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A comparative study of effect of complex conjugate terms in Fabry-Perot and Zeeman Laer cavity

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ABSTRACT

In this work, we have derived amplitude and frequency determining equations using complex conjugate terms of electric field and polarization in Maxwell's equation. It is observed that the derived basics equations are different from the original equations derived by Lamb and his coworkers. The derived additional terms have physical significance related to the lasing action in Fabry-Perot type and Zemann laser cavities.

Key words: Complex conjugate term, amplitude, frequency, Fabry-Perot, Zeeman laser cavity.

INTRODUCTION

The semiclassical theory [1] prescribed by Lamb and his coworkers [1-5] have successfully derived different parameters of laser cavities. Semiclassical theory has been applied to various types of cavities. The purpose of the present work is to present a comparison of amplitude and frequency determining equations using complex conjugate terms with the help of semiclassical theory of laser which was left out in original work of Lamb. Lamb and coworkers [6, 7] had worked out the theory of Zeeman Laser and explained about Electromagnetic field equations, polarization of the medium [8, 9], equation of motion, cavity anisotropy, transverse magnetic field, atomic decay rate, Lande's factor etc in laser.

MATERIALS AND METHODS

The active medium consists of thermally moving atoms of varying isotopic abundance which have two electric states with arbitrary angular momenta. The electromagnetic field is treated classically for a general state of polarization in a cavity with any desired degree of cavity anisotropy. The self-consistency requirement is that a quasi-stationary field should be sustained by the induced polarization lead to the equations which determine the amplitude and frequencies of multimode oscillations as functions of the laser parameters. The Maxwell's equations in mks unit as, neglecting vector properties

$$\frac{\partial^2 E(z,t)}{\partial t^2} + \mu_0 \sigma \frac{\partial E(z,t)}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 E(z,t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P(z,t)}{\partial t^2} \tag{1}$$

Here P, polarization is used to describe the induced atomic polarization of the active medium. It is desirable to provide for different cavity resonant frequencies for the linearly polarized radiation along orthogonal Cartesian axes transverse to the maser axis.

Inside the cavity only certain discrete modes achieve appreciable magnitude whose circular frequency is

$$\Omega_n = \frac{n\pi c}{L} = K_n c$$

Where L is the length of the cavity, c is the velocity of light, n is a large integer in our discussion we take normal modes to have sinusoidal z dependence. The electric field can be expressed as a sum of modes as:

$$U_n(z) = \exp(i K_n z) \quad , K_n = n\frac{\pi}{L}$$

The single polarization component of the electric field is written as

$$E(z,t) = \frac{1}{2}\sum_n \mathbf{E}_n(t)\exp\{-i(\upsilon_n t + \phi_n)\}U_n(z) + c.c \qquad (2)$$

$$P(z,t) = \frac{1}{2}\sum_n \mathbf{P}_n(t)\exp\{-i(\upsilon_n t + \phi_n)\}U_n(z) + c.c \qquad (3)$$

Here, the amplitude coefficient E_n and complex polarization component P_n very small in optical frequency range. The real part of polarization is in phase with the electric field leads to dispersion due to medium. The imaginary part is in quadrature with the electric field and results in gain or loss.

Now putting values of (2) and (3), neglecting complex conjugate terms, in equation (1),

$$\Rightarrow \Omega_n^2 \mathbf{E}_n - i\frac{\sigma}{\varepsilon_0}\upsilon_n \mathbf{E}_n - 2i\upsilon_n \dot{\mathbf{E}}_n - (\upsilon_n + \dot{\phi}_n)^2 \mathbf{E}_n = \upsilon_n^2 \varepsilon_0^{-1}\mathbf{P}_n \qquad (4)$$

Here, we neglect the slowly varying terms with frequency. Adjusting the fictional conductivity σ to create the desired value of Q of the mode

$$\sigma = \varepsilon_0 \frac{\upsilon_n}{Q_n}$$

From equation (4), by comparing real and imaginary part, we get

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = -\frac{1}{2}\upsilon_n \varepsilon_0^{-1} \text{ Im part of } \mathbf{P}_n \qquad (5)$$

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\upsilon_n \varepsilon_0^{-1}\mathbf{E}_n^{-1} \text{ Re part of } \mathbf{P}_n \qquad (6)$$

Which are basic equations of the semiclassical theory of laser.

