to the Deccan. But the British already had a concrete plan in their minds and soon things began to take shape. The line in Bombay was ready by November 1852 and a few engineers and directors of the GIP (Great Indian Peninsula Railway) Company had a trial run between Bombay and Thane. However officially, the first train in India (and in Asia) was flagged off on 16 April 1853, Saturday, between Mumbai and Thane, a distance of 34 Kilometers. The importance of the day can be gauged from the fact that the Bombay government declared the day as a public holiday. The train, hauled by three engines Sindh, Sahib and Sultan carried as many as 400 passengers in its 14 coaches on its debut run. The Great Indian Peninsula Railway had ordered a set of eight locomotives from Vulcan Foundry, England, for the purpose. A suit of Durbar Tents erected at Thane welcomed the first train and a cover for four hundred persons was built with tables laid with menu literally groaning under every delicacy of the season. The Indian railways are considered to be having the largest network of rails and having the busiest rail network in all over the world. Even today there is a continuous growth in the network of the railways. Reserved travel by Indian Railways is facilitated by the Passenger Reservation System (PRS). PRS provides reservation services to nearly 1.5 to 2.2 million
passengers a day on over 2500 trains running throughout the country. The PRS Application CONCERT (Countrywide Network of Computerized Enhanced Reservation and Ticketing) is the world’s largest online reservation application, developed and maintained by CRIS. The system currently operates from five data centers. The server clusters are connected together by a core network that enables universal terminals across country, through which the travelling public can reserve a berth on any train, between any pair of station for any date and class. Tatkal booking is offered in Indian Railways to help the passengers to travel in urgency. A specific number of tickets are kept aside exclusively for this purpose. Tatkal ticket can be booked only one day in advance prior to your journey. Usually railway tickets can be booked as early as two months in advance for a planned journey. But there are occasions one has to travel on an emergency such as for a hospital visit, to attend an interview, or for any other personal or business purpose, and the traveler could not plan it in advance and reserve the railway ticket. Tatkal is a blessing in such occasions. Tatkal bookings are available only in certain trains, not all. Tatkal tickets are allowed in all classes except First AC. Tatkal Booking starts one day in advance (reduced from 2 days) excluding the day of journey e.g. for a journey on third day, bookings would open at 10 am on second day. However, the day of journey is defined as the day of chart preparation. So if the train starts e.g. on third day, and reaches the desired boarding station on fourth day the Tatkal booking will start on second and not third. About 1.16 million berths and seats are booked a day, out of which Tatkal accounts for 0.17 million seats and berths on 2677 trains. Tatkal booking yielded revenue of Rs. 8470 million during 2011-12. This paper presents the Railway Reservation System which is at Bapat Chourah, Indore, M.P, India, after studying advantage and disadvantage of system structure, this paper recommends changing the present queuing system to alternate queuing system, to avoid the inconvenience of passengers.

MATERIALS AND METHODS

A. The basic indexes of the queuing systems

\[ n = \text{Number of passengers in the system at time } t \]
\[ \lambda = \text{Mean arrival rate (number of arrival per unit time)} \]
\[ \mu = \text{Mean service rate per busy server (number of passengers served per unit time)} \]
\[ \rho = \text{Expected fraction of time for which server is busy} \]
\[ P_n = \text{Steady state probability of exactly } n \text{ passengers in the system} \]
\[ L_q = \text{Expected number of passengers in the queue} \]
\[ L_s = \text{Expected number of passengers in the system (waiting + being served)} \]
\[ w_q = \text{Expected waiting time for a passenger in the queue} \]
\[ W_e = \text{Expected waiting time for a passenger in the system (waiting + being served)} \]

B. Queuing Models

Figure 1: Single Queue – Single server model

C. M/M/1 Model (Single server queuing model)

In this model Arrivals are described by Poisson probability distribution and come from infinite population, queue discipline is first come first serve, single server and service time follows exponential distribution. This queuing system can be applied to a wide variety of problems as any system with a large number of independent passengers, and can be approximated as a Poisson process: \( P_n \) (\( n = 0, 1, 2, \ldots \)) is the probability distribution of the queue length \([1, 2, 3, 5]\).

