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# A comparative study of induced transition in a ring cavity

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**ABSTRACT**

*The wave analysis and semiclassical theory of laser can be used to find out some related parameters of any Quantum devices. The power spectrum for the induced transition in a Fabry -Perot cavity with two and three mirrors has been worked out with the help of wave analysis involving only a few parameters like transition, absorption and diffraction. The results obtained by wave analysis have been compared with that obtained in the semiclassical theory of Laser. The salient point in the comparative study is that in wave analysis, the nature of the ring cavity is not clearly visualized whereas semiclassical theory throws the much needed light on the interaction of radiation with matter inside the cavity.*

**Key words:** Fabry-Perot cavity, power spectrum, Ring cavity, wave analysis.

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## INTRODUCTION

The semiclassical theory [1] as prescribed by Lamb [1, 2] has successfully explained a large number of laser systems. Although the assumptions in semiclassical theory are not particularly valid for TEA and GDL laser [3], they are quite good for typical operations of He- Ne and other gas lasers. Semiclassical theory has been applied to various types of cavities. The ring cavity [4-8] is one of very important cavities with considerable interest in science and engineering. It is worthwhile to note that the wave analysis to consider the problem of Laser oscillation was originally proposed by George Birnbaum [4]. The method contains some salient features which describe the laser behavior (induced and spontaneous emission) in an adequate way, while discussing about wave analysis, we have noted few interesting features which may be compared with those in a semiclassical theory. The purpose of the present work is present a comparison of the steady state theory of optical maser and the semiclassical theory of laser. It must be noted that the comparison is only for historical interest because we believe that the semiclassical and Quantum of Laser are considered as nearly complete theory.

## MATERIALS AND MATHODS

It is natural to consider the maser as an amplifier driven by spontaneous emission. In particular, consider the open sided resonator of the Fabry-Perot type, consisting of plane parallel, partially transmitting mirrors whose complex transmission and reflection coefficients are $t'$ and $r'$. Let the resonator be uniformly filled with a material whose complex propagation constant is $K = {(k-\alpha)}/{2+i\beta}$ , where $\alpha$ is the negative absorption, k is the dielectric loss of the host material and $\beta$ is the phase shift due o the maser atoms and the host material. To calculate the amplitudes of the waves, in a single cavity mode, generated by the spontaneous emission noise from a slab of active material. The reflected waves emanating from the mirror second to first sides of the slab are respectively,

$$\frac{t_2 A e^{-k(D-Z)}}{1-r_1 r_2 e^{-2kD}} \qquad \text{--- (1)}$$

$$\frac{r_1 t_2 A e^{-k(D+Z)}}{1-r_1 r_2 e^{-2kD}} \qquad \text{--- (2)}$$

Multiplying (1) and (2) by their complex conjugates and adding we find that the power per unit length of material per unit frequency range transmitted though mirror second is given by

$$P_{\nu,z} = \frac{t_2 A e^{-k(D-Z)}}{1-r_1 r_2 e^{-2kD}} \times \frac{t_2 A e^{-k^*(D-Z)}}{1-r_1 r_2 e^{-2k^* D}} \times \frac{r_1 t_2 A e^{-k(D+Z)}}{1-r_1 r_2 e^{-2kD}} \times \frac{r_1 t_2 A e^{-k^*(D+Z)}}{1-r_1 r_2 e^{-2k^* D}}$$

$$= \frac{A^2 t_2^2 e^{(\alpha-k)D} e^{-(\alpha-k)Z} + A^2 t_2^2 r_1^2 e^{(\alpha-k)D} e^{(\alpha-k)Z}}{1-r_1^2 r_2^2 e^{-2(k-\alpha)D} - 2 r_1 r_2 e^{-(k-\alpha)D} \cos 2\beta D}$$

$$P_{\nu,z} = \frac{A^2 t_2^2 e^{(\alpha-k)D} [e^{-(\alpha-k)Z} + r_1^2 e^{(\alpha-k)Z}]}{[1+r_1^2 r_2^2 e^{-2(k-\alpha)D}]^2 + 4 r_1 r_2 e^{-(k-\alpha)D} \sin^2 \beta D} \qquad \text{--- (3)}$$

The quantity $A^2$ is defined by

$$A^2 dz d\nu = \frac{n_2 \ a \ dz \ h\nu}{P'\tau} \ \frac{1}{\pi} \ \frac{\Delta\nu_a d\nu}{(\nu-\nu_a)^2 + \Delta\nu_a^2} \qquad \text{--- (4)}$$

Where $P'$ is the number of modes in cross sectional area **a** and is given by – $P' = \frac{a\, 4\pi\nu^2}{V^2}$

The quantity $\beta D$ is the single pass phase shift and for a standing wave to build up in the cavity. i.e. to obtain resonance, $\beta D$ must be an integral multiple of $\pi$. The power reflection coefficient $r_2^2 = R_2$, and the power transmission coefficient $t_2^2 = T_2$ are related by- $R_2 + T_2 + A_2 + F_2 = 1$; where $A_2$ and $F_2$ are respectively the fraction of light absorbed and diffracted by the mirror.

