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A numerical model for the effect of stenosis shape on blood flow through an artery using power-law fluid

Sapna Singh*and Rajeev Ratan Shah

**Department of Mathematics, Harcourt Butler Technological Institute, Kanpur, India*
Divisional Planning Office, LDA, Complex, Kapoorthala, Lucknow, India

ABSTRACT

The objective of this present numerical model is to investigate the effect of shape of stenosis on blood flow through an artery using power-law fluid model. Blood is modeled as power-law fluid in a uniform circular tube with an axially non-symmetric but radially symmetric stenosis. The expressions for dimensionless resistance to flow, wall shear stress and apparent viscosity have been obtained. The variation of resistance to flow, wall shear stress and apparent viscosity with stenosis shape parameter, stenosis length and stenosis size has been shown graphically. It has been found that the resistance to flow, wall shear stess and apparent viscosity decreases as stenosis shape parameter increases but increases as stenosis size and stenosis length increases. The significance of the present model over the existing models has been pointed out by comparing the results with other theories both analytically and numerically. This information of blood could be useful in the development of new diagnosis tools for many diseases.

Keywords: Stenosed artery, Power-law fluid model, Resistance to flow, Wall shear stress, Apparent viscosity, Stenosis shape parameter.

INTRODUCTION

The hemodynamics behavior of the blood flow is influenced by the presence of the arterial stenosis. If the stenosis is present in an artery, normal blood flow is disturbed. The intimal thickening of stenotic artery was understood as an early process in the beginning of atherosclerosis. Atherosclerosis is the leading cause of death in many countries. There is considerable evidence that vascular fluid dynamics plays an important role in the development and progression of arterial stenosis, which is one of the most widespread diseases in human beings. The fluid mechanical study of blood flow in artery bears some important aspects due to the engineering interest as well as the feasible medical applications. Various investigators [1-4] have emphasized that the formation of intravascular plaques and the impingement of ligaments and spurs on the blood vessel wall are some of the major factors for the initiation and development of this vascular disease. The fruitful study of [5, 6] has pointed out that the variation of resistance to flow and the wall shear stress with the axial distance are physiologically important quantities. [7,8] have shown theoretical results of for the velocity profiles, pressure drop, wall shearing stress and separation phenomena for special geometries for Newtonian model of blood. In the series of the papers [9-12] the effects on the cardiovascular system can be understood by studying the blood flow in its vicinity. In these studies the behavior of the blood has been considered as a Newtonian fluid. However, it may be noted that the blood does not behave as a Newtonian fluid under certain conditions. It is generally accepted that the blood, being a suspension of cells, behaves as a non-Newtonian fluid at low shear rate [13, 14]. It has been pointed out by [15] that the flow behaviour of blood in a tube of small diameter (less than 0.2 mm) and at less than 20sec^{-1} shear rate, can be represented by a power-low fluid model. In these discussed models, the investigators have not dealt with the effect of stenosis shapes on resistance to flow wall shear stress and apparent viscosity. The published literature on the stenosis further reveals that very few studies are concerned with the problem of symmetric stenosis. In an actual situation, however, the increase in the arterial wall thickness would not be symmetrical. To generalize the problem further, an attempt is therefore made in the present investigation. Keeping these in view, in this paper, we have investigated the effects of stenosis shape parameter on resistance to flow, wall shear stress and apparent viscosity with stenosis size and stenosis length, in an artery by introducing blood as Power-law fluid model.

Formulation of the problem

In the present analysis, it is assumed that the stenosis develops in the arterial wall in an axially non-symmetric but radially symmetric manner and depends upon the axial distance z and the height of its growth. In such a case the radius of artery, R(z) can be written as follows [Fig (1)]:

$$\frac{R(z)}{R_0} = \begin{cases} 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], & d \le z \le d + L_0 \\ 1, & \text{otherwise,} \end{cases} \qquad (1)$$

where R(z) and R_0 is the radius of the artery with and without stenosis, respectively. L_0 is the stenosis length and d indicates its location, $m \ge 2$ is a parameter determining the stenosis shape and is referred to as stenosis shape parameter. Axially symmetric stenosis occurs when m = 2, and a parameter A is given by;

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)} \qquad (2)$$

where δ denotes the maximum height of stenosis at $z = d + L_0 / m^{1/(m-1)}$. The ratio of the stenosis height to the radius of the normal artery is much less than unity.