Expected number of passengers in the system (waiting + being served)

\[ L_s = \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n, \]
\[ = \frac{\lambda}{(1-\rho)} = \frac{\lambda}{\mu-\lambda} \]

(1)

Expected number of passengers waiting in the queue (i.e. queue length)

\[ W_q = \frac{L_q}{\lambda} = \frac{\mu}{\lambda} \]
\[ L_q = \sum_{n=1}^{\infty} (n-1)P_n = \frac{j^2}{\mu(\mu - \lambda)} \]  

Expected waiting time for a passenger in the queue
\[ w_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{I_q}{\lambda} \]  

Expected waiting time for a passenger in the system (waiting and service)
\[ W' = \text{Expected waiting time in queue + Expected service time} \]
\[ = W_q + \frac{1}{\mu} \]
\[ = \frac{1}{\mu - \lambda} = \frac{I_q}{\lambda} \]

\section{D. \textit{Ek}/\textit{M}/1 model}
Consider a queuing system in which the inter-arrival times follows \( k \)-Erlang distribution with mean \( 1/\lambda_2 \) in which an arrival has to pass through \( k \) phases, each with a mean time \( 1/\lambda_2 \) prior to entering the service the mean service time follows an exponential distribution with mean \( 1/\mu_2 \)

Let \( p_n \) be the probability that there are \( n \) passengers in \textit{Ek}/\textit{M}/1 queuing system \[ P_n = \sum_{j=nk}^{nk+k-1} P_j^{(p)} \]  

Where \( P_j^{(p)} \) is the probability of completion of \( j \) phases and is given by
\[ P_j^{(p)} = \frac{k\lambda_2^{p_j}}{\mu_2} r_0^{j-k} \]  

Where \( r_0 \in (0,1) \)
\[ \mu_2 z^{k+1} - (k\lambda_2 + \mu_2)z + k\lambda_2 = 0 \]

Since \( P_0^{(p)} = 1 - r_0 \), taking \( z_2 = \lambda_2/\mu_2 \), equation (6) becomes
\[ P_j^{(p)} = \rho(1 - r_0)^{j-k} \]

Therefore equation (5), reduces to
\[ P_n = \rho(1 - r_0^k)(r_0^k)^{n-1} \]  

which is a geometric distribution

The performance measures of \textit{Ek}/\textit{M}/1 queuing model
The expected number of passengers in the queue \[ L_q = \frac{\rho n_k}{(1-r_0^k)} \]  

The expected number of passengers in the system \[ L_s = \frac{\rho}{(1-r_0^k)} \]  

The expected waiting time of a passenger in the queue \[ W_q = \frac{r_0^k}{\mu_2(1-r_0^k)} \]  

Therefore equation (10), reduces to
\[ W_s = \frac{\rho}{\lambda_2(1-r_0^k)} \]

\section{E. Proposed Alternative Queuing Model}
Here is the proposing alternate queuing model. When the passenger arrive at reservation counter, service provider will provide a appointment slip (which is mentioned with serial number, date and time of the service on that) to the passenger, as soon as passenger gets the appointment slip, he/she will be free from the waiting line, and again he/she joins the queue for service according to the mentioned time on the appointment slip, with this passengers need not to wait from the early morning to till 10:00AM for the service. The proposing queuing model consisting of two
queuing models one is $M/M/1$ queuing model and the other one is $E_k/M/1$ queuing model. These two queuing models are working at two different times the $M/M/1$ queuing model will work for appointment request few hours before (one day before for two hours from 6:00 pm to 8:00 pm) starting reservation system $E_k/M/1$ queuing model which will starts next day from 10:00 am, assuming that $M/M/1$ queuing model follows the mean arrivals $\frac{1}{\lambda_1}$ are described by Poisson distribution and come from infinite population, queue discipline is first come first serve, single server and mean service time $\left(\frac{1}{\mu_1}\right)$ follows exponential distribution and $E_k/M/1$ queuing model follows the inter-arrival times follow k-Erlang distribution with mean $(1/\lambda_2)$ in which an arrival has to pass through k phases, each with a mean time $1/k\lambda_2$ prior to entering the service the mean service time follows an exponential distribution with mean $1/\mu_2$.

A.  Case I

Analysis of the system service provider point of view, the data was collected from the Railway reservation counter at Bapat Chourah, Indore, M.P, India. Tatkal booking starts from 10:00 am, the data was collected before starting the booking 25 ($L_q$) passengers are already waiting in the queue, after that 20 passengers are join the queue in between 10:00 am to 11:00 am, here it is little difficult to decide arrival rate of passengers because some passengers are joining in the waiting line at 5:00 am, some are at 6:00 am some are at 7:00 am and soon, total 37 passengers are booked their tickets from 10:00 am to 11:00 am (i.e. service rate is 37 passengers per hour ($\mu$), here arrival rate can estimate theoretically using by the formula but practically it is not considerable