With the assumption that $k = 0^*$, $\alpha D \langle\langle 1$, $r_{21} r_2 \approx 1$, we obtain from (3) by performing the integration over z,

$$\int \frac{dP_\nu}{dz} = \int_o^D \frac{A^2 t_2^2 e^{\alpha D} \{e^{-\alpha Z} + r_1^2 e^{\alpha Z}]}{\{1-r_1 r_2 e^{2D}\}^2 + 4 r_{21} r_2 e^{\alpha D} \sin^2 \beta D} \, dz$$

$$= \frac{A^2 t_2^2 e^{\alpha D} \{-e^{-\alpha D} + r_1^2 e^{\alpha D}\}}{\alpha[(1-r_1 r_2) - \alpha D]^2 + (2\Delta\beta D)^2} d\nu$$

$$P_\nu d\nu = \frac{A^2 t_2^2[2\alpha D]d\nu}{\alpha[(1-r_1 r_2)-\alpha D]^2 + (2\Delta\beta D)^2} \quad \text{--- (5)}$$

We have used thee relation that the phase shift $\phi = \beta D$, near resonance is $\Delta\beta D = \beta D - \beta_c D$; where $\beta_c D = q\pi$ and $\Delta\beta D \langle\langle 1$. Now the phase shift for a wave traveling once through a resonator length D and waveguide wavelength, $\lambda_g$ is

$$\phi = \frac{2\pi D}{\lambda_g} \approx \frac{2\pi \nu D n}{c} \quad \text{--- (6)}$$

We have taken $\lambda_g$ to be $c/\nu n$ and n is the refractive index of medium. Hence the cavity has resonance frequencies separated by $c/2Dn$. Small changes in phase measured from the phase at the cavity frequency are given by

$$\frac{\partial\phi}{\partial\nu}(\nu - \nu_c) + \Delta\phi = 0 \quad \text{--- (7)}$$

From (6) and (7) we have for the change in phase shift due to the dispersion of the resonator

$\Delta\phi_c = (\nu_c - \nu)\, 2\pi n_0 D/C$; Where $\nu_n = n_0$ = total population ($n_1 + n_2$)

To calculate the phase change due to the dispersion of the amplifying medium, we write for a Lorentzian line, $\alpha(\nu) = \alpha(\nu_a)[1 + (\nu_a - \nu)^2 / \Delta\nu_a^2]^{-1}$ and

$\Delta\phi(\nu) = (n_\nu - 1)2\pi\nu D/C = -\alpha(\nu)D(\nu_a - \nu)/2\Delta\nu_a$; Where $\alpha(\nu\nu \langle\langle 1, \nu + \nu_a \approx 2\nu$ and $\alpha(\nu_a)$ is the absorption coefficient at the atomic resonance. Now, from (5) and (4)

$$P_\nu d\nu = \frac{t_2^2}{1 - r_1 r_2} \frac{G N_2 h\nu_a}{Y} \frac{8\pi^2\mu^2\nu_a}{3hV\Delta\nu_a} \frac{1}{\pi} (\Delta\nu_c/GY)d\nu \Big/ (\Delta\nu_c/GY)^2 + (\nu - \nu_c)^2 \quad \text{--- (8)}$$

$$Y = [1 + (\Delta\nu_c/\Delta\nu_a)] \text{ and } G = [1 - \alpha(\nu_a)D/(1 - r_1 r_2)]^{-1} \quad \text{--- (9)}$$

In obtaining (8) we have neglected $Y = (\nu_a - \nu)^2 / \Delta\nu_a^2$ in comparison with 1 on the basis that $G \rangle\rangle 1$. The limiting case $G \to \infty$ will be recognized as the oscillation condition.

For half cavity width, at half amplitude $\Delta\nu_c$, is $\Delta\nu_c = c(1 - r_1 r_2)/4\pi n_0 D$

For half width $\Delta\nu_{oc}$, the power spectrum has a Lorentzian shape

$$\Delta\nu_{os} = \Delta\nu_c / G(1 + \Delta\nu_c/\Delta\nu_a \quad \text{--- (10)}$$

When $\Delta\nu_c \langle\langle \Delta\nu_a$, the equation (10) will be $\Delta\nu_{os} = \Delta\nu_c / G$

In equation (8) we see that the term $\frac{1}{\pi}(\Delta\nu_c/GY)d\nu \Big/ (\Delta\nu_c/GY)^2 + (\nu - \nu_c)^2$ is the atomic resonance has the Lorentz shape. So, this term equal to 1.