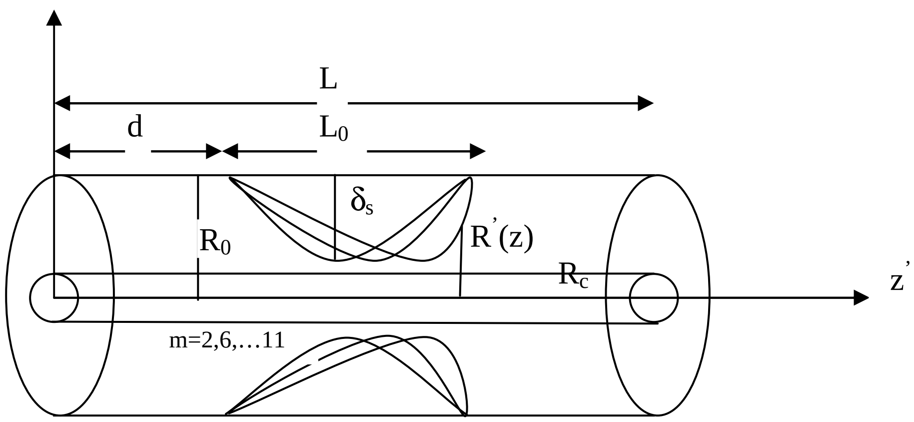

Fig (1) Geometry of Stenosis

Conservation equation and boundary condition

The equation of motion for laminar and incompressible, steady, fully-developed, one-dimensional flow of blood whose viscosity varies along the radial direction in an artery reduces to [4]:

$$\left.\begin{aligned} 0 &= -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\tau)}{\partial z}, \\ 0 &= -\frac{\partial P}{\partial r}, \end{aligned}\right\} \tag{3}$$

where (z, r) are co-ordinates with z measured along the axis and r measured normal to the axis of the artery.

Following boundary conditions are introduced to solve the above equations,

$$\left.\begin{aligned} &\partial u/\partial r = 0 && \text{at } r = 0 \\ &u = 0 && \text{at } r = R(z) \\ &\tau \text{ is finite} && \text{at } r = 0 \\ &P = P_0 && \text{at } z = 0 \\ &P = P_L && \text{at } z = L \end{aligned}\right\} \tag{4}$$

Analysis of the problem

Power-law fluid: Non-Newtonian fluid is that of power-law fluid which have constitutive equation,

$$\left.\begin{aligned} &\left(-\frac{du}{dr}\right) = \left(\frac{\tau}{\mu}\right)^{1/n} = f(\tau), \\ &\text{where } \tau = \left(-\frac{dp}{dz}\right)\frac{R_c}{2} \end{aligned}\right\} \tag{5}$$

Where u is the axial velocity, μ is the viscosity of fluid, (-dp/dz) is the pressure gradient and n is the flow behaviour index of the fluid.

Solving for u from equation (3), (5) and using the boundary conditions (4), we have,

$$\frac{du}{dr} = \left(\frac{P}{2\mu}\right)^{1/n} [(r - R_C)^{1/n}], \tag{6}$$

The volumetric flow rate Q can be defined as,

$$Q = \int_0^R 2\pi u r \, dr = \pi \int_0^R r\left(-\frac{du}{dr}\right) dr, \tag{7}$$

By the help of equations (6) and (7) we have,

$$Q = \left(\frac{P}{2\mu}\right)^{1/n}\left(\frac{n\pi}{(3n+1)}\right)(R)^{[(1/n)+1]} \tag{8}$$

From equation (8) pressure gradient is written as follows,

$$\frac{dp}{dz} = -2\mu\left(\frac{(3n+1)}{n\pi}Q\right)^{n}\frac{1}{(R)^{3n+1}} \tag{9}$$

Integrating equation (9) using the condition $P = P_0$ at $z = 0$ and $P = P_L$ at $z = L$. We have,

$$P_L - P_0 = \left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{2\mu}{(R_0)^{3n+1}} \int_0^L \frac{dz}{\left(R/R_0\right)^{1+3n}} \tag{10}$$