From equation (2) we can estimate the arrival rate of passengers per hour

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$25 = \frac{\lambda^2}{37(37-\lambda)}$$

$$\lambda^2 + 925\lambda - 34225 = 0,$$

$$\lambda = 35.63$$

The arrival rate of passengers is 35.6 per hour

$$\rho = \frac{\lambda}{\mu} = \frac{35.63}{37} = 0.963$$

$$P_0 = 1 - \rho = 1 - 0.96297 = 0.03703$$

$$L_2 = 26.00730, \ W_q = 0.70290 \ hr, \ W_s = 0.72993 \ hr$$
B. Case II
Analysis of the system passenger point of view. In case-I the performance measures are calculated service provider point of view using formula but practically the problem is arises from the passengers those who are waiting from 5:00 am to 10:00 am, if take note of this then the arrival rate of passengers become 45 per 6 hours (i.e. 7.5 passengers per hour)

\[ \rho = \frac{\lambda}{\mu} = \frac{7.5}{37} = 0.203 \]
\[ p_0 = 1 - \rho = 1 - 0.2027 = 0.7973 \]

\[ L_q = 0.05153 \quad L_s = 0.25424, \quad W_q = 0.00687 \text{ hr}, \quad W_s = 0.0339 \text{ hr} \]

From the analysis of case –I and case – II the utilization factors are 0.963 in case –I and 0.203 in case – II, service provider thinks that utilization factor of the system is good but passenger (service receiver) thinks that utilization of the system is very poor, both thinking are contradicts each other, the important point is that the service provider not consider the waiting time of the passengers from early morning to till 10:00 am for service. This is the major drawback of the present queuing system. For all age groups of passengers it is very difficult to wait from early morning to till 10:00 am for service. This reason would be cause of passenger’s dissatisfaction.

C. Case – III
Analysis of alternative queuing model

a. Example for \( M/M/1 \) model
The mean arrival rate of the passengers \( \lambda_1 = 36/\text{hr} \) and the mean service rate \( \mu_1 = 40/\text{hr} \) then \( \rho = 0.9 \)

The performance measures of \( M/M/1 \) queuing model as follows

\[ L_{q_1} = 8 \quad L_{s_1} = 9 \quad W_{q_1} = 0.222 \text{ hr} = 13 \text{ min.} \quad \text{and} \quad W_{s_1} = 0.25 \text{ hr} = 15 \text{ min} \]

b. Example for \( E_k/M/1 \) model
Arrivals are \( E_{36} \) with mean inter arrival time of 3 min, service time is Exponential with mean service time of 2 min, \( \lambda_2 = 20/\text{hr} \quad \mu_2 = 30/\text{hr} \) and \( \rho_2 = 0.6666 \quad k = 36 \)

\[ 30r^{36} - 750z + 720 = 0 \]
By using Newton’s Iteration method the real root of the above equation \( r = 0.9767 \)

(i.e \( r_0 = 0.9767 \))

The performance measures of \( E_k/M/1 \) queuing model as follows

\[ L_{q_2} = 0.499 \quad L_{s_2} = 1.17 \quad W_{q_2} = 0.025 \text{ hr} = 1.5 \text{ min.} \quad \text{and} \quad W_{s_2} = 0.0582 \text{ hr} = 3.5 \text{ min}. \]

c. The total performance measures of alternate queuing models
The average utilization factor of the system is 0.7833

The total expected number of passengers in the queue \( L_Q = L_{q_1} + L_{q_2} = 8.499 \cong 9 \)

The total expected number of passengers in the system \( L_s = L_{s_1} + L_{s_2} = 10.17 \cong 10 \)

The total expected waiting time of a passenger in the queue

\[ W_Q = W_{q_1} + W_{q_2} = 14.5 \text{ min.} \cong 15 \text{ min}. \]

The total expected waiting time of a passenger in the queue

\[ W_s = W_{s_1} + W_{s_2} = 18.5 \text{ min.} \cong 19 \text{ min}. \]
CONCLUSION

From the analysis of case –I and case – II the utilization factors are 0.9629 in case –I and 0.2027 in case – II, service provider thinks that utilization factor of the system is good but passenger (service receiver) thinks that utilization of the system is very poor, both thinking are contradicts each other, the important point is that the service provider not consider the waiting time of the passengers from early morning to till 10:00 am for service. This is the major drawback of the present queuing system. For all age groups of passengers it is very difficult to wait from early morning to till 10:00 am for service. This reason would be cause of passenger’s dissatisfaction. In the case of proposed alternate queuing system there will be no contradiction of the utilization of the system between the service provider and passengers. The average utilization factor of the system is also considerable and most important thing is that passengers need not to wait from early morning to till 10:00 am for service this will be cause of passenger satisfaction too. After studying advantages and disadvantages of system structure of the Railway Reservation System which is at Bapat Chourah, Indore, M.P, India, the paper recommends changing the present queuing system to proposed alternate queuing system to avoid the inconvenience of passengers. It was proved that this model of the queuing system is feasible and the results are effective and practical.

REFERENCES