Since $S(\nu) = \frac{1}{\pi} \Delta\nu / (\nu_0 - \nu)^2 + \Delta\nu^2$ , $\therefore \int S(\nu)\, d\nu = 1$

Upon performing the integration in (8) and omitting the term $t_2^2 / (1 - r_1 r_2)$, then the power is given by

$$P\nu\, d\nu = [GN_2 h\nu_a / Y][8\pi^2\mu^2\nu_a / 3hV\Delta\nu_a]d\nu$$

$$P = \frac{N_2 h\nu G}{Y}\frac{8\pi^2\mu^2\nu_a}{3hV\Delta\nu_a}$$

$$P = [N_2 h\nu G \frac{8\pi^2\mu^2\nu_a}{3hV\Delta\nu_a}]/[1/(1+\frac{\Delta\nu_c}{\Delta\nu_a}]^{-1} \quad \text{--- (11)}$$

The term $\frac{8\pi^2\mu^2\nu_a}{3hV\Delta\nu_a}$ is the induced transition rate resulting from one photon per unit volume and when multiplied by $N_2 h\nu$ represents the emitted power for $N_2$ atoms in the excited state.

## RESULT AND DISCUSSION

**Semiclassical theory of laser and wave analysis:**
From the semiclassical theory of Laser (5) the expression for population difference necessary for inversion is given in density matrix formulize as,

$$\rho_{aa} - \rho_{bb} = N(z,t)/[1+\frac{R}{R_s}] \quad \text{--- (12)}$$

Where, R= rate constant $= \frac{1}{2}(\frac{\wp E_n}{h})^2 |U_n(Z)|^2 \gamma^{-1} L(\omega - \nu_n)$ --- (13)

And $R_s$ = Saturation parameter $= \frac{\gamma_a \gamma_b}{2\ \gamma_{ab}}$ --- (14)

Equation (12) in fact include the gain coefficient given in semi classical theory as

$$a_n = L(\omega - \nu_n)F_1 - \nu/2Q_n \quad \text{--- (15)}$$

$F_1 = \frac{1}{2}\ \nu\ \wp^2 (\varepsilon_0\ \hbar\ \gamma)^{-1} N$ and Lorentzian $L(\omega - \nu_n) = \frac{\gamma^2}{\gamma^2 + (\omega + \nu_n)^2}$ --- (16)

It is worthwhile to compare equation (9) representing gain and wave analysis with equation (12) and (13) representing population inversion as well as gain coefficient. Further we note that the $\alpha$ gain coefficient is written as

$$\alpha = \frac{\pi^2 r_0 c\lambda}{2}(\frac{g_b}{g_a})(\frac{m}{2\pi kT})^{\frac{1}{2}} N_a[1 - \frac{N_b}{N_a}\frac{g_a}{g_b}]\ f \quad \text{--- (17)}$$

$$\alpha \cong KN_a[1 - \frac{N_b}{Na}] \quad \text{--- (18)}$$

Where, K includes every term in the RH S excepting square bracket.

## CONCLUSION

An evaluation of gain coefficient as given by equation (9) shows that gain is dependent on reflectivity and absorption coefficient. The salient point of discussion of equation (9) is that gain increases as reflectivity increases as seen in the **Fig1**.

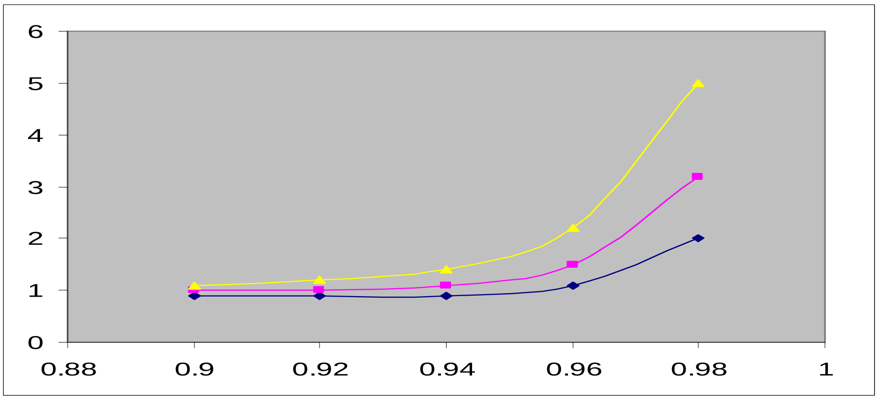


**Fig 1 Variation of reflectivity at a particular value of absorption co-efficient**
*(Y- axis) against gain co- efficient (X-axis)*

But this is applicable only when the reflectivity ($r = r_1 r_2$) is above 0.96, below this value significance gain is not observed. This is considered as general rule because we observed that to achieve laser action; we need dielectric mirror of extremely high reflectivity.

But a singular situation arises when $r_1 r_2 = 1$, this indicates the limitation of wave analysis. Also the oscillating condition is given by $G >> 1$. This is an ideal situation only. From Equations (12), (15) and (18) we may conclude that gain co-efficient includes the term for inversion whereas in the wave analysis the expression for population inversion is somehow missing.

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