Using the same procedure for the complex conjugate terms, we get the gain and dispersion relations as

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = \frac{1}{2}\upsilon_n \varepsilon_0^{-1} \text{ Im part of } \mathbf{P}_n \qquad (7)$$

$$\{2\upsilon_n - \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\} = \Omega_n + \frac{1}{2}\upsilon_n \varepsilon_0^{-1}\mathbf{E}_n^{-1} \text{ Re part of } \mathbf{P}_n \qquad (8)$$

In case of Zeeman laser, the vectorial electric field with two transverse degrees of freedom is particularly convenient choice of representation with circularly polarized components.

$$\vec{E}(z,t) = \frac{1}{2}\{\hat{\varepsilon}_+ E_+(t)\exp[-i(\upsilon_+ t + \phi_+) + \hat{\varepsilon}_- E_-(t)\exp[-i(\upsilon_- t + \phi_-)]\}U(z) + cc \quad (9)$$

Where, the complex circularly polarized unit vectors $\hat{\varepsilon}_\pm = 2^{-\frac{1}{2}}(\hat{x} \mp i\,\hat{y})$, and amplitudes E_+ and E_- and phases ϕ_+, ϕ_- are slowly varying functions of time. The induced polarization of the medium corresponding to field has the form

$$\vec{P}(z,t) = \frac{1}{2}\{\hat{\varepsilon}_+ P_+(t)\exp[-i(\upsilon_+ t + \phi_+) + \hat{\varepsilon}_- P_-(t)\exp[-i(\upsilon_- t + \phi_-)]\}U(z) + cc \quad (10)$$

Where, the complex Fourier components of polarization P_+, P_- are slowly varying functions of time. Putting the values from (9) & (10) in (1), ignoring complex conjugate term and also neglecting slowly varying terms with frequency, $\dot{\phi}_+, \sigma\dot{\phi}_+, \sigma\dot{E}_+$ and $\dot{E}_+$, we get

$$(\upsilon_+ + \dot{\phi}_+ - \Omega)E_+ + i\{\dot{E} + \frac{1}{2}\upsilon[g_{++}E_+ + g_{--}E_- \exp(i\,\psi)]\} = \frac{1}{2}\frac{\upsilon}{\varepsilon_0}P_+ \quad (11)$$

Where, the relative phase angle is

$$\psi = \upsilon_+ t + \dot{\phi}_+ - \upsilon_- t + \dot{\phi}_-$$

And the conductivity matrix

$$G = \begin{pmatrix} g_{++} & g_{+-} \\ g_{-+} & g_{--} \end{pmatrix} = (\varepsilon_0 \upsilon)^{-1}\sigma$$

The frequency $\upsilon \cong \upsilon_+ \cong \upsilon_-$ and g $g_{++} = Q^{-1}{}_+, g_{--} = Q^{-1}{}_-$

Equating the real and imaginary part of equation (11) separately to zero, the self consistency equations

$$E_+ + \frac{1}{2}\upsilon(g_{++}E_+) + \mathrm{Im}[ig_{+-}E_- \exp(i\psi)] = \frac{1}{2}\frac{\upsilon}{\varepsilon_0}\mathrm{Im}(P_+) \quad (12)$$

$$(\upsilon_+ + \phi_+ - \Omega)E_+ + \frac{1}{2}\upsilon\mathrm{Re}[ig_{+-}E_- + \exp(i\psi)] = \frac{1}{2}\frac{\upsilon}{\varepsilon_0}\mathrm{Re}(P_+) \quad (13)$$

Using the complex conjugate terms and following the same procedure we get

$$(2\upsilon_+ - \frac{\Omega_+}{\upsilon_+})\dot{\phi}_+ - \Omega_+]E_+ + i\{\frac{1}{2}\upsilon[g_{++}E_+ + g_{+-}E_- \exp(i\psi)]\} = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0}(P_+) \quad (14)$$

Equating the real and imaginary parts these equations to zero; we obtain the self-consistency equations

$$\dot{E}_+ + \frac{1}{2}\upsilon[g_{++}E_{++} + \mathrm{Im}\{ig_{+-}E_- + \exp(i\,\psi)] = \frac{1}{2}\frac{\upsilon}{\varepsilon_0}\mathrm{Im}(P_+) \quad (15)$$

$$(2\upsilon_+ - \frac{\Omega_+}{\upsilon_+})\dot{\phi}_+ - \Omega_+]E_+ + \frac{1}{2}\upsilon\mathrm{Re}[ig_{+-}E_- \exp(i\psi)] = \frac{1}{2}\frac{\upsilon}{\varepsilon_0}\mathrm{Re}(P_+) \quad (16)$$