The resistance to flow (resistive impedance) is denoted by λ and defined as follows,

$$\lambda = \frac{P_L - P_0}{Q} \tag{11}$$

The resistance to flow from equation (11) using equations (10) can write as:

$$\lambda_0 = \left(\frac{(3n+1)Q}{n\pi}\right)^n \frac{2\mu}{Q R_0^{3n+1}} \left(\int_0^d dz + \int_0^{d+L_0} \frac{dz}{\left(R/R_0\right)^{3n+1}} + \int_{d+L_0}^{L} dz \right) \tag{12}$$

When there is no stenosis in artery then $R = R_0$, the resistance to flow,

$$\lambda_N = \left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{2\mu}{Q R_0^{3n+1}} L \tag{13}$$

From equation (11) and (12) the ratio of (λ_0 / λ_N) is given as;

$$\lambda = \frac{\lambda_0}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1}{L} \int_d^{d+L_0} \frac{dz}{(R/R_0)^{3n+1}} \tag{14}$$

Now the ratio of shearing stress at the wall can be written as;

$$\frac{\tau_R}{\tau_N} = \left(\frac{R_0}{R}\right)^{-3n} \tag{15}$$

$$\tau = \frac{\tau_R}{\tau_N} = \frac{1}{\left(1-\frac{\delta}{R_0}\right)^{3n}} \tag{16}$$

The apparent viscosity (μ_0/μ) is defined as follows ;

$$\frac{\mu_0}{\mu} = \frac{1}{(R/R_0)^4 f(\bar{y})} \tag{17}$$

RESULTS AND DISCUSSION

In order to have estimate of the quantitative effects of stenosis shape parameter (m= 2...11), stenosis size, stenosis length on resistance to flow, wall shear stress and apparent viscosity, computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, wall shear stress apparent viscosity for diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Fig 2-10 by using the values of parameter based on experimental data in stenosed artery.

Fig.2 reveals the variation of resistance to flow (λ) with stenosis shape parameter (m). It is observed that the resistance to flow (λ) decreases as stenosis shape parameter (m) increases, maximum resistance to flow (λ) occurs at (m = 2), i. e. in case of symmetric stenosis. The result is consisting with the result of [14]. Fig.3 consists the variation of resistance to flow (λ) with stenosis size (δ/R_0). It is evident that resistance to flow increases as stenosis size increases. Resistance to flow increase as stenosis grows or radius of artery decreases (this referred to as Fahraeus-Lindquist effect in very thin tubes). It is also shown in fig.4 that the resistance to flow increases with increasing value of stenosis length. Pontrelli [9] has found that the resistance of blood in diabetic patients is higher than in non-diabetic patients, resulting higher resistance to blood flow in the presence of magnetic effect. Thus diabetic patients with higher resistance to flow are more prone to high blood pressure. Therefore the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma. These results are consistent with the observation of [12]. In Fig.5 the variation of wall shear stress (τ) with stenosis shape parameter (m) has been shown. This figure depicts that wall shear stress (τ) decreases as stenosis shape parameter (m) increases. As the stenosis grows, the wall shearing stress (τ) increases in the stenotic region. These results are similar with the results of [16]. Fig.6 describes the variation of wall shear stress (τ) with stenosis size. This figure depicts that wall shear stress (τ) increases as stenosis size increases. These results are consistent to the observation of [12].

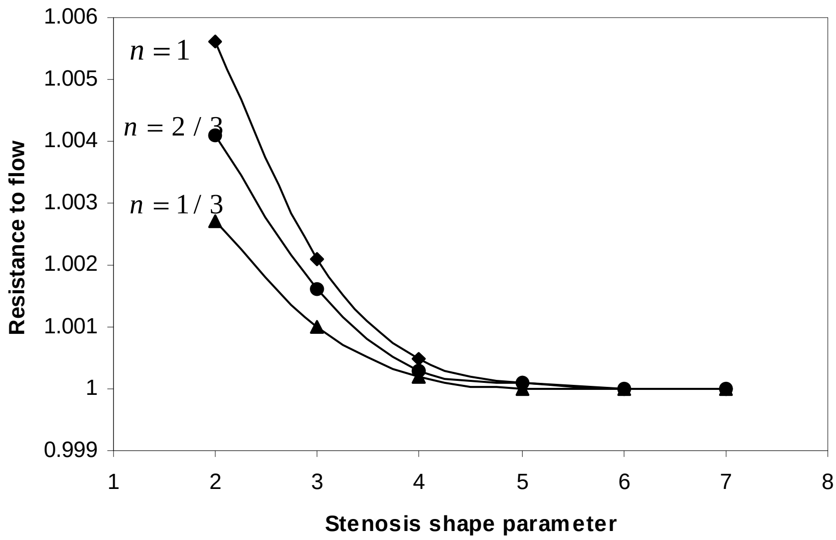

Fig.2 Variation of resistance to flow with stenosis shape parameter

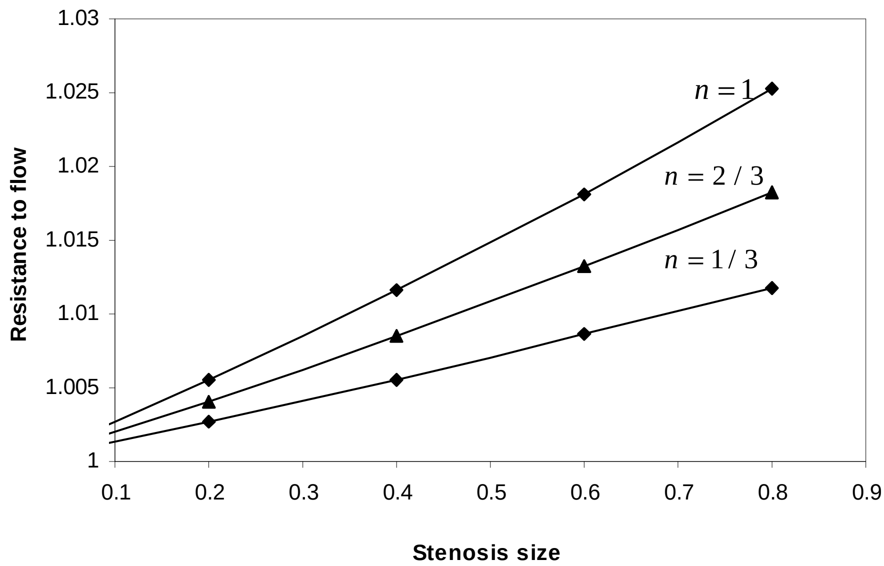

Fig.3 Variation of resistance to flow with stenosis size

Fig.7 reveals the variation wall shear stress with stenosis length for different values of stenosis size (δ/R_0). Graph depicts that wall shear stress increases as stenosis length increases. To capture the results for apparent viscosity with stenosis shape parameter, stenosis size, stenosis length, the graphs have been plotted in fig.8, fig.9, and fig.10. Apparent viscosity decreases as stenosis shape parameter increases, it is shown in fig.8, but decreases as stenosis size and stenosis length increases it is shown in fig. 9 and fig.10.