For diagonal losses, these become

$$\dot{E}_+ + \frac{1}{2}\upsilon/Q_+ E_+ = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0}\text{Im}(P_+) \qquad (17)$$

and $$(2\upsilon_+ - \frac{\Omega_+}{\upsilon_+})\dot{\phi}_+ - \Omega_+] = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0}E^{-1}{}_+\text{Re}(P_+) \qquad (18)$$

RESULTS AND DISCUSSION

For Fabry-Perot type cavity, in absence of active medium, $P_n = 0$, the amplitude and frequency determining equations with real terms becomes

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = 0 \quad \text{and} \quad \upsilon_n + \dot{\phi}_n = \Omega_n$$

The equations (5) and (7) are same except the negative sign. But equations (6) and (8) are different which represents dispersion of the medium. In absence of active medium, the equations using complex conjugate terms become

$$\dot{\mathbf{E}}_n = \frac{\upsilon}{2Q_n}\mathbf{E}_n \quad \text{and} \quad \{2\upsilon_n - \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\} = \Omega_n$$

From equation (8), we get two relations, one part is

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t)\ \text{Re}\ \mathbf{P}_n(t)$$

This equation is same as the real part of the original basic equation of laser derived by Lamb. The another part of the equation is

$$[\upsilon_n - \{\dot{\phi}_n + \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\}] = \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t)\ \text{Re}\ \mathbf{P}_n(t) + \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t)\ \text{Re}\ \mathbf{P}_n(t)$$

or, $$\upsilon_n - \dot{\phi}_n = \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n + \frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t)\ \text{Re}\ \mathbf{P}_n(t)$$

Which represents dispersion but with different form.

Physical significance:
The complex polarization for complex susceptibility

$$\mathbf{P}_n(t) = \varepsilon_0\chi_n = \varepsilon_0(\chi'_n + i\chi''_n)\ \mathbf{E}_n(t)$$

As

$$\dot{\mathbf{E}}_n(t) = -\frac{\upsilon}{2Q_n}\mathbf{E}_n(t) - \frac{1}{2}\upsilon\chi''_n\mathbf{E}_n(t) \quad \text{and} \quad \upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\upsilon\chi'_n$$

Or, $$\upsilon_n + \dot{\phi}_n = \frac{\Omega_n}{\eta(\upsilon_n)}$$

Thus the frequency determining equations with real and that of complex conjugate term show a distinct difference between gain and classical problem.

In case of Zeeman laser cavity, for diagonal losses, the self-consistency equations (12) & (13) reduces to

$$\dot{E}_+ + \frac{1}{2}\upsilon/Q_+ E_+ = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0}\mathrm{Im}(P_+) \text{ and } (\upsilon_+ + \phi_+ - \Omega) = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0} E^{-1}{}_{+} \mathrm{R}e(P_+)$$

These equations are the same as equations of semiclassical theory of laser of which represents the oscillation conditions of laser and the dispersion of the medium.

In absence of active medium $P_+ = 0$, we get

$$\dot{E}_+ = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0} E_+ \text{ and } \quad (2\upsilon_+ - \frac{\Omega_+}{\upsilon_+})\dot{\phi}_+ = \Omega_+$$

Which are also different as derived for Fabry-Perot type cavity.

CONCLUSION

For Zeeman laser, the equation (13) can be divided as

$$(\upsilon_+ + \dot{\phi}_+) = \Omega_+ - \frac{1}{2}\frac{\upsilon}{\varepsilon_0} E^{-1}{}_{+} \mathrm{R}e(P_+)$$

This represents dispersion as that derived for Fabry-Perot type cavity. And the other part is

$$[\upsilon_+ - \dot{\phi}_+] = \frac{\Omega_+}{\upsilon_+}\dot{\phi}_+$$

This equation represents dispersion in different form. It is reasonable to believe that the additional term will be contributed in the determination of atomic decay rates and g values for the laser medium. These equations are similar as the equations in absence of active medium in the former calculations with real or complex conjugate term only. Thus it is observed that the complex conjugate term has overall affect on the gain and dispersion relations.

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