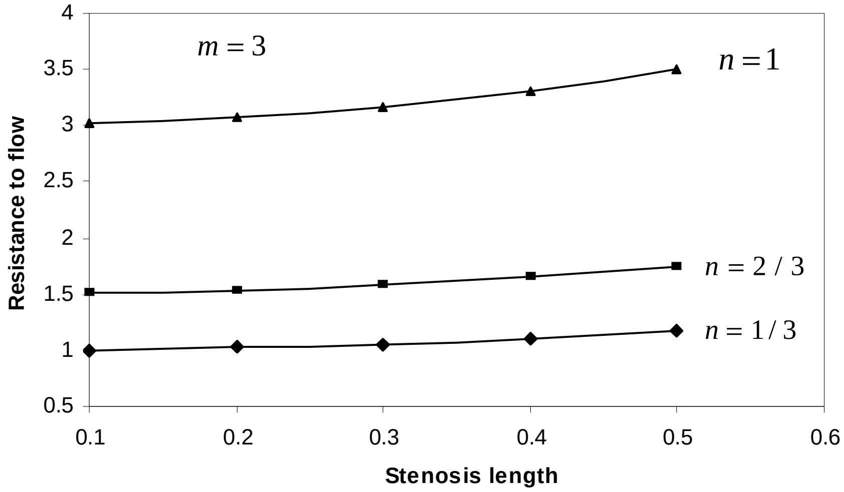

Fig.4 Variation of resistance to flow with stenosis length

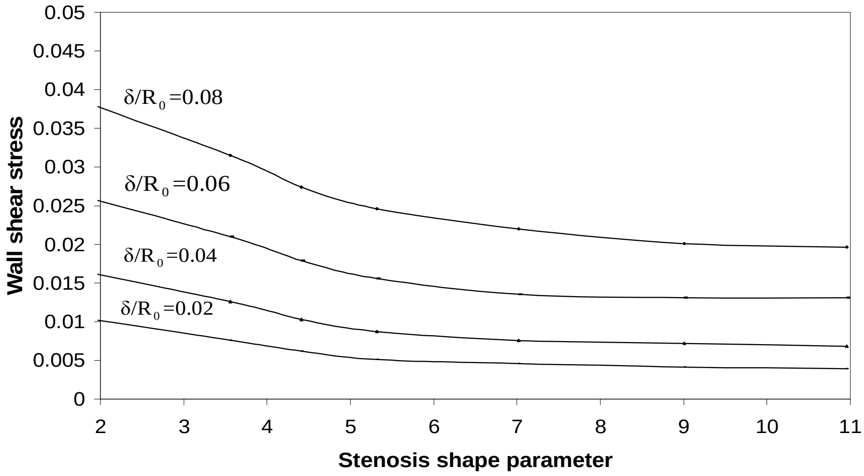

Fig.5 Variation of Wall shear stress with stenosis shape parameter

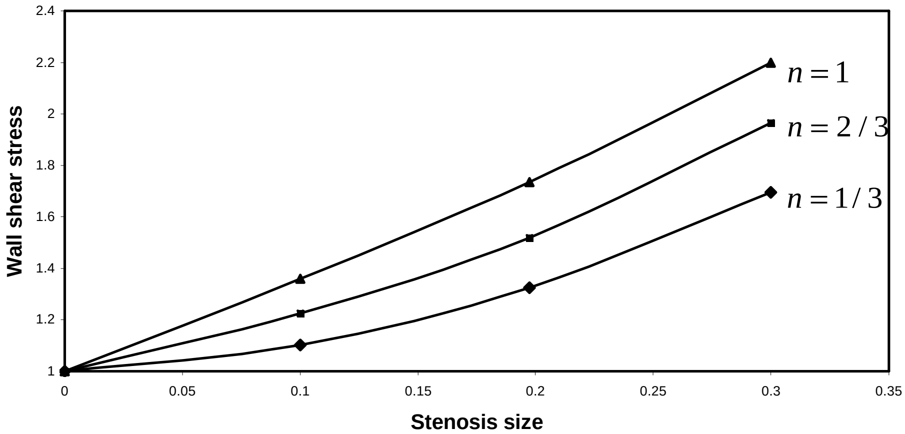

Fig.6 Variation of wall shear stress with stenosis size

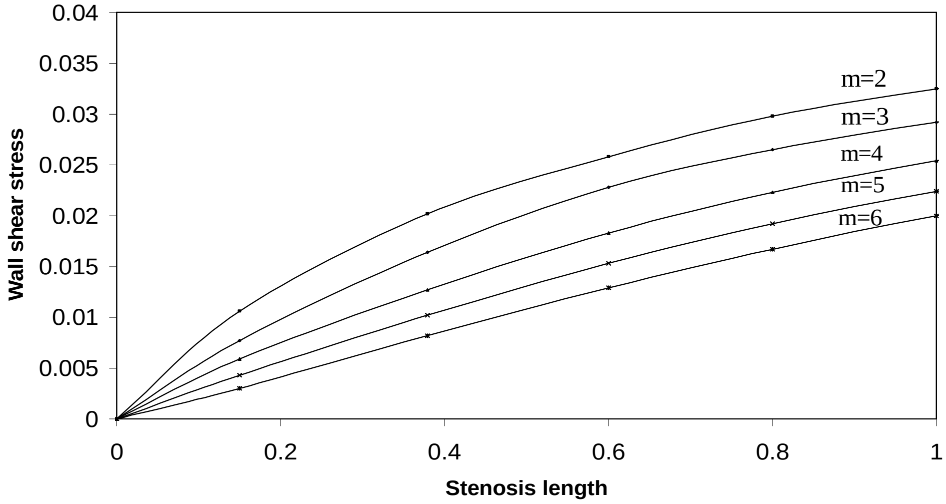

Fig.7 Variation of wall shear stress with stenosis length

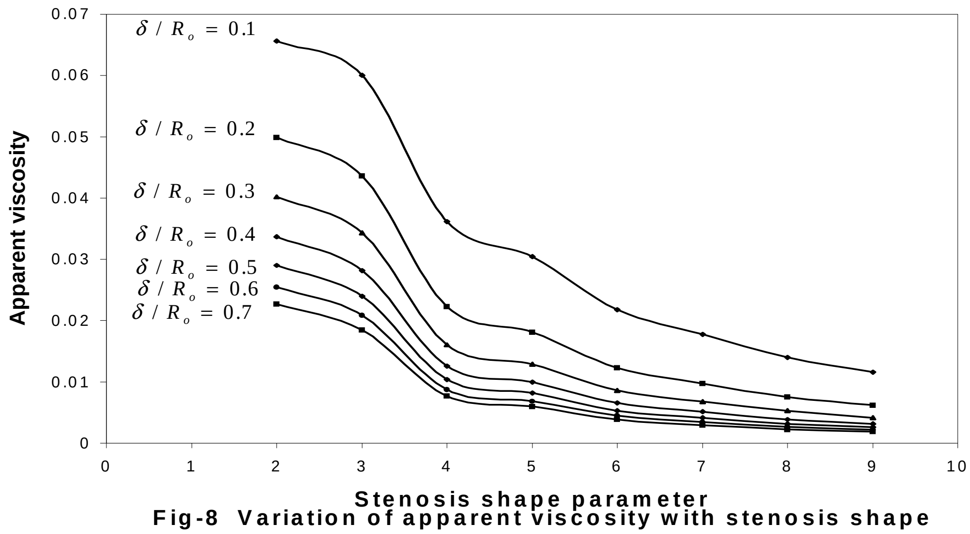

Fig-8 Variation of apparent viscosity with stenosis shape parameter

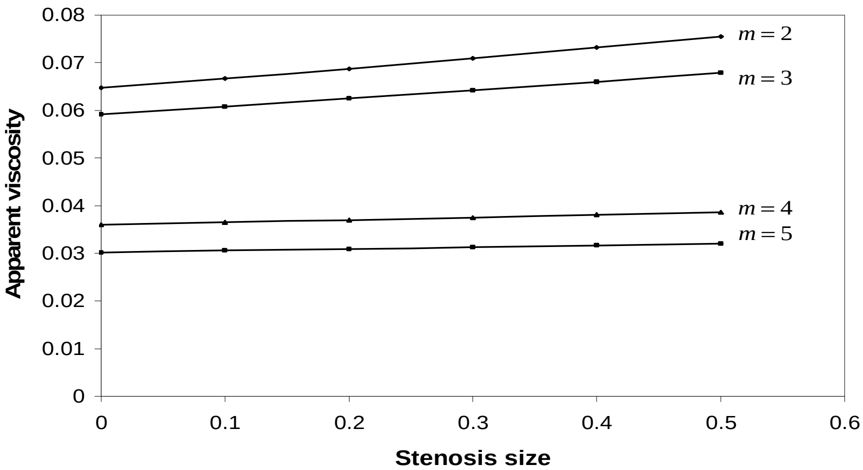

Fig.9 Variation of apparent viscosity with stenosis size

CONCLUSION

In his paper, we have studied the effect of stenosis shape parameter on resistance to blood flow, wall shear stress and apparent viscosity in an artery by introducing blood as Power-law fluid model. It has been concluded that the resistance to blood flow, wall shear stress and apparent viscosity increases as stenosis size and stenosis length increases but decreases as stenosis shape parameter increases. So it has shown that the results were greatly influenced by the change of stenosis shape parameter. In an artery flow, the viscosity of blood found to vary with the arterial radius decreasing with it. One may recollect that the diabetic patients are more prone to the various types of cardiovascular diseases. The viscosity of the diabetic patients is higher than that of normal. Therefore the blood viscosity of diabetic patients is lowered by regular dose of aspirin or injecting saline water in order to dilute the blood. This model is able to predict the main characteristics of the physiological flows and may have some interest in biomedical application.

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